Exact Correlation Functions in $SU(2)$ $\mathcal{N}=2$ Superconformal QCD

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(Received 6 October 2014; published 15 December 2014)

We report an exact solution of 2- and 3-point functions of chiral primary fields in $SU(2)$ $\mathcal{N}=2$ super-Yang-Mills theory coupled to four hypermultiplets. It is shown that these correlation functions are nontrivial functions of the gauge coupling, obeying differential equations which take the form of the semi-infinite Toda chain. We solve these equations recursively in terms of the Zamolodchikov metric that can be determined exactly from supersymmetric localization on the four-sphere. Our results are verified independently in perturbation theory with a Feynman diagram computation up to 2 loops. This is a short version of a companion paper that contains detailed technical remarks, additional material, and aspects of an extension to the $SU(N)$ gauge group.

DOI: 10.1103/PhysRevLett.113.251601 PACS numbers: 11.15.−q, 11.25.Hf, 11.30.Pb

Introduction.—Quantum field theories often possess exactly marginal deformations along which the data of the theory (spectrum, correlation functions, etc.) may change continuously. A characteristic well-studied example in four dimensions is the $\mathcal{N}=4$ super-Yang-Mills (SYM) theory. In this case an exactly marginal deformation interpolates between weak coupling (where the theory can be analyzed with standard perturbation theory) and strong coupling (where standard perturbative methods are inadequate). It is of great interest to develop nonperturbative techniques that allow us to describe (analytically) properties of the theory at any value of the marginal couplings.

Supersymmetric theories are an opportune context for the development of such techniques. They often possess special sectors that exhibit dynamics with nontrivial, but exactly computable, coupling constant dependence. An exact solution in these sectors can provide useful intuition, or a solid starting point, towards an analysis of the more general properties of the theory.

In this Letter, we will concentrate on a specific example of a four-dimensional conformal field theory with $\mathcal{N}=2$ supersymmetry: $\mathcal{N}=2$ SYM theory with gauge group $SU(2)$ coupled to 4 hypermultiplets in the fundamental representation (in short, $SU(2)$ $\mathcal{N}=2$ superconformal QCD, or simply SCQCD). By definition, this theory is invariant under 8 real supercharges. The special sector of interest is comprised of (scalar) superconformal chiral primary fields ϕ_I (to be specified explicitly in a moment) annihilated by the four supercharges of right chirality. The conjugate fields annihilated by the supercharges of left chirality will be denoted as $\overline{\phi}_I$. $\mathcal{N}=2$ superconformal field theories (SCFTs) are also invariant under the global $SU(2)_R\times U(1)_R$ R symmetry. The chiral primaries ϕ_I are singlets of the $SU(2)_R$, but have nonzero $U(1)_R$ charge R [1]. Their scaling dimension Δ obeys the relation $\Delta=R/2$. (For antichiral primaries $\Delta=-(R/2)$).

It is well known that the operator product expansion (OPE) of chiral primary fields is nonsingular

$$\phi_I(x)\phi_J(0)=C_{IJ}^K\phi_K(0)+\cdots. \tag{1}$$

It forms a ring structure known as the chiral ring [2]. Two important sets of data in the chiral ring are the 2-point functions

$$\langle\phi_I(x)\overline{\phi}_J(0)\rangle=\frac{g_{I\bar{J}}}{|x|^{2\Delta}} \tag{2}$$

and the 3-point functions

$$\langle\phi_I(x)\phi_J(y)\overline{\phi}_K(z)\rangle=\frac{C_{IJ\overline{K}}}{|x-y|^{\Delta_{IJ,K}}|x-z|^{\Delta_{IK,J}}|y-z|^{\Delta_{JK,I}}}, \tag{3}$$

where $\Delta_{IJ,K}=\Delta_I+\Delta_J-\Delta_K$. There is an obvious relation between the OPE and 2- and 3-point function coefficients $C_{IJ\overline{K}}=C_{IJ}^L g_{L\overline{K}}$.

