

Fractional Quantum Hall Effect and Wigner Crystal of Interacting Composite Fermions

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In two-dimensional electron systems confined to GaAs quantum wells, as a function of either tilting the sample in a magnetic field or increasing density, we observe multiple spin-polarization transitions of the fractional quantum Hall states at filling factors $\nu = 4/5$ and $5/7$. The number of observed transitions provides evidence that these are fractional quantum Hall states of interacting two-flux composite fermions. Moreover, the fact that the reentrant integer quantum Hall effect near $\nu = 4/5$ always develops following the transition to full spin polarization of the $\nu = 4/5$ fractional quantum Hall state links the reentrant phase to a pinned ferromagnetic Wigner crystal of composite fermions.

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Fractional quantum Hall states (FQHSs) are among the most fundamental hallmarks of ultraclean interacting two-dimensional electron systems (2DESs) at a large perpendicular magnetic field (B_{\perp}) [1]. These incompressible quantum liquid phases, signaled by the vanishing of the longitudinal resistance (R_{xx}) and the quantization of the Hall resistance (R_{xy}), can be explained by mapping the interacting electrons to a system of essentially noninteracting, $2p$ -flux composite fermions (${}^{2p}\text{CFs}$), each formed by attaching $2p$ magnetic flux quanta to an electron (p is an integer). The ${}^{2p}\text{CFs}$ have discrete energy levels, the so-called Λ -levels, and the FQHSs of electrons seen around Landau level (LL) filling factor $\nu = 1/2$ ($1/4$) would correspond to the integer quantum Hall states (IQHSs) of noninteracting ${}^2\text{CFs}$ (${}^4\text{CFs}$) at integral ν^{CF} [2]. In state-of-the-art, high-mobility 2DESs, FQHSs also develop at unusual fillings, such as $\nu = 4/11$ [3]. These states cannot be explained in a non-interacting CF picture, and might signal the formation of fractional QHSs of interacting CFs [4]. If so, then theory also predicts that these states would possess different spin configurations and therefore should exhibit spin-polarization transitions [5]; however, such transitions have not been seen experimentally.

Here we report our extensive study of the FQHSs near $\nu = 3/4$ (at $\nu = 4/5$ and $5/7$) which are usually understood as the particle-hole counterparts of the FQHSs near $\nu = 1/4$ through the relation $\nu \leftrightarrow (1 - \nu)$ [6]. For example, the $\nu = 4/5$ FQHS would be equivalent to the FQHS seen at $\nu = 1/5$, correspond to a fully spin-polarized IQHS of noninteracting ${}^4\text{CFs}$, and exhibit no spin-polarization transitions [7]. Alternatively, these states might be the FQHSs of interacting ${}^2\text{CFs}$ at $\nu^{\text{CF}} = \nu/(1 - 2\nu)$. If this is true, then the $\nu = 4/5$ FQHS would be the $\nu^{\text{CF}} = -4/3$ FQHS of interacting ${}^2\text{CFs}$, and have the same origin as the unconventional FQHS seen at $\nu = 4/11$ ($\nu^{\text{CF}} = 4/3$) [3,4]. In our experiments, via either increasing the 2DES density or the tilt angle (θ) between the magnetic field and the sample normal, we increase the ratio of the Zeeman energy

($E_Z = |g|\mu_B B$, where B is the total magnetic field) to Coulomb energy ($V_C = e^2/4\pi\epsilon l_B$, where $l_B = \sqrt{\hbar/eB_{\perp}}$ is the magnetic length), and demonstrate that the $\nu = 4/5$ and $5/7$ FQHSs indeed undergo transitions as they become spin-polarized. The number of observed transitions, one for the FQHS at $\nu = 4/5$, and two for the FQHS at $\nu = 5/7$, is inconsistent with non-interacting ${}^4\text{CFs}$ but agrees with what is expected for polarizing the spins of interacting ${}^2\text{CFs}$, which form FQHSs at fractional CF fillings $\nu^{\text{CF}} = -4/3$ and $-5/3$.

