Chiral Anomaly and Diffusive Magnetotransport in Weyl Metals

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We present a microscopic theory of diffusive magnetotransport in Weyl metals and clarify its relation to the chiral anomaly. We derive coupled diffusion equations for the total and axial charge densities and show that the chiral anomaly manifests as a magnetic-field-induced coupling between them. We demonstrate that a universal experimentally observable consequence of this coupling in magnetotransport in Weyl metals is a quadratic negative magnetoresistance, which will dominate all other contributions to magnetoresistance under certain conditions.

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Weyl semimetals have attracted considerable attention recently as the first realization of a metallic, yet topologically nontrivial state of matter [1–4], as anticipated some time ago by Volovik [5]. Observation of the closely related Dirac semimetals [6–13] clearly makes the experimental realization of Weyl semimetals only a matter of time.

The most distinctive observable spectroscopic feature of Weyl semimetals is the presence of the so-called Fermi-arc surface states [1]. It is of great interest, however, to find similar smoking-gun features of Weyl semimetals in response, especially in transport. These do exist and have been described as being consequences of a chiral anomaly, i.e., anomalous nonconservation of the numbers of Weyl fermions of distinct chiralities [14–24]. Notably, Son and Spivak [17] have proposed that in nonmagnetic inversion-asymmetric Weyl semimetals the chiral anomaly leads to a novel kind of weak-field magnetoresistance: negative and quadratic in the magnetic field.

However, while the chiral anomaly is a well-defined concept in the context of relativistic field theory [25,26], where massless fermions in unbounded momentum space possess exact chiral symmetry, violated by the anomaly, the situation is less clear in the condensed matter context. Even though chiral symmetry may be formally defined in a low-energy model of a Weyl semimetal, in which the band dispersion is approximated as being exactly linear and unbounded, no real microscopic model of Weyl semimetal actually possesses such a symmetry, simply because the momentum space in this case is compact, being confined to the first Brillouin zone (BZ). Since the chiral symmetry is not present to begin with, it is then unclear how meaningful it is to speak of its violation by the chiral anomaly and the physical consequences of this violation.

In this Letter we clarify the issues raised above. Starting from a microscopic model of a Weyl semimetal [3], which does not possess chiral symmetry, we demonstrate that one may, nevertheless, define a microscopic quantity, which we call axial charge density in analogy to the corresponding concept in relativistic field theory, and show that this quantity may be expected to be conserved or nearly conserved in the absence of an external magnetic field, when one is not too close to the phase boundaries at which the Weyl semimetal phase disappears. We then derive hydrodynamic (diffusion) equations, which govern coupled evolution of the axial and the total charge densities in the presence of an external magnetic field. The near conservation of the axial charge density at the microscopic level translates into long relaxation time at the level of hydrodynamic equations. We demonstrate that when the axial charge relaxation time is long, any Weyl metal indeed possesses a large negative magnetoresistance, which is quadratic in the magnetic field, in agreement with Ref. [17]. We show, however, that this effect is in fact even more universal than suggested in Ref. [17], and characterizes magnetic Weyl semimetals just as well as the inversion-asymmetric ones. In this sense, quadratic negative weak-field magnetoresistance may be regarded as a universal smoking-gun transport signature of Weyl semimetals and Weyl metals.

We start from the microscopic model of a Weyl semimetal in a magnetically doped topological insulator (TI) and normal insulator (NI) multilayer, introduced by us before [3], which has the important virtue of being the simplest realistic model of a Weyl semimetal

$$\mathcal{H}(\mathbf{k}) = v_F \tau^z (\hat{z} \times \boldsymbol{\sigma}) \cdot \mathbf{k} + b\sigma^z + \hat{\Delta}(k_z), \qquad (1)$$

where $\hat{\Delta}(k_z) = \Delta_S \tau^x + (\Delta_D/2)(\tau^+ e^{ik_z d} + \text{H.c.})$. $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$ in Eq. (1) are Pauli matrices, describing the spin and the *which surface* pseudospin degrees of freedom, b is the spin splitting due to magnetized impurities, $\Delta_{S,D}$ are tunneling matrix elements, describing tunneling between TI surface states in the same or neighboring TI layers, and d is the superlattice period in the growth (z) direction. We will take $\Delta_{S,D}$ to be non-negative for concreteness.

