Emergent Gravity Requires Kinematic Nonlocality

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This Letter refines arguments forbidding nonlinear dynamical gravity from appearing in the low energy effective description of field theories with local kinematics, even for those with instantaneous long-range interactions. Specifically, we note that gravitational theories with universal coupling to energy—an intrinsically nonlinear phenomenon—are characterized by Hamiltonians that are pure boundary terms on shell. In order for this to be the low energy effective description of a field theory with local kinematics, all bulk dynamics must be frozen and, thus, irrelevant to the construction. The result applies to theories defined either on a lattice or in the continuum, and requires neither Lorentz invariance nor translation invariance.

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Introduction.—Attempts to directly quantize the gravitational field encounter well-known difficulties associated with a lack of perturbative renormalizability, the black hole information problem, and the lack of local observables due to invariance under diffeomorphism gauge symmetry (which, from the active point of view, moves spacetime points from one location to another). While it remains possible that any or all of these issues may one day be surmounted, it is, nevertheless, interesting to ask whether diffeomorphism-invariant gravity—sometimes called background-independent gravity—might emerge an effective approximate description of a system that is inherently better behaved at the microscopic level.

Most leading approaches to quantum gravity embody ideas along these lines. String theory, loop quantum gravity, causal sets [1], and causal dynamical triangulations [2] (see, e.g., [3] for a recent overview) all propose that smooth classical geometries arise only in appropriate semiclassical limits. (Asymptotic safety is the most prominent exception; see, e.g., [4,5] for recent reviews.) But the structures underlying these theories again involve novel physics that is difficult to control. So, it is natural to ask if gravity can arise from more familiar systems such as field theories with local kinematics. Examples of such proposals include [6–22]. Below, we argue that such scenarios can succeed only if the map to gravitational degrees of freedom involves long-range nonlocality, i.e., only if the notions of locality are very different in the two descriptions. We emphasize that we focus here on whether gravitational theories with an appropriate form of diffeomorphism-invariance (or what is often called independence from background structures) can emerge as effective descriptions of theories built on familiar background structures such as fixed (nondynamical) spacetime lattices or smooth spacetimes with a metric; we make no comment on the possible emergence of general relativity from discrete theories of quantum gravity or on scenarios, such as in [23,24], where the entire notion of time evolution is, itself, emergent.

As has been well known for some time, the (spin-2) Weinberg-Witten theorem [25] already excludes the emergence of gravity from local Poincaré-invariant field theories. In particular, it forbids such theories from containing an interacting massless spin-2 degrees of freedom in its spectrum of asymptotic states. While clear and concise, the technical assumption of Poincaré invariance appears to leave open many doors for exploration. For example, one might attempt to evade the theorem by working on a lattice as in, e.g., [9,15,16,19,20], or by using other structures that break this symmetry.

However, as noted in, e.g., [26,27], the lack of local observables in quantum gravity suggests a more general result forbidding diffeomorphism-invariant gravity arising as the effective description of any theory with sufficiently interesting local observables. Our purpose, here, is to make this precise. Since any theory can be made diffeomorphism invariant via a process known as parametrization (see, e.g., [28,29,29–37]), we follow [38] in using the gravitational Gauss law to distinguish theories with sufficiently "interesting" diffeomorphism invariance. The desired theories roughly correspond to what are often called "background-independent" theories of gravity.

Before stating our technical result in the next section, let us, therefore, take a moment to explain this idea in broadly accessible terms. First, we recall that (nonrelativistic) Newtonian gravity can be formulated in terms of a gravitational potential ϕ that satisfies a Poisson equation $\nabla^2\phi = 4\pi G\rho_M$ sourced by the mass density ρ_M. As a result, the total mass $M = \int_V \rho_M dV$ inside a volume V (with volume element dV) can be expressed as the boundary term $M = (1/4\pi G)\int_{\partial V} dSn^i\partial_i\phi$ where $\partial_i = (\partial/\partial x^i)$ denotes derivatives with respect to spatial coordinates x^i, n^i is the unit (outward pointing) normal to the boundary ∂V, and dS is the area element on ∂V. This is just the Newtonian gravity analogue of Gauss's law from electrostatics. Now, in relativistic theories, the gravitational field couples not just to mass, but to all forms of energy. As a result, in the presence of appropriate boundary conditions, one finds a corresponding Gauss-law-like boundary integral that encodes the total energy E; see, e.g., [39] for a recent review. This Gauss law for energy will turn out to be the critical feature that forbids the theory from arising as an effective description; the full Lorentz-invariance that originally motivated the coupling to energy is not required. (Lorentz-violating theories that couple universally to energy may be constructed in analogy with Hořava-Lifhshitz gravity [40], interchanging the roles of space and time and replacing the extrinsic curvature of a preferred foliation with the proper acceleration of a preferred family of worldlines. The Einstein-Aether theory [41,42] also has universal coupling and, at low energies, might be considered to violate Lorentz invariance.) It is useful to mention here that the Gauss-law property is inherently nonlinear due to the fact that the energy source term also receives contributions from the gravitational field. As a result, in parallel with the Weinberg-Witten theorem [25], our arguments place no constraints on the emergence of strictly linear spin-2 degrees of freedom.