In our Letter, we also address another hallmark of clean 2DESs at high B_{\perp} , namely the insulating phase (IP) that terminates the series of FQHSs at low fillings, near $\nu = 1/5$ [8,9]. This IP is generally believed to be a Wigner crystal, pinned by the small but ubiquitous disorder potential [10]. Recently, an IP was observed near $\nu = 4/5$ in clean 2DESs confined to relatively wide GaAs quantum wells (QWs) [11,12]. This phase, which is signaled by a reentrant IQHS (RIQHS) near $\nu = 1$, was interpreted as the particle-hole symmetric state of the Wigner crystal seen at very small ν [11,12]. In this picture, the holes, unoccupied states in the lowest LL, have filling factor $\nu^h \sim 1/5$ ($= 1 - 4/5$) and form a liquid phase when the short-range interaction is strong; see the left panel of Fig. 1(a). They turn into a solid phase when the thickness of the 2DES increases and the long-range interaction dominates [right panel of Fig. 1(a)]. This interpretation is plausible since the RIQHS only appears when the QW width (W) is more than about five times larger than the magnetic length, but such an interpretation does not predict or allow for any transitions of the $\nu = 4/5$ FQHS. However, our experiments reveal that, whenever the RIQHS near $\nu = 4/5$ develops, it is preceded by a transition of the FQHS at $\nu = 4/5$ to a fully spin-polarized ${}^2\text{CF}$ state. This provides evidence that the RIQHS is the manifestation of a ferromagnetic ${}^2\text{CF}$ Wigner crystal [see Fig. 1(b)].

We studied 2DESs confined to wide GaAs QWs bounded on each side by undoped $\text{Al}_{0.24}\text{Ga}_{0.76}\text{As}$ spacer

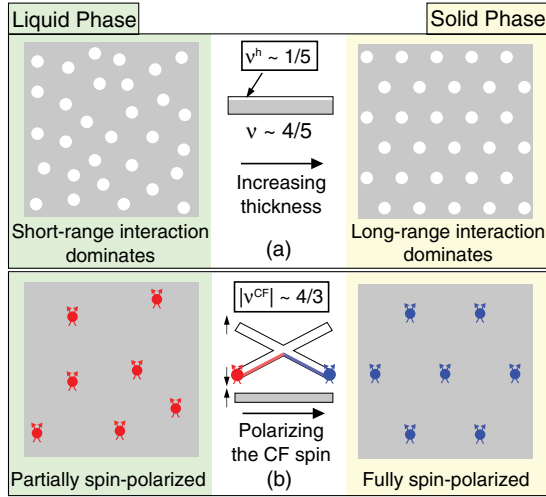


FIG. 1 (color online). Schematic figures illustrating two different explanations of the RIQHS near $\nu = 4/5$. (a) The hole Wigner crystal picture. The electrons at $\nu \sim 4/5$ are equivalent to holes at $\nu^h \sim 1/5$. These holes condense into a liquid phase when the short-range interaction dominates (left), and into a crystal phase when the long-range interaction dominates in thicker 2DESs (right). The gray background represents electrons, and white circles represent the holes. (b) The ${}^2\text{CF}$ Wigner crystal picture. The electrons at $\nu \sim 4/5$ are equivalent to ${}^2\text{CF}$ s at $|\nu^{\text{CF}}| \sim 4/3$. There is one fully filled, spin-up Λ -level (the gray background), and the rest of the ${}^2\text{CF}$ s can either be spin-down (red) and form a liquid phase when the Zeeman energy (E_Z) is small (left panel), or be spin-up (blue) at large E_Z and form a ferromagnetic crystal phase of ${}^2\text{CF}$ s (right panel).

layers and Si δ -doped layers, grown by molecular beam epitaxy. We report here data for two samples, with $W = 65$ and 60 nm, and as-grown densities of $n \approx 1.4$ and 0.4 , in units of 10^{11} cm^{-2} which we use throughout this report. The low-temperature mobility for these samples is $\approx 5 \times 10^6 \text{ cm}^2/\text{Vs}$. The samples have a van der Pauw geometry with InSn contacts at their corners, and each is fitted with an evaporated Ti/Au front gate and an In back gate. We carefully control n while keeping the charge distribution symmetric. The measurements were carried out in dilution refrigerators, and using low-frequency ($\lesssim 40$ Hz) lock-in technique.