Equation (1) has a Weyl semimetal phase when $b_{c1} \leq b \leq b_{c2}$, where $b_{c1} = |\Delta_S - \Delta_D|$ and $b_{c2} = \Delta_S + \Delta_D$. The two Weyl nodes are located on the z axis in momentum

space at points $k_{\pm}^{z} = \pi/d \pm k_0$, where $k_0 = d^{-1}\arccos[(\Delta_S^2 + \Delta_D^2 - b^2)/2\Delta_S\Delta_D]$. The Weyl nodes are interchanged by the spatial inversion transformation, with the inversion center placed midway between the top and bottom surfaces of any TI or NI layer, $\mathcal{I}:\mathcal{H}(\mathbf{k}) \to \tau^x\mathcal{H}(-\mathbf{k})\tau^x$.

We now introduce the axial charge density operator, which is analogous to the total charge density in every aspect, except changes sign when the chiralities of the Weyl nodes are interchanged (generalization of this concept to multiple Weyl node pairs is obvious). It can be defined rigorously and uniquely based on symmetry considerations. Namely, we define the axial charge density n_a as a local operator, that is odd under inversion $\mathcal{I}$ and $z \to -z$ reflections, even under time reversal, but odd under time reversal, combined with rotation of the spin quantization axis by π around either the x or y axis. This uniquely determines the explicit representation of the axial charge density operator to be

$$\hat{n}_a = \tau^y\sigma^z. \tag{2}$$

One may easily check [27] that adding the term $-\mu_a\hat{n}_a$, where μ_a is the axial chemical potential, to the Hamiltonian Eq. (1), shifts the Weyl nodes in opposite directions in energy, giving rise to the energy difference

$$\Delta\epsilon = \frac{2\mu_a\tilde{v}_F}{\Delta_S d}, \tag{3}$$

where

$$\tilde{v}_F = \frac{d}{2b}\sqrt{(b^2 - b_{c1}^2)(b_{c2}^2 - b^2)}, \tag{4}$$

is the z component of the Fermi velocity at the location of the Weyl nodes.

We now ask the following question: does $\hat{n}_a$ represent a conserved quantity, as it would in a low-energy model of the Weyl semimetal? To answer this we need to evaluate the commutator of $\hat{n}_a$ with the Hamiltonian $\mathcal{H}(\mathbf{k})$. It is convenient at this point to apply the following canonical transformation to all the operators: $\sigma^{\pm} \to \tau^z\sigma^{\pm}$, $\tau^{\pm} \to \sigma^z\tau^{\pm}$ [3]. Evaluating the commutator at the Weyl node locations, we now obtain

$$[\mathcal{H}(\mathbf{k}), \hat{n}_a]_{k_{\pm}^z} = i\frac{b^2 - \Delta_D^2 + \Delta_S^2}{\Delta_S}\tau^z\sigma^z. \tag{5}$$

This means that n_a may indeed be a conserved quantity in the Weyl semimetal or weakly doped Weyl metal, provided $\Delta_D \geq \Delta_S$ and $b = \sqrt{b_{c1}b_{c2}}$; i.e., the magnitude of the spin splitting is exactly the geometric mean of its lower- and upper-critical values, at which the transitions out of the Weyl semimetal phase occur. Otherwise, the commutator is nonzero and n_a is not conserved. However, as will be shown below, the relevant relaxation time may still be long, even when the above condition is not exactly satisfied, in which case the axial charge density is still a physically meaningful quantity.