As we explain below, the Gauss-law property will imply that gravity can be a good effective description of a theory with local kinematics only in limits where the bulk dynamics freezes out away from the boundary. (The dynamics is required to be local in time and generated by a Hamiltonian. However, the Hamiltonian can be nonlocal in space. We explicitly allow instantaneous long-range interactions.) While there is nothing wrong with such freeze out in and of itself, an interesting effective gravitational description should remain nontrivial in the bulk of the spacetime. (There are, of course, many theories where bulk excitations are gapped at low energy.) Consistency then requires that bulk gravitational physics be the effective description of purely boundary dynamics in the original theory. Modulo anomalies, the original bulk theory served no purpose in the construction and may be discarded. As a result, the notions of bulk vs boundary are completely different in the original kinematically local theory and the effective gravitational description. This is the requisite nonlocality referred to in the title. The reader will note that it also describes a paradigm embodied in string theory by gauge-gravity duality (e.g., [43,44]).

Definitions and Results.—We begin the technical treatment with two definitions that will allow us to sharply state our result. Each definition is followed by comments to provide clarity. Discussion of the main result will appear in the Discussion section.

Definition 1.—A gravitational theory with universal coupling to energy is one for which, in the presence of any boundary conditions for which a Hamiltonian exists, the total energy can be written as the integral over the boundary of space at each time of some local function of the gravitational field and its derivatives. Below, we refer to the integrand of this boundary integral as the gravitational flux. We require the gravitational flux to be an observable (i.e., it is invariant under gauge transformations allowed by the given boundary conditions).

We now make several remarks to clarify this definition. See, e.g., [39] for any definitions and for further discussion of the examples below. We will use the term Riemann-curvature gravity theories to refer to Einstein-Hilbert gravity together with its higher-derivative generalizations described by Lagrangians that are local scalar functions of the Riemann tensor and its derivatives.

Gravitational field: Note that we have not specified any particular variables in terms of which this field is to be expressed; our discussion is, thus, invariant under local field redefinitions and is not restricted to metric theories.

Simultaneity: The phrasing implies that the spacetime boundary admits a notion of "each time," i.e., of which points on the boundary are simultaneous. This notion need not be unique; e.g., for Riemann-curvature gravity with either anti–de Sitter (AdS) boundary conditions or Dirichlet boundary conditions at a finite wall, any time function on the boundary may be used to define simultaneity so long as all pairs of points on its level surfaces are spacelike separated. The notion of simultaneity is also allowed to be trivial as in asymptotically flat Riemann-curvature gravity where the boundary of space should be interpreted as spacelike infinity (i^0) and, in the usual representation, all points at spacelike infinity are simultaneous. Indeed, the entire notion of "boundary of space" can be trivial so long as the total energy vanishes identically in such cases; Riemann-curvature gravity for closed cosmologies provides an example.

Total energy: This quantity is defined to be the generator of (asymptotic) time evolution; i.e., it is the Hamiltonian. This time evolution need not be a symmetry, so the Hamiltonian may have explicit time dependence.

Observable gravitational flux: We remind the reader that the gravitational flux at the boundary is, indeed, gauge invariant in Riemann-curvature gravity since it can be defined as the variation of the action (see, e.g., [45–49]) with respect to boundary conditions (which are by definition gauge invariant). It may also be useful to mention that, while often not presented in this form, in Einstein-Hilbert gravity with asymptotically flat [50] or asymptotically AdS boundary conditions [51,52], the gravitational flux may be written in terms of the Weyl tensor at the boundary. In this form, it more closely resembles the familiar electric flux computed from the field strength of a vector gauge field.

