Coupling Hydrodynamics to Nonequilibrium Degrees of Freedom in Strongly Interacting Quark-Gluon Plasma

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(Received 6 October 2014; revised manuscript received 3 December 2014; published 24 December 2014)

Relativistic hydrodynamics simulations of quark-gluon plasma play a pivotal role in our understanding of heavy ion collisions at RHIC and LHC. They are based on a phenomenological description due to Müller, Israel, Stewart (MIS) and others, which incorporates viscous effects and ensures a well-posed initial value problem. Focusing on the case of conformal plasma we propose a generalization which includes, in addition, the dynamics of the least damped far-from-equilibrium degree of freedom found in strongly coupled plasmas through the AdS/CFT correspondence. We formulate new evolution equations for general flows and then test them in the case of $\mathcal{N}=4$ super Yang-Mills plasma by comparing their solutions alongside solutions of MIS theory with numerical computations of isotropization and boost-invariant flow based on holography. In these tests the new equations reproduce the results of MIS theory when initialized close to the hydrodynamic stage of evolution, but give a more accurate description of the dynamics when initial conditions are set in the preequilibrium regime.

DOI: 10.1103/PhysRevLett.113.261601 PACS numbers: 11.25.Tq, 12.38.Mh

Introduction.—The successful phenomenological description of soft observables in heavy ion collisions at RHIC and LHC asserts that the quark-gluon plasma phase is formed and in less than one Fermi after the collision subsequent evolution till hadronization, is governed by hydrodynamic expansion with a very small shear viscosity [1,2]. Finding an explanation for the emergence of this collective behavior under experimentally viable conditions based on the microscopic theory, QCD, poses a timely theoretical challenge. In consequence, much attention has recently been devoted to the studies of equilibration processes of non-Abelian gauge fields in a few known tractable situations, such as at strong coupling using holography and a dual gravitational description.

Within this approach it has been shown that viscous hydrodynamics can work remarkably well already after a time of order of the inverse of the local effective temperature despite significant pressure anisotropy in the local rest frame [3–6]. [The hydrodynamization in $1/T$ is phenomenologically attractive, as ballpark quantities characterizing initialization of hydrodynamics codes, $\tau = 0.5$ Fermi and $T = 500$ MeV, obey $\tau = \mathcal{O}(1)/T$.] This finding suggests that the applicability of hydrodynamics is not limited by the size of gradient corrections to the perfect fluid stress tensor, but rather by the presence of degrees of freedom not described by hydrodynamics. This implies that any phenomenological attempts to capture features of preequilibrium dynamics in heavy ion collisions need to incorporate effects of these degrees of freedom.

The holographic AdS/CFT description of $\mathcal{N}=4$ supersymmetric Yang-Mills (SYM) theory provides a direct handle on both hydrodynamic and nonhydrodynamic degrees of freedom in strongly coupled plasma. Understanding the dynamics of these modes generically requires solving numerically five-dimensional Einstein equations, which is a formidable endeavor. The goal of this Letter is to extract the dynamics of the least damped nonhydrodynamic modes from AdS/CFT and to incorporate them in a *four-dimensional* description in which they are coupled to conventional hydrodynamic quantities: local temperature T and flow velocity u^μ. Such a four-dimensional description should be of definite practical utility. Moreover, its novel structural form should have quite general applicability.

The precise distinction between hydrodynamic and nonhydrodynamic modes is hard to make in general, but in the hydrodynamic phase and its vicinity equilibrium concepts are expected to approximately apply. A natural definition of excitations of the equilibrium plasma comes from linear response theory and is expressed in terms of singularities of the retarded stress tensor correlator in the complex frequency plane.