We now want to derive hydrodynamic transport equations (diffusion equations) for both the axial charge density $n_a(\mathbf{r}, t)$ and the total charge density $n(\mathbf{r}, t)$. As will be shown below, what is known as the chiral anomaly will be manifest at the level of these hydrodynamic equations as a coupling between n_a and n in the presence of an external magnetic field. This coupling leads to significant observable magnetotransport effects, provided the axial charge relaxation time, calculated below, is long enough.

To proceed with the derivation, we add a constant uniform magnetic field in the $\hat{z}$ direction $\mathbf{B} = B\hat{z}$ and a scalar impurity potential $V(\mathbf{r})$, whose precise form will be specified later. Adopting the Landau gauge for the vector potential $\mathbf{A} = xB\hat{y}$, the second-quantized Hamiltonian of our system may be written as

$$H = \sum_{nak_yk_z} \epsilon_{na}(k_z)c^{\dagger}_{nak_yk_z}c_{nak_yk_z} + \sum_{nak_yk_z,n'a'k_y'k_z'} \langle n, a, k_y, k_z|V|n', a', k_y', k_z'\rangle c^{\dagger}_{nak_yk_z}c_{n'a'k_y'k_z'}. \tag{6}$$

Here $\epsilon_{na}(k_z)$ are Landau-level (LL) eigenstate energies of a clean multilayer in magnetic field, $n = 0, 1, 2, \ldots$ is the main LL index, k_y is the Landau-gauge intra-LL orbital quantum number, k_z is the conserved component of the crystal momentum along the z direction, and $a = (s, t)$ is a composite index (introduced for compactness of notation), consisting of $s = \pm$, which labels the electronlike ($s = +$) and holelike ($s = -$) sets of Landau levels, and $t = \pm$, which labels the two components of a Kramers doublet of LLs, degenerate at $b = 0$. Explicitly we have

$$\epsilon_{na}(k_z) = s\sqrt{2\omega_B^2 n + m_t^2(k_z)} \equiv s\epsilon_{nt}(k_z), \tag{7}$$

where $\omega_B = v_F/\ell_B$ is the Dirac cyclotron frequency and $\ell_B = 1/\sqrt{eB}$ is the magnetic length. We will use units in which $\hbar = c = 1$ throughout. The "Dirac masses" $m_t(k_z)$ are given by $m_t(k_z) = b + t\Delta(k_z)$ where $\pm\Delta(k_z) = \pm\sqrt{\Delta_S^2 + \Delta_D^2 + 2\Delta_S\Delta_D\cos(k_z d)}$ are the two eigenvalues of the $\hat{\Delta}(k_z)$ operator.

The LL eigenstates have the following form, typical for LLs in Dirac systems:

$$|n, a, k_y, k_z\rangle = \sum_{\tau}[z^a_{n\uparrow\tau}(k_z)|n-1, k_y, k_z, \uparrow, \tau\rangle + z^a_{n\downarrow\tau}(k_z)|n, k_y, k_z, \downarrow, \tau\rangle]. \tag{8}$$

Here

$$\langle \mathbf{r}|n,k_y,k_z,\sigma,\tau\rangle = \frac{1}{\sqrt{L_z}} e^{ik_z z}\phi_{nk_y}(\mathbf{r})|\sigma,\tau\rangle, \tag{9}$$

$\phi_{nk_y}(\mathbf{r})$ are the Landau-gauge orbital wave functions, and σ,τ are the spin and pseudospin indices, respectively. Finally, the four-component eigenvector $|z_n^a(k_z)\rangle$ may be written as a tensor product of the two-component spin and pseudospin eigenvectors, i.e., $|z_n^a(k_z)\rangle = |v_n^a(k_z)\rangle \otimes |u^a(k_z)\rangle$, where

$$|v_n^{st}(k_z)\rangle = \frac{1}{\sqrt{2}}\left(\sqrt{1+s\frac{m_t(k_z)}{\epsilon_{nt}(k_z)}}, -is\sqrt{1-s\frac{m_t(k_z)}{\epsilon_{nt}(k_z)}}\right),$$
$$|u^t(k_z)\rangle = \frac{1}{\sqrt{2}}\left(1, t\frac{\Delta_S+\Delta_D e^{-ik_z d}}{\Delta(k_z)}\right). \tag{10}$$