In our example there is a single exactly marginal deformation labeled by a complex parameter τ (the complexified gauge coupling constant). The 2- and 3-point function coefficients $g_{I\bar{J}}$, $C_{IJ\overline{K}}$ are nontrivial functions of τ, receiving corrections at all orders in perturbation theory as well as from instanton effects. (The scaling dimensions Δ_I are fixed by the nonrenormalized $U(1)_R$ charge R_I as described above). We will present exact formulae for these data combining methods of supersymmetric localization (in particular, Refs. [3,4]) with certain exact relations between chiral ring correlation functions [5] that are four-dimensional analogs of the tt^* equations in two dimensions [6,7]. We have verified the resulting expressions with an independent computation in perturbation theory up to 2 loops [8].

We point out that analogous correlation functions in $\mathcal{N}=4$ SYM theory are nonrenormalized [9–19] and are therefore trivial functions of the gauge coupling that can be determined at tree level. $\mathcal{N}=2$ dynamics is clearly more interesting and the results in this Letter indicate that there is a considerable amount of new data that are tractable analytically compared to previous knowledge. The techniques presented here are useful in $\mathcal{N}=2$ theories with exactly marginal deformations beyond the specific example analyzed in this Letter. A detailed explanation of the general properties of these techniques and extensions to more general examples are discussed in a companion paper [8].

$SU(2)$ $\mathcal{N}=2$ SCQCD.—The main example of this Letter is $\mathcal{N}=2$ SYM theory with gauge group $SU(2)$ coupled to 4 hypermultiplets (at the origin of the Coulomb branch). This is a gauge theory whose field content includes (a) the $\mathcal{N}=2$ vector multiplet fields, namely, the gauge boson A_μ, a complex scalar field φ and four Weyl fermions (all in the adjoint representation), and (b) the 4 $\mathcal{N}=2$ hypermultiplets that are comprised of 4 complex bosons and 8 Weyl fermions (all in the fundamental representation). The global symmetry group is $SO(8)\times SU(2)_R\times U(1)_R$. $SO(8)$ is a flavor symmetry rotating the hypermultiplets. The standard Yang-Mills Lagrangian of this theory is summarized, for example, in appendix B of Ref. [8] whose conventions we are also following here.

The single exactly marginal coupling of this theory is the complexified Yang-Mills coupling $\tau=(\theta/2\pi)+(4\pi i/g_{\rm YM}^2)$, where θ is the θ angle and $g_{\rm YM}$ is the Yang-Mills coupling. We will work in conventions where the infinitesimal exactly marginal deformation of the action takes the form

$$S\to S+\frac{\delta\tau}{4\pi^2}\int d^4x\mathcal{O}_\tau(x)+\frac{\delta\bar{\tau}}{4\pi^2}\int d^4x\overline{\mathcal{O}}_\tau(x), \qquad (4)$$

where the $\Delta=4$ operators $\mathcal{O}_\tau$, $\overline{\mathcal{O}}_\tau$ are descendants of $\Delta=2$ (anti)chiral primary fields

$$\mathcal{O}_\tau=\mathcal{Q}^4\phi_2, \qquad \overline{\mathcal{O}}_\tau=\overline{\mathcal{Q}}^4\overline{\phi}_2. \qquad (5)$$

The notation $\mathcal{Q}^4\phi_2$ is shorthand notation for the nested (anti)commutator of four supercharges of left chirality. The Lorentz and $SU(2)_R$ indices of the supercharges are combined to give a Lorentz and $SU(2)_R$ singlet. ϕ_2 is the lowest dimension $\mathcal{N}=2$ chiral primary field

$$\phi_2=\frac{\pi}{8}\mathrm{Tr}[\varphi^2]. \qquad (6)$$

The overall normalization in Eq. (5) is fixed so that $\langle\mathcal{O}_\tau(x)\overline{\mathcal{O}}_\tau(0)\rangle=\nabla_x^2\nabla_x^2\langle\phi_2(x)\overline{\phi}_2(0)\rangle$.