Figure 2(a) shows R_{xx} and R_{xy} traces near $\nu = 3/4$ measured in a 65-nm-wide QW, at $n \approx 1.00$ to 1.54 . The deep R_{xx} minimum at $\nu = 4/5$ in the lowest density ($n = 1.00$) trace disappears at $n = 1.13$ and reappears at higher densities. With increasing n , an R_{xx} minimum also develops to the left of $\nu = 4/5$, and merges with the $\nu = 1$ $R_{xx} = 0$ plateau at the highest density $n = 1.54$ [see down arrows in Fig. 2(a)]. Meanwhile, when the $\nu = 4/5$ FQHS reappears, R_{xy} starts to dip below its classical Hall value on the sides of $\nu = 4/5$ [up arrows in Fig. 2(a)]. These two R_{xy} dips become deeper at higher n and, at $n \approx 1.54$, the R_{xy} minimum on the left side of $\nu = 4/5$ merges into the $\nu = 1$ $R_{xy} = h/e^2$ plateau [13].

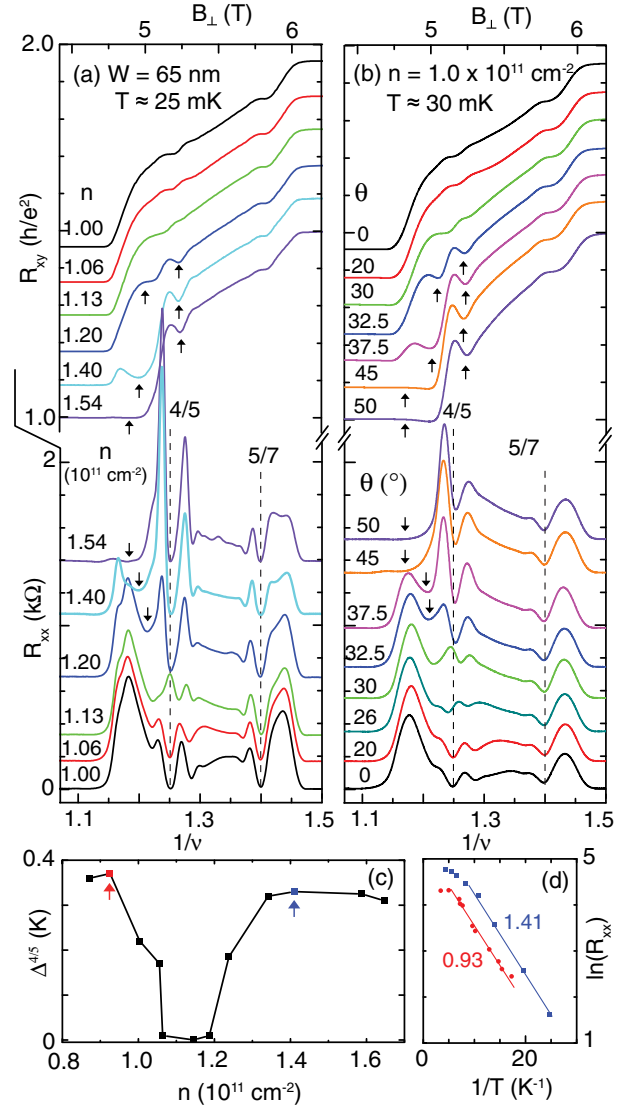


FIG. 2 (color online). Longitudinal (R_{xx}) and Hall (R_{xy}) magnetoresistance traces for 2D electrons confined to a 65-nm-wide GaAs QW near $\nu = 3/4$ as a function of (a) increasing density, and (b) tilting the sample in the magnetic field. The density n (in units of 10^{11} cm^{-2}) or tilting angle θ for each trace is indicated, and traces are shifted vertically for clarity. The top (B_{\perp}) axis in (a) is valid only for the $n = 1.00 \times 10^{11} \text{ cm}^{-2}$ trace. Note also that the angular-dependent traces in (b) were taken in a dilution refrigerator with a slightly warmer base temperature ($T \approx 30$ mK) compared to the one used to take traces in (a) ($T \approx 25$ mK). (c) Energy gap of the $\nu = 4/5$ FQHS as a function of n . (d) Arrhenius plot of R_{xx} vs $1/T$ at $n = 0.93$ and $1.41 \times 10^{11} \text{ cm}^{-2}$.