As in all Dirac systems, the lowest $n=0$ LL is special and needs to be considered separately. The s quantum number is absent in this case and taking $B>0$ for concreteness, we have $\epsilon_{nt}(k_z) = -m_t(k_z)$, and $|v_0^t(k_z)\rangle = (0,1)$.

To proceed, we will make the standard assumption that the impurity potential obeys Gaussian distribution, with $\langle V(\mathbf{r})V(\mathbf{r}')\rangle = \gamma^2\delta(\mathbf{r}-\mathbf{r}')$. To simplify calculations further we will also assume that the momentum transfer due to the impurity scattering is smaller than the size of the BZ, i.e., $|k_z - k_z'|d \ll 1$. In this case $\langle u^t(k_z)|u^{t'}(k_z')\rangle \approx \delta_{tt'}$, i.e., the t quantum number may be assumed to be approximately preserved during the impurity scattering.

We treat the impurity scattering in the standard self-consistent Born approximation (SCBA). The retarded SCBA self-energy satisfies the equation

$$\Sigma^R_{nak_yk_z}(\omega) = \frac{1}{L_z}\sum_{n'a'k_y'k_z'}\langle|\langle n,a,k_y,k_z|V|n',a',k_y',k_z'\rangle|^2\rangle \times G^R_{n'a'k_y'k_z'}(\omega). \tag{11}$$

We will assume that the Fermi energy ϵ_F is positive; i.e., the Weyl semimetal is electron doped, and large enough that the impurity-scattering-induced broadening of the density of states is small on the scale of the Fermi energy ϵ_F [28]. We can then restrict ourselves to the electronlike states with $s=+$ (we will drop the s index henceforth for brevity), and easily solve the SCBA equation analytically. We obtain

$$\mathrm{Im}\Sigma^R_{ntk_z} \equiv -\frac{1}{2\tau_t(k_z)} = -\frac{1}{2\tau}\left[1+\frac{m_t(k_z)\langle m_t\rangle}{\epsilon_F^2}\right], \tag{12}$$

where $1/\tau = \pi\gamma^2 g(\epsilon_F)$ and

$$g(\epsilon_F) = \frac{1}{2\pi\ell_B^2}\int_{-\pi/d}^{\pi/d}\frac{dk_z}{2\pi}\sum_{nt}\delta[\epsilon_{nt}(k_z)-\epsilon_F], \tag{13}$$

is the density of states at Fermi energy. We have also introduced the Fermi-surface average of $m_t(k_z)$ as

$$\langle m_t\rangle = \frac{1}{2\pi\ell_B^2 g(\epsilon_F)}\int_{-\pi/d}^{\pi/d}\frac{dk_z}{2\pi}\sum_{nt} m_t(k_z)\delta[\epsilon_{nt}(k_z)-\epsilon_F]. \tag{14}$$

All the necessary information about the density response of our system is contained in the diffusion propagator, or diffuson $\mathcal{D}$, given by the sum of ladder impurity-averaging diagrams [29]. This is evaluated in the standard manner and we obtain

$$\mathcal{D}^{-1}(\mathbf{q},\Omega) = 1 - I(\mathbf{q},\Omega), \tag{15}$$

where I is a 16×16 matrix, given by

$$I_{\alpha_1\alpha_2,\alpha_3\alpha_4}(\mathbf{q},\Omega) = \frac{\gamma^2}{L_xL_yL_z}\int d^3r d^3r' e^{-i\mathbf{q}\cdot(\mathbf{r}-\mathbf{r}')} \times G^R_{\alpha_1\alpha_3}(\mathbf{r},\mathbf{r}'|\Omega)G^A_{\alpha_4\alpha_2}(\mathbf{r}',\mathbf{r}|0), \tag{16}$$