In the case of strongly coupled holographic plasma, the singularities are single poles leading to nonequilibrium excitations characterized via complex dispersion relations $\omega(k)$. In a dual gravitational picture, $\omega(k)$ are the quasinormal mode (QNM) frequencies of the black brane representing equilibrium plasma [7,8]. Nonhydrodynamic modes are those which are exponentially damped for any value of momentum and, if excited, typically become physically negligible after time of order of $1/\Im(\omega)$. One also finds that QNMs have $\Re(\omega) \neq 0$ and $\Im(\omega) = \mathcal{O}(T)$, in line with the typical hydrodynamization scale being $\mathcal{O}(1/T)$.

In this Letter, we focus on the dynamics of the mode with the smallest nonvanishing $\Im(\omega)$, as it generically governs the direct approach to the hydrodynamic phase. Incorporating it in a phenomenological description including hydrodynamic modes should improve the range of applicability of such a description. We explicitly address the case of $\mathcal{N} = 4$ SYM theory, but our considerations should carry over verbatim to any conformal theory (or QCD in the conformal approximation) with appropriate values of $\Re(\omega)$ and $\Im(\omega)$ for the least damped mode.

Evolution equations for quasinormal modes.—In strongly coupled field theories, expectation values of local operators, e.g., $O = \mathrm{tr}F^2$ or $T^{\mu\nu}$, typically decay exponentially when the system is perturbed out of global thermal equilibrium, with the exception of hydrodynamic modes. The characteristic frequencies governing this behavior can be computed as poles of the retarded Green's function and depend on momentum. At sufficiently late times, only the lowest mode gives a physically relevant contribution, e.g.,

$$\langle O \rangle = \int d^3k A(k) e^{-\omega_I T t} \cos\left(\omega_R T t + \vec{k}\cdot\vec{x} + \phi(k)\right), \qquad (1)$$

where A and ϕ are some functions and we have defined

$$\omega/T = \omega_R(k/T) + i\omega_I(k/T). \qquad (2)$$

In the case of holographic plasma, $\omega_{R/I}$ are given by the quasinormal frequencies of the black brane appearing in the dual gravitational description. Their momentum dependence has been computed numerically [8] and it is apparent that both for O and $T^{\mu\nu}$ they exhibit very weak dependence on k up to $k \approx 2\pi T$. As far as we know, this important feature has not been emphasized so far. This suggests neglecting this dependence entirely as a first approximation, which we do in the rest of the text. Under this assumption, which we will refer to as ultralocality, the expectation value $\langle O \rangle$ given above satisfies the following second order differential equation:

$$\left(\frac{1}{T}\frac{\partial}{\partial t}\right)^2 \langle O \rangle + 2\omega_I \frac{1}{T}\frac{\partial}{\partial t}\langle O \rangle + |\omega|^2 \langle O \rangle = 0, \qquad (3)$$

where $|\omega|^2 \equiv \omega_I^2 + \omega_R^2$. Equation (3) is formally the equation of motion of a damped harmonic oscillator. For $\mathcal{N} = 4$ SYM (here and in the following this will always mean $\mathcal{N} = 4$ SYM theory at large N_c and strong 't Hooft coupling) and $O = \mathrm{tr}F^2$ the frequencies (the QNM frequencies at zero momentum) are [8]

$$\omega_R \approx 9.800 \quad \text{and} \quad \omega_I \approx 8.629. \qquad (4)$$

The focus of interest here is the analog of Eq. (3) for the expectation value of the energy momentum tensor, which would be a step toward writing phenomenological equations describing the interactions of the lowest stress tensor QNM with the hydrodynamic degrees of freedom. To this end, note that for sufficiently near-equilibrium situations (but not limited to hydrodynamics) the stress tensor can be decomposed in the following way:

$$\langle T^{\mu\nu} \rangle = \mathcal{E} u^\mu u^\nu + \mathcal{P}(\mathcal{E})(\eta^{\mu\nu} + u^\mu u^\nu) + \Pi^{\mu\nu}, \qquad (5)$$

where $u_\nu u^\nu = -1$ and the symmetric tensor $\Pi^{\mu\nu}$ obeys the Landau frame condition $u_\mu \Pi^{\mu\nu} = 0$. For conformal field theories considered here one also has $\mathcal{P}(\mathcal{E}) = 1/3\mathcal{E}$, $\Pi^\mu_\mu = 0$. Furthermore, one defines also the "effective temperature" T in any state as the temperature of an equilibrium state with the same energy density.