Heuristics, examples, and contrasting theories: The idea behind calling the above property "universal coupling to energy" is that there is an aspect of the gravitational field (namely, the above boundary integral) which directly gives the total energy of the system. Any Riemann-curvature theory satisfies this definition (see, e.g., [53]). In contrast, scalar (Nordström) gravity, Hořava-Lifshitz gravity [40], and massive gravity theories (see, e.g., [54] for a recent review) do not have universal coupling to energy in this sense; none of these theories have a Gauss law. (An alternative definition of Hořava-Lifshitz gravity can be given by imposing a hypersurface-orthogonality constraint on the aether field of the Einstein-aether theory [55]. This formulation inherits the Gauss law equation of motion of the Einstein-aether theory. But it is now second class in the sense of Dirac [56]. This requires the commutators to be modified as in [56] so that the resulting theory is no longer kinematically local in the sense of Definition 2. This illustrates that Definitions 1 and 2 are most meaningful when considered together.) We will not discuss such theories further except to note that their behavior is, generally, rather distinct [57–59] from theories with universal coupling and that Nordström gravity does appear to emerge from the dynamics of Bose-Einstein condensates [60].

Definition 2.—Whether defined in the continuum or on a spatial or space-time lattice, a theory will be said to be kinematically local if and only if the commutator of two gauge-invariant local Heisenberg-picture operators (with at least one bosonic) vanishes when evaluated at different spatial locations at a common time. We also assume that time evolution is generated by some Hamiltonian. We again provide the following clarifying comments.

Simultaneity: We require the theory to have a concept of bulk simultaneity (i.e., when two spacetime events occur at the same time). We assume this to be a background structure independent of dynamical fields. As above, this notion need not be unique; i.e., in a relativistic theory, it will suffice to choose any time function that is constant on spacelike surfaces. (By which we mean that there is no causal connection between any two points on the surface.) So, any theory built in the usual local way from scalar, spinor, or vector fields is local in this sense.

Heisenberg picture: We assume the existence of a Heisenberg picture, in which gauge-invariant operators at each position $\vec{x}$ satisfy $-i\hbar(\partial/\partial t)\mathcal{O}(\vec{x},t) = [H(t), \mathcal{O}(\vec{x},t)]$ for some (perhaps time-dependent) Hamiltonian $H(t)$. In this sense, the dynamics is local in time, though $H(t)$ may be arbitrarily nonlocal in space. In particular, instantaneous long-range interactions are allowed. So long as the commutation relations satisfy Definition 2 at some initial time, any unitary evolution ensures that they continue to hold at all other times.

Bosonic operators: It is sufficient, for our purposes, to define gauge-invariant operators to be bosonic when they commute with all local gauge-invariant operators located at different positions in space at the same time. While we referred to local operators above, in a lattice theory, it is natural to also consider bosonic operators B built from multiple nearby lattice sites; e.g., the product of two free fermions at adjacent sites may be considered a local bosonic operator. In that case, we require it to commute with all operators whose support does not contain the lattice sites from which our operator B was built.

Combining the above definitions leads quickly to the desired result. We begin by assuming the theory with local kinematics to admit some limit where it is effectively described by a gravitational theory with universal coupling to energy. We take this to mean that notions of time evolution embodied by the above two definitions coincide. The time evolution of the local theory is then generated by a Hamiltonian which, by Definition 1, can be written as an integral over the boundary gravitational flux.

There is, in principle, some change of variables that writes this boundary integral in terms of variables in the original local theory. Since the gravitational flux is a (gauge-invariant) bosonic observable at the boundary of the gravity theory, we assume that the result in the kinematically local theory is again the integral of a bosonic gauge-invariant operator supported only on (or near) the boundary. Failure of this property to hold would mean that the two theories have radically different notions of bulk vs boundary; we, therefore, refer to the above property as the assumption that the two theories have compatible notions of locality. But having expressed the Hamiltonian in terms of boundary operators in the local theory, it must commute with all local observables in the interior. So, interior local observables must be time independent in the limit where the effective gravitational description applies; i.e., the local interior dynamics has become frozen. We restate this conclusion as the following theorem.