where we have introduced a composite index $\alpha=(\sigma,\tau)$ to simplify the notation. The impurity-averaged Green's functions $G^{R,A}$ are given by

$$G^{R,A}_{\alpha\alpha'}(\mathbf{r},\mathbf{r}'|\Omega) = \sum_{ntk_yk_z}\frac{\langle \mathbf{r},\alpha|n,t,k_y,k_z\rangle\langle n,t,k_y,k_z|\mathbf{r}',\alpha'\rangle}{\Omega-\xi_{nt}(k_z)\pm i/2\tau_t(k_z)}, \tag{17}$$

where $\xi_{nt}(k_z) = \epsilon_{nt}(k_z)-\epsilon_F$.

In general, the evaluation of Eq. (16) is a rather complicated task, primarily due to the fact that the impurity scattering will mix different LLs. At this point we will thus specialize to the case of transport along the z direction only, as this is where we can expect the chiral anomaly to be manifest. In this case the contributions of different LLs to Eq. (16) decouple. Setting $\mathbf{q}=q\hat{z}$, we obtain

$$I_{\alpha_1\alpha_2,\alpha_3\alpha_4}(q,\Omega) = \frac{\gamma^2}{2\pi\ell_B^2 L_z} \times\sum_{ntt'k_z}\frac{\langle\alpha_1|z_n^t(k_z+q/2)\rangle\langle z_n^t(k_z+q/2)|\alpha_3\rangle}{\Omega-\xi_{nt}(k_z+q/2)+i/2\tau_t(k_z+q/2)} \times\frac{\langle\alpha_4|z_n^{t'}(k_z-q/2)\rangle\langle z_n^{t'}(k_z-q/2)|\alpha_2\rangle}{-\xi_{nt'}(k_z-q/2)-i/2\tau_{t'}(k_z-q/2)}. \tag{18}$$

As mentioned above, I and $\mathcal{D}^{-1}$ are large 16×16 matrices, which contain a lot of information of no interest to us. We are interested only in hydrodynamic physical quantities, with long relaxation times. All such quantities need to be identified, if they are expected to be coupled to each other. One such quantity is obviously the total charge density $n(\mathbf{r},t)$, which has an infinite relaxation time due to the exact conservation of the particle number. Another is the axial charge density $n_a(\mathbf{r},t)$, which, as discussed above,

may be almost conserved under certain conditions. On physical grounds, we expect no other hydrodynamic quantities to be present in our case. We are thus only interested in the 2×2 block of the matrix $\mathcal{D}^{-1}$, which corresponds to the coupled evolution of the total and the axial charge densities. To separate out this block, we apply the following transformation to the inverse diffusion matrix

$$\mathcal{D}^{-1}_{a_1 b_1, a_2 b_2} = \frac{1}{2}(\sigma^{a_1}\tau^{b_1})_{\alpha_2\alpha_1}\mathcal{D}^{-1}_{\alpha_1\alpha_2,\alpha_3\alpha_4}(\sigma^{a_2}\tau^{b_2})_{\alpha_3\alpha_4}, \quad (19)$$

where $a_{1,2}$, $b_{1,2} = 0$, x, y, z. The components of interest to us are $a_{1,2} = b_{1,2} = 0$ which corresponds to the total charge density, $a_{1,2} = 0$, $b_{1,2} = y$, which corresponds to the axial charge density, and the corresponding cross terms.

We will be interested in the hydrodynamic regime, which corresponds to low frequencies and long wavelengths, i.e., $\Omega\tau \ll 1$ and $v_F q\tau \ll 1$. We will also assume that the magnetic field is weak, so that $\omega_B \ll \epsilon_F$. Finally, we will assume that the Fermi energy is close enough to the Weyl nodes, so that only the $t = -$ states participate in transport and $\langle m_-\rangle \approx 0$, since $m_-(k_z)$ changes sign at the nodes [24].