In equilibrium $\Pi^{\mu\nu} = 0$ and the system can always be described in the global rest frame, i.e., $u^\mu = 0$ for $\mu \neq t$ and $u^t = 1$. Perturbations near equilibrium are thus δT, δu^μ with $\delta u^t = 0$ and $\delta\Pi^{\mu\nu}$ with $\delta\Pi^{t\mu} = 0$. Note that the conservation equation of the stress tensor

$$\partial_\mu \langle T^{\mu\nu} \rangle = 0 \qquad (6)$$

always allows one to solve for the four variables given by δT and δu^μ.

At nonzero momentum, different combinations of components of $\delta\Pi^{\mu\nu}$ (different channels) oscillate with different frequencies. However, ultralocality implies that for momenta smaller than $k \approx 2\pi T$ this effect is negligible and the oscillation frequencies in all channels are approximately the same and coincide with the frequencies in Eq. (4). Because of this, each component of $\delta\Pi^{\mu\nu}$ satisfies the same equation as Eq. (3),

$$\left(\frac{1}{T}\frac{\partial}{\partial t}\right)^2 \delta\Pi^{\mu\nu} + 2\omega_I \frac{1}{T}\frac{\partial}{\partial t}\delta\Pi^{\mu\nu} + |\omega|^2 \delta\Pi^{\mu\nu} = 0. \qquad (7)$$

Equation (7) together with Eq. (6) describe the evolution of the lowest nonhydrodynamic degree of freedom for small deviations from global thermal equilibrium.

Quasinormal modes in a hydrodynamic background.—In generic situations one expects that the lowest nonhydrodynamic degree of freedom interacts with hydrodynamic modes and properly accounting for these interactions turns out to require nontrivial modifications of Eq. (7). Part of these modifications can be motivated by generalizing Eq. (3) to describe late time equilibration of $\langle O \rangle$ on top of the plasma described by hydrodynamics, i.e., with

$$\Pi^{\mu\nu} = \Pi^{\mu\nu}_{\mathrm{hydro}} = -\eta(T)\sigma^{\mu\nu} + \ldots, \qquad (8)$$

where $\eta(T)$ is the shear viscosity, $\sigma^{\mu\nu}$ is the shear tensor, and the ellipsis denotes terms containing two and more derivatives of the hydrodynamic fields.

The naive covariantization of Eq. (3) by taking $\partial_t \to u^\mu \partial_\mu$ and using T and u^μ solving Eq. (6) with $\Pi^{\mu\nu}$ in hydrodynamic form, does not preserve the Weyl covariance of the microscopic theory. (We neglect here the effects of the Weyl anomaly, as in Refs. [9,10].) The latter is the statement that, under Weyl rescaling of the background metric,

$$\eta_{\mu\nu} \to e^{-2\omega(x)}\eta_{\mu\nu} \tag{9}$$

both O and $T^{\mu\nu}$ transform homogeneously. In general, a field ϕ is said to transform with Weyl weight w if

$$\phi \to e^{w\omega(x)}\phi. \tag{10}$$

Thus, for example, the metric components $g_{\mu\nu}$ transform with weight -2, while $T^{\mu\nu}$ transform with weight 6.