The data of Fig. 2(a) provide evidence for the development of a RIQHS between $\nu = 4/5$ and 1, as reported recently and attributed to the formation of a pinned Wigner crystal [11,12]. Note that at the onset of this development, the $\nu = 4/5$ FQHS shows a transition manifested by the disappearance and reappearance of its R_{xx} minimum. The central questions we address here are: What is the source of

this transition, and what does that imply for the origin of the $\nu = 4/5$ FQHS and the nearby RIQHS?

As Fig. 2(b) illustrates, R_{xx} and R_{xy} measured in the same QW at a fixed density $n = 1.00$ and different θ reveal an evolution very similar to the one seen in Fig. 2(a). At $\theta = 0^\circ$, a strong FQHS is seen at $\nu = 4/5$. It disappears at $\theta \approx 30^\circ$ but reappears at higher θ . Two minima in R_{xy} on the sides of $\nu = 4/5$, marked by the up arrows, develop at $\theta > 30^\circ$. As θ is further increased, the R_{xy} minimum to the left of $\nu = 4/5$ deepens and an R_{xx} minimum starts to appear at the same ν [see down arrows in Fig. 2(b)]. At the highest tilting angles $\theta > 40^\circ$, these minima merge into the $\nu = 1$ R_{xy} and R_{xx} plateaus, respectively. This evolution with increasing θ is very similar to what is seen in Fig. 2(a) as a function of increasing n . Moreover, the $\nu = 4/5$ FQHS transition in Fig. 2(b) is induced by increasing E_Z suggesting that it is spin related, similar to the spin-polarization transitions observed for other FQHSs [14–17].

Observing a spin-polarization transition for the $\nu = 4/5$ FQHS, however, is surprising as this state is usually interpreted as the particle-hole counterpart of the $\nu = 1/5$ FQHS, which is the $\nu^{\text{CF}} = 1$ of four-flux ^4CFs . Such a state should be always fully spin-polarized and no spin-polarization transition is expected. On the other hand, the $\nu = 4/5$ FQHS can be interpreted as the $\nu^{\text{CF}} = -4/3$ FQHS of interacting ^2CFs , which has two possible spin configurations, as shown in Fig. 3(c). The system has one fully occupied, spin-up, Λ -level and one $1/3$ -occupied Λ -level. Depending on whether E_Z is smaller or larger than the Λ -level separation ($\hbar\omega_C^{\text{CF}}$) of the ^2CFs , the $1/3$ -filled Λ -level may be either spin-down or spin-up [see Fig. 3(c)] [18].

To further test the validity of the above interpretation, we measured R_{xx} in a 60-nm-wide QW at $n = 0.44$ and different θ in Fig. 3(a). The $\nu = 4/5$ FQHS exhibits a clear transition at $\theta = 60^\circ$, manifested by a weakening of the R_{xx} minimum. Note that the transition of the $\nu = 4/5$ FQHS appears in Figs. 2(a), 2(b), and 3(a) when the ratio of the Zeeman to Coulomb energies (E_Z/V_C) is about 0.0145, 0.0157, and 0.0177, respectively. The electron layer-thicknesses at these three transitions, parameterized by the standard deviation (λ) of the charge distribution in units of l_B , are 1.66, 1.52, and 0.75, respectively. The softening of the Coulomb interaction due to the finite-layer-thickness effect is less and the spin-polarization energy should be higher for smaller λ/l_B (see [17] for the dependence of spin-polarization energy on the finite-layer-thickness). Therefore, these values are consistent with each other.

In Fig. 3(a), we also observe two transitions for the $\nu = 5/7$ FQHS at $\theta = 37.5^\circ$ and 50° , suggesting three different phases. This observation is consistent with the $\nu = 5/7$ FQHS being the $\nu^{\text{CF}} = -5/3$ FQHS of the interacting ^2CFs , which has three different possible spin configurations, as shown in Fig. 3(d). Similar to Fig. 3(c), the lowest spin-up Λ -level is always fully occupied. The second