In accordance with the above assumptions, we expand the inverse diffusion propagator to leading order in $\Omega\tau$, $v_F q\tau$ and ω_B/ϵ_F and obtain after a straightforward but lengthy calculation

$$\mathcal{D}^{-1}(q,\Omega) = \begin{pmatrix} -i\Omega\tau + Dq^2\tau & -iq\Gamma\tau \\ -iq\Gamma\tau & -i\Omega\tau + Dq^2\tau + \tau/\tau_a \end{pmatrix}. \quad (20)$$

Here $D = \tilde{v}_F^2\tau\langle m_-^2\rangle/\epsilon_F^2$ is the charge diffusion constant, associated with the diffusion in the z direction, $\Gamma = eB/2\pi^2 g(\epsilon_F)$ is the total charge-axial charge coupling coefficient, and

$$\frac{1}{\tau_a} = \frac{1-(\tilde{v}_F/\Delta_S d)^2}{(\tilde{v}_F/\Delta_S d)^2\tau} \quad (21)$$

is the axial charge relaxation rate. Several comments are in order here. First, note that the axial charge relaxation rate $1/\tau_a \geq 0$, as it should be, and vanishes when $\tilde{v}_F = \Delta_S d$. It is easy to see that this is identical to the condition of the vanishing of the commutator of the axial charge operator with the Hamiltonian Eq. (5). Henceforth we will assume that this condition is nearly satisfied so that $\tau_a \gg \tau$. Second, the situation when $\tilde{v}_F = \Delta_S d$ and thus $1/\tau_a$ appears to vanish, actually needs to be treated with some care. Namely, the condition $\tilde{v}_F = \Delta_S d$ may be satisfied exactly only in the limit $\epsilon_F \to 0$. The Fermi velocity depends on the Fermi energy as [23]

$$\tilde{v}_F(\epsilon_F) = \frac{d}{2(b+\epsilon_F)}\sqrt{[(b+\epsilon_F)^2 - b_{c1}^2][b_{c2}^2 - (b+\epsilon_F)^2]}. \quad (22)$$

When $b = \sqrt{b_{c1}b_{c2}}$ and thus $\tilde{v}_F(0) = \Delta_S d$, the Fermi energy dependence of $\tilde{v}_F$ needs to be taken into account. Expanding to leading nonvanishing order in ϵ_F we obtain in this case

$$\frac{1}{\tau_a} = \frac{\epsilon_F^2}{\Delta_S^2\tau}; \quad (23)$$

i.e., $1/\tau_a$ is in fact always finite, but may be very small. We can estimate the minimal value of the axial charge relaxation rate by setting $\epsilon_F \approx 1/\tau$ in Eq. (23), which gives $(\tau/\tau_a)_{\min} \approx 1/(\Delta_S\tau)^2$.

We may now write down the coupled diffusion equations for the total and axial charge densities, which correspond to the propagator Eq. (20). These equations read

$$\frac{\partial n}{\partial t} = D\frac{\partial^2 n}{\partial z^2} + \Gamma\frac{\partial n_a}{\partial z},$$
$$\frac{\partial n_a}{\partial t} = D\frac{\partial^2 n_a}{\partial z^2} - \frac{n_a}{\tau_a} + \Gamma\frac{\partial n}{\partial z}. \quad (24)$$