These properties have led to the development of the Weyl-covariant formulation [11], in which the equations of conformal hydrodynamics assume a very compact form. This formalism makes use of the (nondynamical) "Weyl connection,"

$$\mathcal{A}_\mu = u^\lambda \nabla_\lambda u_\mu - \frac{1}{3}\nabla_\lambda u^\lambda u_\mu, \tag{11}$$

to define a derivative operator, denoted here by $\mathcal{D}_\mu$, which is covariant under Weyl transformations. (A general formula can be found in Ref. [11].) We have checked, by performing an explicit gravitational calculation of the lowest quasinormal mode in the viscous fluid background [12], that the covariantization of Eq. (3) with the use of the Weyl-covariant derivative, i.e., $\partial_t \to \mathcal{D} \equiv u^\mu \mathcal{D}_\mu$, reproduces the correct result. Hence, the natural generalization of Eq. (7) is

$$\left(\frac{1}{T}\mathcal{D}\right)^2 \tilde{\Pi}_{\mu\nu} + 2\omega_I \frac{1}{T}\mathcal{D}\tilde{\Pi}_{\mu\nu} + |\omega|^2 \tilde{\Pi}_{\mu\nu} = 0, \tag{12}$$

where the role of $\delta\Pi$ is now taken on by

$$\tilde{\Pi}^{\mu\nu} = \Pi^{\mu\nu} - \Pi^{\mu\nu}_{\text{hydro}} \tag{13}$$

and

$$\mathcal{D}\tilde{\Pi}_{\mu\nu} = u^\lambda(\nabla_\lambda + 4\mathcal{A}_\lambda)\tilde{\Pi}_{\mu\nu} - 2\mathcal{A}_\lambda u_{(\mu}\tilde{\Pi}^\lambda_{\nu)}. \tag{14}$$

This formula also defines the action of $\mathcal{D}$ on $(1/T)\mathcal{D}\tilde{\Pi}^{\mu\nu}$, since the latter object has the same Weyl weight as $\tilde{\Pi}_{\mu\nu}$.

Equation (12) has two key features. First, it is consistent with $\Pi^{\mu\nu}$ transforming homogeneously under Weyl transformations. Second, it preserves its transversality and tracelessness due to the fact that $\mathcal{D}u^\mu = 0$.

As a nontrivial test of Eq. (12) we have checked that it is obeyed by the QNM computed in [13] for the strongly coupled plasma undergoing Bjorken expansion [14]. Even though this is a special flow with a high degree of symmetry, already in this case the terms coming from the Weyl connection are nontrivial.

Generalized theories of hydrodynamics.—The Landau-Lifschitz theory of relativistic viscous hydrodynamics is defined by adopting as the evolution equation the conservation of the stress tensor [Eq. (5)] with $\Pi^{\mu\nu}$ given by Eq. (8). However, this system of differential equations is not hyperbolic and in general does not have a well-posed initial value problem [15,16].

Hyperbolic theories of hydrodynamics, postulated by Müller, Israel, Stewart, (MIS) and others [17,18], instead of using Eq. (8) assume that the shear tensor is replaced by a new dynamical object, $\Pi^{\mu\nu}_{\text{MIS}}$, which obeys an evolution equation involving additional phenomenological parameters. A prototypical example of such an equation is

$$\left(\hat{\tau}_\Pi \frac{1}{T}\mathcal{D} + 1\right)\Pi^{\mu\nu}_{\text{MIS}} = -\eta\sigma^{\mu\nu}, \tag{15}$$

where $\hat{\tau}_\Pi$ is a dimensionless constant and the combination $\hat{\tau}_\Pi(1/T)$ has been referred to in the literature as the relaxation time. Equation (15) can be supplemented with terms quadratic in $\Pi^{\mu\nu}$ and gradients of hydrodynamic fields in such a way that solving it recursively in the gradient expansion gives the correct form of the hydrodynamic stress tensor up to second order in derivatives [10] (when referring to MIS theory in the following we will always mean this formulation). In this approach the relaxation time is identified with one of the second order transport coefficients. Assuming $\eta/s = 1/(4\pi)$, the linearized theory is causal as long as $\hat{\tau}_\Pi \geq 1/(2\pi)$. The drawback of the MIS formulation, however, is that it introduces a spurious nonphysical decaying mode with a frequency given by the relaxation time: $\omega = iT/\hat{\tau}_\Pi$.