The chiral ring of the $SU(2)$ theory can be freely generated by the chiral primary field ϕ_2 by repeated multiplication. The explicit checks reported below verify the consistency of this picture. We will normalize the generic chiral primary $\phi_{2n}\propto(\mathrm{Tr}[\varphi^2])^n$ by requiring the OPE

$$\phi_2(x)\phi_{2n}(0)=\phi_{2n+2}(0)+\cdots. \qquad (7)$$

This choice fixes all the nonvanishing OPE coefficients

$$C_{2n2m}^{2(n+m)}=1 \qquad (8)$$

and the normalization of all the higher order chiral primaries ϕ_{2n} $(n>1)$ which are multitrace.

To summarize, the (chiral ring) sector of interest in this Letter is comprised of a sequence of fields ϕ_{2n} with scaling dimensions $\Delta_{2n}=2n$.

We will denote the 2-point functions of these fields as

$$\langle\phi_{2n}(x)\overline{\phi}_{2n}(0)\rangle=\frac{g_{2n}(\tau,\bar{\tau})}{|x|^{4n}}. \qquad (9)$$

The 2-point function coefficients g_{2n} (as well as the corresponding 3-point function coefficients $C_{2m2n\overline{2m+2n}}$) are nontrivial functions of the complexified coupling τ that we will determine exactly.

Notice that g_2 is directly related to the coefficient G_2 of the 2-point function $\langle\mathcal{O}_\tau(x)\overline{\mathcal{O}}_\tau(0)\rangle$. G_2 is the so-called Zamolodchikov metric on the space of exactly marginal couplings. For $\mathcal{N}=2$ theories this space is known to be a complex Kähler manifold. Hence, (specializing to the case at hand) there is a scalar function $\mathcal{K}$, the Kähler potential, such that

$$G_2=\partial_\tau\partial_{\bar{\tau}}\mathcal{K}=192g_2. \qquad (10)$$

Exact correlation functions.—Reference [5] formulated a set of exact relations between the OPE and 2-point function coefficients for general four-dimensional $\mathcal{N}=2$ theories with exactly marginal directions. These relations, which take the form of systems of differential equations on the marginal couplings, are direct analogs of the tt^* equations in two-dimensional $\mathcal{N}=(2,2)$ superconformal theories derived in Refs. [6,7] with the method of topological-antitopological fusion. Reference [5] derived such relations in four dimensions with the judicious use of superconformal Ward identities.

Applying the general tt^* equations of Ref. [5] in the case of interest here in the so-called holomorphic gauge and the related above-mentioned normalization conventions (see Ref. [8] for an exposition of all the pertinent details), we arrive at the following relations for the 2-point function coefficients g_{2n} (9):

$$\partial_\tau \partial_{\bar{\tau}} g_{2n} = \frac{g_{2n+2}}{g_{2n}} - \frac{g_{2n}}{g_{2n-2}} - g_2, \tag{11}$$

where $n = 1, 2, \ldots$ and $g_0 = 1$ by definition. By unitarity all $g_{2n} > 0$ and this infinite sequence of differential equations can be recast as the more familiar semi-infinite Toda chain

$$\partial_\tau \partial_{\bar{\tau}} q_n = e^{q_{n+1} - q_n} - e^{q_n - q_{n-1}}, \qquad n = 2, \ldots \tag{12}$$

by setting $g_{2n} = \exp[q_n - \log(\mathcal{K}/192)]$. $\mathcal{K}$ is the Kähler potential in Eq. (10) and the factor of 192 follows from the normalization conventions of the previous section.

It is interesting to ask what is the general solution of the system [Eq. (11)] subject to positivity over the entire space of marginal couplings and whether positivity and some other "boundary conditions" from perturbation theory at weak coupling can fix the solution uniquely. We will not try to answer this question here. Instead, we will use the system of equations [Eq. (11)] recursively, writing

$$g_{2n+2} = g_{2n} \partial_\tau \partial_{\bar{\tau}} \log g_{2n} + \frac{g_{2n}^2}{g_{2n-2}} + g_2 g_{2n}, \quad n = 1, 2, \ldots, \tag{13}$$

to determine all the higher 2-point functions g_{2n} $(n > 1)$ from the lowest one g_2.