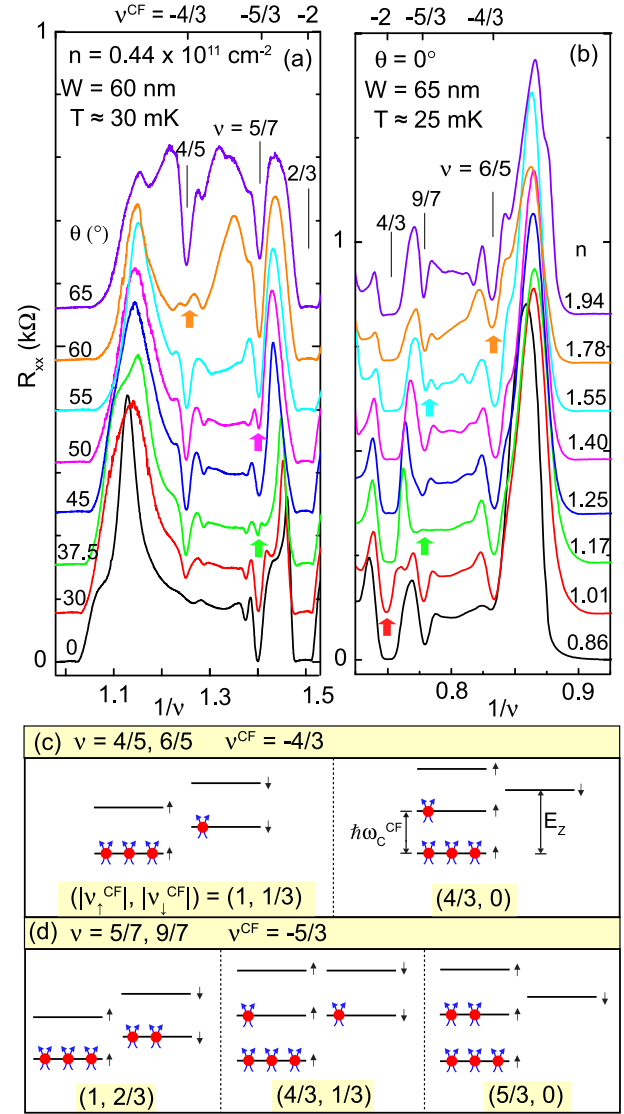


FIG. 3 (color online). (a) R_{xx} measured in a 60-nm-wide QW near $\nu = 3/4$, at a fixed density $n = 0.44 \times 10^{11} \text{ cm}^{-2}$ and different tilting angles θ . (b) R_{xx} for a 65-nm-wide QW near $\nu = 5/4$, at $\theta = 0^\circ$ and different densities $n = 0.86$ to $1.94 \times 10^{11} \text{ cm}^{-2}$. (c),(d) Schematic plots showing multiple configurations of the $\nu = 4/5$ and $6/5$, and $5/7$ and $9/7$ FQHSs with different spin-polarizations.

Λ -level is $2/3$ -occupied spin-up (spin-down), if E_Z is larger (smaller) than $\hbar\omega_C^{\text{CF}}$. If $E_Z \approx \hbar\omega_C^{\text{CF}}$, the ^2CFs form a novel spin-singlet state when the spin-up and spin-down Λ -levels are both $1/3$ -occupied; see the middle panel of Fig. 3(d).

Data near $\nu = 5/4$ measured in the 65-nm-wide QW at different densities, shown in Fig. 3(b), further confirm our picture. The $\nu = 6/5$ and $9/7$ FQHSs exhibit transitions similar to their particle-hole conjugate states at $\nu = 4/5$ and $5/7$, respectively [6]. The $\nu = 6/5$ FQHS shows a transition at $n = 1.78$. At this transition, $E_Z/V_C \approx 0.0149$ and $\lambda/l_B \approx 1.86$, very similar to the corresponding values

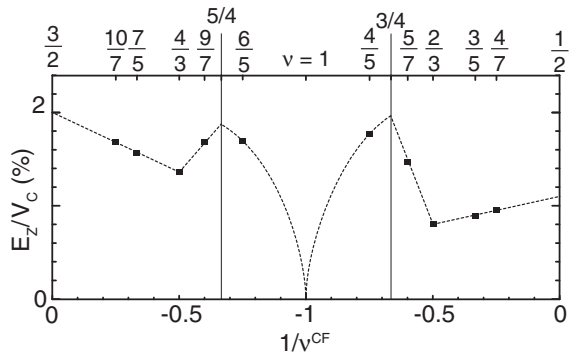


FIG. 4. Summary of the spin-polarization energy in units of the Coulomb energy, E_Z/V_C , at different filling factors (ν). Dotted lines are guides to the eye. All data points were measured in a symmetric 60-nm-QW. The transitions at $\nu = 2/3, 3/5$, and $4/5$ were measured in perpendicular magnetic field by changing n , and the rest at a fixed density $n = 0.44$ by changing θ . For each filling, only the (last) transition into a fully spin-polarized configuration is shown.