The simplest way to incorporate additional *physical* nonequilibrium degrees of freedom into a causal hyperbolic description is to set

$$\Pi^{\mu\nu} = \Pi^{\mu\nu}_{\text{MIS}} + \tilde{\Pi}^{\mu\nu}, \tag{16}$$

with $\Pi^{\mu\nu}_{\text{MIS}}$ satisfying Eq. (15) and $\tilde{\Pi}^{\mu\nu}$ obeying Eq. (12). These traceless and transverse quantities are coupled together by the conservation law Eq. (6).

The resulting theory satisfies the same causality and stability properties as the MIS formulation. At the linearized level, in addition to the standard hydrodynamic modes it contains the damped modes corresponding to the QNM as seen in AdS/CFT. However, as a by-product of using the MIS formulation we have in addition the spurious decaying mode of MIS theory discussed above. In order to minimize its impact, we always set $-\eta(T)\sigma^{\mu\nu}$ as the initial condition for $\Pi^{\mu\nu}_{\text{MIS}}$. Moreover, we set the τ_Π parameter to the smallest value allowed by causality in order to maximize the damping of this mode.

The above formulation is the simplest generalization of MIS hydrodynamics. The equations presented here should provide a useful extension of hydrodynamics in situations where only a single QNM dominates the approach to equilibrium. Setting vanishing initial conditions for $\tilde{\Pi}^{\mu\nu}$ reduces the theory to standard MIS, while incorporating some nontrivial initial conditions allows us to examine the physical effects of the least damped nonhydrodynamic degrees of freedom. This theory could be used as an alternative to MIS hydrodynamics in situations, when an account of early preequilibrium dynamics including modes with $\Re(\omega) \neq 0$ is relevant. We perform various tests of this theory in the following section.

Before that, however, we would like to mention a possible alternative which aims to get rid of the nonphysical MIS mode altogether and use the physical nonequilibrium degrees of freedom as a means of ensuring hyperbolicity.

Note that since the QNMs have a sizable real frequency, one can never describe them using the MIS decaying mode. This has already been emphasized in Ref. [19].

Heuristically one could proceed by using Eqs. (13) and (8) in Eq. (12) to find

$$\left(\left(\frac{1}{T}\mathcal{D}\right)^2 + 2\omega_I \frac{1}{T}\mathcal{D} + |\omega|^2\right)\Pi^{\mu\nu} = -\eta|\omega|^2\sigma^{\mu\nu} - c_\sigma \frac{1}{T}\mathcal{D}(\eta\sigma^{\mu\nu}) + \ldots, \qquad (17)$$

where the ellipsis denotes contributions of second and higher order in gradients. Of all possible second order terms only one term has been kept, with a coefficient c_σ, which is treated as an arbitrary parameter. [Solving Eq. (8) in the gradient expansion shows that c_σ contributes to second order transport coefficients.] This term is included explicitly, since it improves the stability of Eq. (17).

The key property of Eq. (17) is that linearization around an equilibrium background leads to a system of partial differential equations which is hyperbolic for $c_\sigma \geq 0$. The characteristic velocity in the sound channel is found to be

$$v = \frac{1}{\sqrt{3}}\left(1 + \frac{c_\sigma}{\pi}\right)^{1/2}, \qquad (18)$$

so for causality one must further impose $c_\sigma \leq 2\pi$ (this in fact ensures causality in all channels).