Exact 2-point functions: Recent work [4] has determined the exact quantum Kähler potential of $\mathcal{N} = 2$ SCFTs in terms of the partition function Z_{S^4} of the theory on the four-sphere S^4. The precise relation is

$$\mathcal{K} = 192 \log Z_{S^4}. \tag{14}$$

Notice that the marginal operators $\mathcal{O}_\tau$ are normalized differently in Ref. [4], i.e., $\mathcal{O}_{\text{here}} = 4\mathcal{O}_{\text{there}}$. This explains the factor $192 = 12 \times 4 \times 4$ as opposed to 12 in Ref. [4]. Combining with Eq. (10) we obtain

$$g_2 = \partial_\tau \partial_{\bar{\tau}} Z_{S^4}. \tag{15}$$

For the $SU(2)$ SCQCD theory there is a well-studied integral expression for the sphere partition function Z_{S^4} that has been determined using supersymmetric localization [3]

$$Z_{S^4}(\tau, \bar{\tau}) = \int_{-\infty}^{\infty} da e^{-4\pi \text{Im}(\tau) a^2} (2a)^2 \times \frac{H(2ia)H(-2ia)}{[H(ia)H(-ia)]^4} |Z_{\text{inst}}(a, \tau)|^2. \tag{16}$$

$H(z) = G(1+z)G(1-z)$ in terms of the Barnes G function [20], and Z_{inst} is the Nekrasov partition function [21] that incorporates the contribution from all the instanton sectors. For further details we refer the reader to Ref. [3].

Combining the expressions in Eqs. (13), (15), and (16), we are able to determine recursively any of the 2-point function coefficients g_{2n} in terms of higher derivatives of the S^4 partition function.

Exact 3-point functions: The nonvanishing 3-point function coefficients $C_{2m2n\overline{2(m+n)}}$ follow immediately from the general relation $C_{IJ\bar{K}} = C_{IJ}^L g_{L\bar{K}}$, Eq. (8), and the above solution of the 2-point function coefficients

$$C_{2m2n\overline{2(m+n)}} = C_{2m2n}^{2(m+n)} g_{2(m+n)} = g_{2(m+n)}. \tag{17}$$

Notice that, although the normalization conventions of the previous sections are very convenient for the above computations, in conformal field theory it is common to work instead with orthonormal primary operators $\hat{\phi}_{2n}$ for which $\langle \hat{\phi}_{2n}(x) \bar{\hat{\phi}}_{2\bar{n}}(0) \rangle = \delta_{n,\bar{n}}/|x|^{2\Delta}$. In these alternative conventions, the 2-point function coefficients are trivial but the OPE coefficients are nontrivial and

$$\hat{C}_{2m2n\overline{2m+2n}} = \sqrt{\frac{g_{2m+2n}}{g_{2m} g_{2n}}}. \tag{18}$$

More general extremal correlators: With a conformal transformation of the form $x'^\mu = x^\mu - y^\mu/|x-y|^2$ it is possible to recast the general extremal correlator

$$\langle \phi_{2m_1}(x_1) \ldots \phi_{2m_n}(x_n) \bar{\phi}_{2\bar{m}}(y) \rangle, \tag{19}$$

with $\bar{m} = \sum_{\ell=1}^n m_n$ as

$$\frac{\langle \phi_{2m_1}(x'_1) \ldots \phi_{2m_n}(x'_n) \bar{\phi}_{2\bar{m}}(\infty) \rangle}{|x_1 - y|^{4m_1} \cdots |x_n - y|^{4m_n}}. \tag{20}$$

Using superconformal Ward identities one can prove that the correlation function on the numerator of Eq. (20) is independent of the positions x_i. Consequently, it can be evaluated in any particular limit; in particular, we can make use of the above known OPEs and 2-point functions g_{2n} to determine the exact τ dependence of such extremal correlators as well, as explained in more detail in Ref. [8].