Equation (24) is our main result. Manifestation of the chiral anomaly in these equations is the coupling between the total and the axial charge densities, proportional to the applied magnetic field. Since the total particle number is conserved, the right-hand side of the first of Eqs. (24) must be equal to minus the divergence of the total particle current. Then we obtain the following expression for the density of the charge current in the z direction

$$j = -\frac{\sigma_0}{e}\frac{\partial\mu}{\partial z} - \frac{e^2 B}{2\pi^2}\mu_a, \quad (25)$$

where $\sigma_0 = e^2 g(\epsilon_F)D$ is the zero-field diagonal charge conductivity, μ and μ_a are the total and axial electrochemical potentials, and we have used $\delta n = g(\epsilon_F)\delta\mu$, $\delta n_a = g(\epsilon_F)\delta\mu_a$. The last relation is valid when $\tilde{v}_F/\Delta_S d$ is close to unity, as seen from Eq. (3). Thus the chiral anomaly manifests in an extra contribution to the charge current density, proportional to the magnetic field and the axial electrochemical potential. This is known as chiral magnetic effect (CME) in the literature [30–32]. Note that the CME contribution to the current exists only away from equilibrium [31,32]; i.e., the second term in Eq. (25) should never be interpreted as an equilibrium current, driven by a static magnetic field [33].

To find measurable consequences of the CME contribution to the charge current, we consider a steady-state situation, with a fixed current density j flowing through the sample in the z direction. We want to find the corresponding electrochemical potential drop and thus the conductivity. Assuming the current density is uniform, we obtain from the second of Eqs. (23)

$$n_a = \Gamma\tau_a\frac{\partial n}{\partial z}, \quad (26)$$

which is the nonequilibrium axial charge density, induced by the current and the corresponding electrochemical

potential gradient. Substituting this into the expression for the charge current density Eq. (25), we finally obtain the following result for the conductivity:

$$\sigma = \sigma_0 + \frac{e^4 B^2 \tau_a}{4\pi^4 g(\epsilon_F)}. \tag{27}$$

In the limit when ϵ_F is not far from the Weyl nodes, such that the dispersion may be assumed to be linear, we have $g(\epsilon_F) = \epsilon_F^2/\pi^2 v_F^2 \tilde{v}_F$, which gives

$$\Delta\sigma = \sigma - \sigma_0 = \frac{e^2 \tilde{v}_F \tau_a}{(2\pi v_F)^2} \left(\frac{e^2 v_F^2 B}{\epsilon_F}\right)^2, \tag{28}$$

which agrees with the Son and Spivak result [17]. Thus we see that a measurable consequence of CME is a positive magnetoconductivity, proportional to B^2 in the limit of a weak magnetic field. This of course needs to be compared with the classical negative magnetoconductivity, which is always present and arises from the B^2 corrections to the diffusion constant D, which we have neglected:

$$\frac{\Delta\sigma_{c\ell}}{\sigma_0} \sim -(\omega_c \tau)^2, \tag{29}$$

where $\omega_c = e v_F^2 B/\epsilon_F$ is the cyclotron frequency. This gives

$$\left|\frac{\Delta\sigma}{\Delta\sigma_{c\ell}}\right| \sim \frac{\tau_a/\tau}{(\epsilon_F \tau)^2}. \tag{30}$$

Thus the CME-related positive magnetoconductivity will dominate the classical negative magnetoconductivity, provided τ_a is long enough.

As a final comment we note that we have so far ignored the Zeeman effect due to the applied magnetic field. Its effect is to modify the spin-splitting parameter b as $b \to b + g\mu_B B/2$. In principle, the dependence on b does enter into our final results through the dependence of the Fermi velocity $\tilde{v}_F$ on b. Naively, this will then generate an additional *linear* magnetoconductivity, which may be expected to dominate the quadratic one at small fields. However, the condition of large τ_a, which is the same as $\tilde{v}_F/\Delta_S d \approx 1$, is equivalent to the condition $b_{c1} \ll b \ll b_{c2}$, in which case the dependence of $\tilde{v}_F$ on b becomes negligible. Thus, in the regime in which the positive magnetoconductivity dominates the negative classical one, and is thus observable, one may also expect a negligible linear magnetoconductivity in any type of Weyl metal.

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