For a numerical treatment of Eq. (17) it is important that exponentially growing modes be absent. Whether Eq. (17) is stable in this sense depends on the values of parameters such as the QNM frequencies and the viscosity to entropy ratio. This is similar to the case of the MIS equations. However, unlike that case, for the values of η/s and $\omega_{R,I}$ characteristic of $\mathcal{N} = 4$ SYM, Eq. (17) contains exponentially unstable modes with high k. This renders these equations (as they stand) unsuitable for numerical evaluation and comparison to the results of simulations based on the AdS/CFT correspondence. Let us emphasize, however, that these unstable modes appear far outside the range of applicability of the long wavelength description (e.g., with wave vectors $k > 18.5T$ if one chooses $c_\sigma = 2\pi$). It would be interesting to investigate whether one could modify Eq. (17) to cure this pathology. This question is set aside for the moment, and we henceforth concentrate on the simplest formulation given by Eq. (16) and Eq. (12).

Tests.—An essential part of this Letter is testing Eq. (16) and Eqs. (12), (15) against microscopic numerical computations of $\mathcal{N} = 4$ SYM plasma based on the AdS/CFT correspondence. This requires setting the parameters to appropriate values, i.e., $\eta/s = 1/4\pi$ and $\omega_{R,I}$ as in Eq. (4). We also set $\tau_\Pi = 1/(2\pi)$, which is the smallest value allowed by causality.

Here we consider two particularly symmetric configurations: homogeneous isotropization and boost-invariant flow. It is worth emphasizing at this point that homogeneous isotropization cannot be described at all by conventional Landau-Lifschitz viscous hydrodynamics.

The AdS/CFT computations are based on numerical solutions of $(4+1)$-dimensional Einstein equations with the negative cosmological constant obtained following the methods developed in Refs. [20,21] and Refs. [5,22]. This we compare to numerical solutions of the new phenomenological equations initialized by specifying just the energy, pressure anisotropy, and its time derivative which we take to agree with the values extracted from a particular numerical solution of Einstein equations at the specific initialization time.