Predictions for perturbation theory.—We can use the above results to make very specific predictions for the weak coupling, $g_{YM} \ll 1$, expansion of 2- and 3-point functions in the chiral ring. As an illustration, here we present explicit examples in the 0-instanton and 1-instanton sectors.

0-instanton sector: Working with the perturbative (0-instanton) part of the S^4 partition function [Eq. (16)]

$$Z_{S^4}^{(0)} = \int_{-\infty}^{\infty} da e^{-4\pi \text{Im}(\tau) a^2} (2a)^2 \frac{H(2ia)H(-2ia)}{[H(ia)H(-ia)]^4} \tag{21}$$

our exact formulas provide, e.g., for the first three chiral primaries, the perturbative expansions

$$g_2^{(0)}=\frac{3}{8}\frac{1}{(\mathrm{Im}\tau)^2}-\frac{135\zeta(3)}{32\pi^2}\frac{1}{(\mathrm{Im}\tau)^4}+\frac{1575\zeta(5)}{64\pi^3}\frac{1}{(\mathrm{Im}\tau)^5}+\cdots,$$
$$g_4^{(0)}=\frac{15}{32}\frac{1}{(\mathrm{Im}\tau)^4}-\frac{945\zeta(3)}{64\pi^2}\frac{1}{(\mathrm{Im}\tau)^6}+\frac{7875\zeta(5)}{64\pi^3}\frac{1}{(\mathrm{Im}\tau)^7}+\cdots,$$
$$g_6^{(0)}=\frac{315}{256}\frac{1}{(\mathrm{Im}\tau)^6}-\frac{76545\zeta(3)}{1024\pi^2}\frac{1}{(\mathrm{Im}\tau)^8}+\frac{1677375\zeta(5)}{2048\pi^3}\frac{1}{(\mathrm{Im}\tau)^9}+\cdots. \tag{22}$$

The superscript 0 denotes that this is the 0-instanton contribution. We wrote down contributions only up to 3 loops, but it is easy to go to any desired order. We have verified the validity of the predicted $g_{2n}^{(0)}$, for all values of the positive integer n, with an independent computation in perturbation theory up to 2 loops [8]. This provides an independent 2-loop perturbative check of the tt^* Eqs. (13), but also a check of the recent proposal of Ref. [4] that identifies the quantum Kähler potential of $\mathcal{N}=2$ theories with the S^4 partition function.

Equivalently, in the alternative basis with orthonormal 2-point functions Eq. (18) provides very specific results for the nontrivial 3-point function coefficients $\hat{C}_{2m2n\overline{2m+2n}}$. As an illustration, the first few coefficients are

$$\hat{C}_{224}^{(0)}=\sqrt{\frac{10}{3}}\left(1-\frac{9\zeta(3)}{2\pi^2}\frac{1}{(\mathrm{Im}\tau)^2}+\frac{525\zeta(5)}{8\pi^3}\frac{1}{(\mathrm{Im}\tau)^3}+\cdots\right),$$
$$\hat{C}_{246}^{(0)}=\sqrt{7}\left(1-\frac{9\zeta(3)}{\pi^2}\frac{1}{(\mathrm{Im}\tau)^2}+\frac{675\zeta(5)}{4\pi^3}\frac{1}{(\mathrm{Im}\tau)^3}+\cdots\right),$$
$$\hat{C}_{268}^{(0)}=\sqrt{12}\left(1-\frac{27\zeta(3)}{2\pi^2}\frac{1}{(\mathrm{Im}\tau)^2}+\frac{2475\zeta(5)}{8\pi^3}\frac{1}{(\mathrm{Im}\tau)^3}+\cdots\right). \tag{23}$$