(0.0145 and 1.66) at $\nu = 4/5$ in Fig. 2(a), suggesting that the particle-hole symmetry $\nu \leftrightarrow (2 - \nu)$ is conserved in this case [19]. Furthermore, the $\nu = 9/7$ FQHS becomes weak twice, at $n = 1.17$ and 1.55 , also consistent with the $\nu = 5/7$ FQHS transitions.

It is instructive to compare the transitions we observe at fractional ν^{CF} with the spin-polarization transitions of other FQHSs at integer ν^{CF} . In Fig. 4, we summarize the critical E_Z/V_C above which the FQHSs between $\nu = 1/2$ and $3/2$ become fully spin-polarized. The measurements were all made on the 60-nm-wide QW. The x axis is $1/\nu^{\text{CF}}$, and we mark the electron LL filling factor ν in the top axis. The dotted lines, drawn as guides to the eye, represent the phase boundary between fully spin-polarized (above) and partially spin-polarized (below) ^2CFs . Note that the system is always fully spin-polarized at $\nu^{\text{CF}} = -1$ ($\nu = 1$) [20,21]. The critical E_Z/V_C of FQHSs with integral ν^{CF} increases with ν^{CF} and reaches maxima at $\nu^{\text{CF}} = -\infty$ ($\nu = 1/2$ and $3/2$). Secondary maxima in the boundaries appear at $\nu^{\text{CF}} = -3/2$ ($\nu = 3/4$ and $5/4$), and seem to have approximately the same height as at $\nu = 1/2$ and $3/2$.

While Fig. 3 data strongly suggest that we are observing spin transitions of various FQHSs, there is also some theoretical justification. It has been proposed that the enigmatic FQHSs observed at $\nu = 4/11$ and $5/13$ in the highest quality samples can be interpreted as the FQHSs of interacting ^2CFs at $\nu^{\text{CF}} = +4/3$ and $+5/3$ [3,4]. A recent theoretical study predicts a transition of the $\nu = 4/11$ FQHS to full spin polarization in an ideal zero-thickness 2DES when E_Z/V_C is about 0.025 [5]. Our observed transition of the $\nu = 4/5$ ($\nu^{\text{CF}} = -4/3$) FQHS appears at $E_Z/V_C \approx 0.015$ to 0.024 , in different QWs with well widths ranging from 65 to 31 nm and corresponding $\lambda/l_B \approx 1.7$ to 1.1 [11], consistent with this theoretically predicted value.

Another useful parameter in characterizing the origin of the $\nu = 4/5$ FQHS and its transition are the energy gaps on the two sides of the transition. We show in Fig. 2(c) the measured excitation gaps for this state at different densities in the 65-nm-wide QW. The $\nu = 4/5$ FQHS transition for this sample occurs at density $n \approx 1.1$; see Fig. 2(a). Before and after the transition, e.g. at $n = 0.86$ and 1.64 , the $\nu = 4/5$ FQHS has very similar energy gaps (~ 0.35 K) although the densities are different by nearly a factor of two. Since the FQHS energy gaps at a given filling are ordinarily expected to scale with $V_C \sim \sqrt{n}$, this observation suggests that the excitation gap at $\nu = 4/5$ is reduced when the FQHS becomes fully spin-polarized.

Finally we revisit the RIQHSs we observe near $\nu = 4/5$ [see, e.g., Figs. 1(a) and 1(b)]. These RIQHSs were interpreted as pinned Wigner crystals [11], and the recent microwave resonance experiments confirm this interpretation [12]. Moreover, the data in Ref. [11] as well as the data we have presented here all indicate that, whenever a transition to a RIQHS occurs, it is initiated by a transition of the $\nu = 4/5$ FQHS. As we have shown here, the transition we see for the $\nu = 4/5$ FQHS is a transition to a fully spin-polarized state of interacting ^2CFs . Combining these observations leads to a tantalizing conclusion: The RIQHS near $\nu = 4/5$ is a pinned, ferromagnetic Wigner crystal of ^2CFs , as schematically illustrated in Fig. 1(b).

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