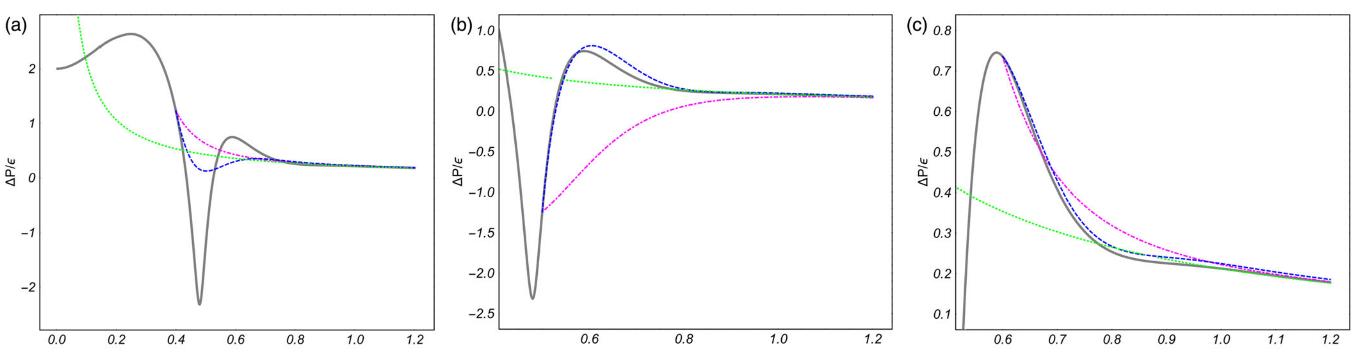

FIG. 1 (color online). Boost-invariant flow is a one-dimensional expansion of plasma in which the late time behavior is dominated by a hydrodynamic tail. In the local rest frame, using proper time (τ)-rapidity (y) coordinates, the stress tensor takes the form $\langle T^{\tau\tau}\rangle = \epsilon(\tau)$, $\langle T^{yy}\rangle = \tau^{-2}P_L(\tau)$ and $\langle T^{\perp\perp}\rangle = P_T(\tau)$. The plots depict the pressure anisotropy $\Delta P \equiv (P_T - P_L)$ normalized by ϵ. Gray curves denote the numerical solution based on AdS/CFT; magenta dash-dotted curves are solutions of MIS theory and the blue dashed curves are solutions of the new theory defined via Eq. (16). For reference, the prediction of first order hydrodynamics is displayed as the dotted green curve. The plots show the results of setting initial conditions at $\tau T = 0.4$ (left), 0.5 (center) and 0.6 (right). One can see that both MIS theory and the new equations converge to the exact curve at late times, which demonstrates the applicability of viscous hydrodynamics. With earlier initialization (center), the new equations lead to a quantitative agreement with the data also in the preequilibrium phase, as opposed to the MIS description.

The results for holographic isotropization (see the Supplemental Material [23]) show that for late enough initialization, Eq. (16) captures both the qualitative and quantitative features of the pressure anisotropy relaxation. Comparison to a solution of linearized Einstein equations, which can be superficially thought of as a sum over all quasinormal modes in this system, demonstrates that the applicability of the new equations is not limited by the far-from-equilibrium nonlinear effects not captured by it, but rather by the presence of the higher quasinormal modes.

The case of boost-invariant flow is presented in Fig. 1, which shows clearly that the MIS approach captures the late time tail very well, as do the new equations proposed here. However, at earlier times Eq. (16) provides a much more accurate picture. Estimates of the final temperature are also more accurate if Eq. (16) is used. For initial conditions involving many QNMs, the agreement at early times should not be as good. Also, for initial conditions where no nonhydrodynamic modes are excited at early times, effects of second and higher order (or possibly resummed [24]) hydrodynamics may become important.

Summary and conclusions.—The new phenomenological equations presented in this Letter generalize the relativistic Navier-Stokes theory by including leading nonhydrodynamic modes expected in theories of strongly coupled plasma with gravity duals. In these theories, the nonhydrodynamic modes correspond to QNMs of black branes in asymptotically anti–de Sitter space. The weak dependence of QNM frequencies on momenta suggests the ultralocality assumption, which we have used to identify the second order equation satisfied by the QNM contribution to the shear stress tensor. This equation is the essential new element, which makes it possible to go beyond the observations made in Refs. [19,25], where generalizations of hydrodynamics were pursued having noted the significance of the analytic structure of retarded correlators in theories with gravity duals.

The use of a conventional hydrodynamic description implicitly assumes that all nonequilibrium collective excitations in the quark-gluon plasma are set to zero. The proposed equations provide a means of relaxing this assumption and exploring their influence on subsequent hydrodynamic evolution. For some observables (such as the final multiplicities) this may not be quantitatively important. However, for observables sensitive to the pre-equilibrium stages of evolution (such as photon [26,27] or dilepton emission [28,29]) capturing the early time dynamics may be valuable. An important step toward such applications will be to develop an effective heuristic for setting initial conditions for the nonhydrodynamic modes in our new evolution equations. One of the possible approaches might be to extract these initial conditions from the early postcollision state following from the numerical simulations of Ref. [30] or Ref. [31].

We would like to thank U. Heinz, M. Martinez, S. Mrówczynski, J. Noronha, R. Peschanski, K. Rajagopal, P. Romatschke, W. van der Schee, M. Strickland, D. Teaney, A. Vuorinen, U. Wiedemann, L. Yaffe, and A. Yarom for valuable discussions and correspondence. M. P. H. is supported by the Netherlands Organization for Scientific Research under the NWO Veni scheme (UvA). This work was supported by the National Science Centre Grants No. 2012/07/B/ST2/03794 (M. P. H. and M. S.), No. 2012/06/A/ST2/00396 (R. J.), and No. 2013/11/N/ST2/03812 (P. W.). Research at Perimeter Institute is supported by the Government of Canada through Industry Canada and by the Province of Ontario through the Ministry of Research and Innovation.

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