Theorem I.—Consider any limit where the effective description of a local theory is a gravitational theory with universal coupling to energy, the same notion of time evolution, and a compatible definition of locality. In this limit, all local observables away from the boundary become independent of time.

Discussion.—We have seen that all bulk dynamics must freeze out in any limit where a kinematically local theory develops an effective gravitational description (and where this gravitational field couples universally to energy, maintains the same notion of time evolution, and contains a compatible definition of locality). We emphasize that only the kinematics need be local for this conclusion to hold. While our definition of kinematic locality requires Hamiltonian evolution, the Hamiltonian may contain instantaneous long-range interactions. The form of the commutation relations is preserved by any unitary notion of time evolution.

As remarked in the Introduction, there is no inherent contradiction in this freeze out on its own. After all, gapped theories are quite common. But an interesting effective gravitational description should remain nontrivial in the bulk of the spacetime, which then requires that its notion of the bulk-boundary distinction be rather different than that of the original kinematically local theory. This constitutes a certain nonlocality intrinsic to the process of emergence—beyond any nonlocality already present in the original dynamics—and is similar to what occurs in string theoretic gauge-gravity duality (e.g., [43,44]). Indeed, modulo anomalies, we may imagine discarding the original bulk and obtaining the gravity theory directly from degrees of freedom at the boundary.

Since partial motivation for this Letter came from the (gravitational) Weinberg-Witten theorem [25], one may recall that Weinberg-Witten has a useful analogue for U(1) vector fields. The corresponding analogue of our result is far less interesting. It states, simply, that all local operators remaining in the limit where the effective U(1) vector description applies must be uncharged.

Returning to the gravitational context, it is clear that the consequences of our theorem can be avoided by introducing *a priori* kinematic nonlocalities violating our assumptions. The gauge-gravity dualities of string theory are examples of this strategy. Indeed, any (Hamiltonian) quantum theory of gravity defined on a separable Hilbert space is completely equivalent to some local field theory—and, in fact, to a quantum mechanical theory describing a single particle in one dimension—via a sufficiently nonlocal map. One simply uses the fact that all separable Hilbert spaces are isomorphic to transcribe the Hamiltonian to the Hilbert space of a single nonrelativistic particle. As a $0+1$-dimensional field theory, the result trivially satisfies Definition 2. The dynamics are also local in time, though when written (perhaps only formally) in terms of the usual position and momentum operators, the Hamiltonian need not bear any resemblance to standard energy functions of Newtonian mechanics.

Of course, the above procedure requires one to first know the exact spectrum of the gravitational Hamiltonian. This is tantamount to solving the theory; and any construction which first requires the theory to be solved will be of very limited use. Allowing the map between theories to be arbitrary nonlocality, thus, seems unproductive. Again, stringy gauge-gravity duality represents a sort of happy medium with enough nonlocality to evade our theorem and enough structure to remain useful.

One might also ask if gravity could be the effective description (via a more local change of variables) of a theory with some special type of kinematic nonlocality over which one might hope to have more control. Noncommutative gauge theories [61] are a natural first category to consider. Since these theories lack local observables, there is no immediate direct transcription to this context of our theorem above; but closely related reasoning indicates failure here as well. In particular, recall that noncommutative gauge theories can be defined on compact spaces with translational symmetry (e.g., tori) where they continue to admit gauge-invariant observables with nonzero momentum [62]. Recall, also, that, like energy, momentum is a source for (other components of) the gravitational field and admits a similar Gauss-law expression as a boundary integral in standard gravitational theories. This motivates a definition of "universal coupling to momentum" in analogy with our Definition 1 above. In theories with this property, the total momentum must vanish on spatially compact manifolds, and operators with nonzero momentum cannot be gauge invariant. However, restricting the noncommutative theory to zero-momentum operators is comparable to freezing out bulk degrees of freedom in a local field theory [63], so this approach seems similarly unproductive.

In closing, we remark that the momentum version of the argument in the above paragraph also constrains the emergence of Hořava-Lifshitz gravity [40], which couples universally to momentum but not to energy (though, see parenthetic note in the text). Again, this universal coupling is an intrinsically nonlinear phenomenon. Thus, the linearized theory is free to appear in an effective description as found in [15,16,19,20].

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