1-instanton sector: As an example, we consider the 1-instanton contribution to the S^4 partition function [Eq. (16)]

$$Z_{S^4}^{(1)}=\cos\theta\exp\left(-\frac{8\pi^2}{g_{YM}^2}\right)\left(-\frac{3}{4\pi(\mathrm{Im}\tau)^{3/2}}\right)\times\left[1-\frac{1}{8\pi\mathrm{Im}\tau}-\frac{45\zeta(3)}{16\pi^2(\mathrm{Im}\tau)^2}+\frac{105(\zeta(3)+10\zeta(5))}{128\pi^3(\mathrm{Im}\tau)^3}+\cdots\right] \tag{24}$$

and from this we obtain expansions of g_{2n} in the 1-instanton sector

$$g_2^{(1)}=\cos\theta\exp\left(-\frac{8\pi^2}{g_{YM}^2}\right)\left[\frac{3}{8(\mathrm{Im}\tau)^2}+\frac{3}{16\pi(\mathrm{Im}\tau)^3}-\frac{135\zeta(3)}{32\pi^2(\mathrm{Im}\tau)^4}+\cdots\right], \tag{25}$$

$$g_4^{(1)}=\cos\theta\exp\left(-\frac{8\pi^2}{g_{YM}^2}\right)\left[\frac{15}{16(\mathrm{Im}\tau)^4}+\frac{15}{32\pi(\mathrm{Im}\tau)^5}-\frac{945\zeta(3)}{32\pi^2(\mathrm{Im}\tau)^6}+\cdots\right]. \tag{26}$$

If desired, it is straightforward to extend these results to higher n, higher instanton number ℓ, and higher order in the perturbative expansion around any given instanton sector. It would be interesting to confirm them with an independent perturbative computation in the general ℓ-instanton sector. Moreover, it would be interesting to verify the expected positivity of the resulting expressions at general n.

Outlook.—We reported exact nonperturbative formulae for 2- and 3-point functions of chiral primary fields in the $SU(2)$ $\mathcal{N}=2$ SCQCD theory. A detailed exposition of the employed technology, of the perturbative 2-loop check, as well as an extension to the $\mathcal{N}=2$ SCQCD theory with more general $SU(N)$ gauge group can be found in the companion paper [8]. The general $SU(N)$ case exhibits a much more involved chiral ring structure with additional generators and the set of independent data needed to determine a complete solution is currently unclear. Nevertheless, we find preliminary signs of an underlying structure [8] that is worth investigating further.

Regardless of the technical possibility to obtain the full solution in more general cases, the real merit of the approach proposed in this Letter lies in the organized use of superconformal Ward identities that reduce the general problem considerably. This reduction combined with the use of data obtained independently, e.g. with localization, provides a promising route towards new highly nontrivial results in four-dimensional $\mathcal{N}=2$ theories.

Beyond the $SU(N)$ SCQCD case it would be interesting to extend the application of the $4d$ tt^* equations to other known classes of $\mathcal{N}=2$ theories and to determine general conditions (e.g., positivity constraints) that fix their solution uniquely. Results along these lines are expected to have wider implications. For example, we have already seen that the explicit knowledge of 2- and 3-point functions implies also the exact form of general extremal correlation functions in the chiral ring. In a different direction one can envision using these results as input in a more general bootstrap program in $\mathcal{N}=2$ SCFTs aiming to determine larger classes of correlation functions, spectral data etc. Clearly, several possibilities remain to be explored.

We would like to thank M. Buican, J. Drummond, M. Kelm, W. Lerche, B. Pioline, M. Rosso, D. Tong, C. Vafa, C. Vergu, C. Vollenweider, and A. Zhedanov for useful discussions. The work of M. B. is supported in part by a grant of the Swiss National Science Foundation. The work of V. N. was supported in part by European Union's 7th Framework Programme under grant agreements (FP7-REGPOT-2012-2013-1) No. 316165, PIF-GA-2011-300984, the EU program "Thales" MIS 375734 and was also cofinanced by the EU (European Social Fund, ESF) and Greek national funds through the Operational Program "Education and Lifelong Learning" of the National Strategic Reference Framework (NSRF) under "Funding of proposals that have received a positive evaluation in the 3rd and 4th Call of ERC Grant Schemes." K. P. would like to thank the Royal Netherlands Academy of Sciences (KNAW).

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