

Nuclear Matter from Effective Quark-Quark Interaction

M. Baldo and K. Fukukawa

INFN, Sezione di Catania, via Santa Sofia 64, I-95123 Catania, Italy

(Received 15 September 2014; published 9 December 2014)

We study neutron matter and symmetric nuclear matter with the quark-meson model for the two-nucleon interaction. The Bethe-Bruckner-Goldstone many-body theory is used to describe the correlations up to the three hole-line approximation with no extra parameters. At variance with other nonrelativistic realistic interactions, the three hole-line contribution turns out to be non-negligible and to have a substantial saturation effect. The saturation point of nuclear matter, the compressibility, the symmetry energy, and its slope are within the phenomenological constraints. Since the interaction also reproduces fairly well the properties of the three-nucleon system, these results indicate that the explicit introduction of the quark degrees of freedom within the considered constituent quark model is expected to reduce the role of three-body forces.

DOI: [10.1103/PhysRevLett.113.242501](https://doi.org/10.1103/PhysRevLett.113.242501)

PACS numbers: 21.65.-f, 13.75.Cs, 21.30.-x, 21.65.Cd

Introduction.—Understanding the properties of the nuclear medium on the basis of the bare interaction among nucleons is one of the fundamental issue of nuclear physics. Several methods have been used to model the nucleon interaction and to develop accurate many-body theory to describe the correlations in nuclear systems. It has been established [1–3] that realistic nucleon-nucleon (NN) potentials based on the meson-exchange interaction model fail to reproduce the correct saturation point and require the introduction of three-body forces (TBFs). The latter can be phenomenological [4,5] or more fundamental [6,7]. In any case, within this framework, the effect of TBFs is moderate, but it is essential to shift the saturation point inside the phenomenological boundaries. The main problem of this approach is that it appears difficult to devise a TBF that describes satisfactorily well few-body systems and at the same time nuclear matter near saturation [8]. The same conclusion has been reached within the variational many-body method [9]. In this types of nuclear forces, nonlocal two-body interactions [10] have been constructed that reproduce closely the binding energy of three and four nuclear systems. However, they fail to reproduce the correct saturation point [11]. The Dirac-Brueckner-Hartree-Fock (DBHF) method introduces relativistic effects in the many-body theory. With only two-body forces and two-body correlations, the saturation point is fairly well reproduced [12], but the problem of the few-body systems remains unsolved. It can also be shown [13] that the relativistic effects introduced by the DBHF scheme are equivalent to a particular TBF at the nonrelativistic level. More recently, the chiral effective forces have been developed [14,15]. These forces were devised to connect the underlying quantum chromodynamics theory of strong interaction among quarks to the low energy interaction among nucleons. However, no explicit quark degrees of freedom are introduced, but it is based on the expansion of the

interaction in the chiral symmetry breaking parameter, i.e., the ρ -meson mass m_ρ . As such, it is an expansion in k/m_ρ , where k is the typical nucleon momentum. It has the fundamental property to classify the forces according to the expected relevance, following a “power counting” rule. Three-body (or higher) forces arise naturally in this expansion. Even if the assignment of the order to the different interaction processes looks tricky [16], this approach has been developed both at the fundamental level [17–19] and in a wealth of applications to nuclei [20] and nuclear matter [21–24]. The parameters of the forces are fixed by fitting the NN phase shifts and eventually the properties of the three-body system. However, the procedure does not look unique. In Ref. [25], it has been shown that it is possible to construct a realistic chiral two-body force that reproduces the spectroscopic data on light nuclei without invoking TBFs. However, symmetric nuclear matter was not considered. In Ref. [26], it has been shown that a version of chiral force was able to reproduce, by a suitable choice of the momentum cutoff parameter, the few-body binding energies and at the same time a fair saturation point. However, this remarkable result needs confirmation, since two- and three-body forces were actually taken at different orders, and some correlation diagrams were neglected. In any case, for all chiral forces, TBFs are the dominant mechanism for saturation. Indeed, with only two-body forces, no saturation is apparent in nuclear matter. NN potentials based on the constituent quark model have been developed for some decades, since the resonating-group-method (RGM) equations were first solved by Oka and Yazaki [27]. In this model, the quark degrees of freedom are explicitly introduced, and the NN potential is derived from the quark-quark (qq) interactions. The resulting interaction is highly nonlocal due to the RGM formalism and contains a natural cutoff in momentum. Realistic quark-model (QM) interactions were proposed by Fujiwara *et al.* [28], in which

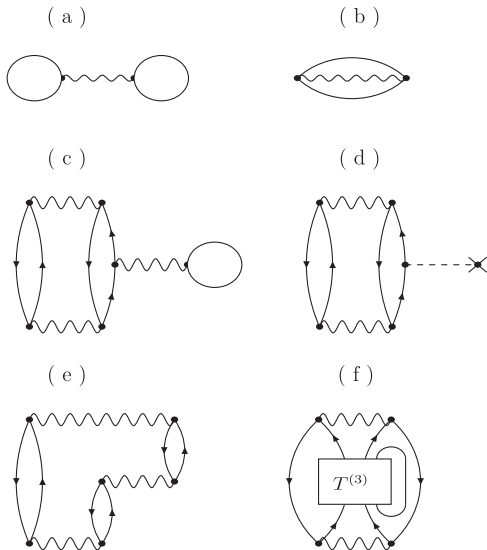


FIG. 1. Different Goldstone diagrams contributing to the nuclear matter equation of state (EOS). The wavy line indicates Brueckner G -matrix. Diagrams corresponding to the two hole-line approximation are labeled (a) and (b). Diagram (d) is the first potential insertion in the hole-line expansion. Diagram (e) is the ring diagram, which includes long range correlations. The box labelled $T^{(3)}$ is the in-medium three-body scattering matrix. Diagram (c), usually indicated as "bubble diagram", is the first term of the set of diagrams obtained once the expansion of $T^{(3)}$ is inserted in (f) and it has been singled out for numerical convenience.

the qq interaction consists of a color analog of the Fermi-Breit interaction and an effective meson-exchange potential. The most recent model fss2 [29] reproduces the experimental data on the few-body systems (triton, hypertriton [30], and α particle [31] and nucleon-deuteron scattering [32]) fairly well without introducing TBFs. In this Letter, we present results for nuclear matter obtained with the QM force within the Bethe-Brueckner-Goldstone (BBG) many-body expansion up to the three hole-line level of approximation. A pedagogical introduction to this many-body method can be found in Ref. [33]. The energy dependence inherent to the RGM formalism is eliminated by the off-shell transformation utilizing the norm kernel as in Refs. [34,35] and the Gaussian representation of fss2

[36] is used. The set of Goldstone diagrams that are used in the calculations is reported in Fig. 1. The BBG expansion classifies the diagrams according to the number of hole lines that they contain. Figures 1(a) and 1(b) (direct and exchange) include two hole lines, and they correspond to the well-known Brueckner-Hartree-Fock (BHF) approximation. The wavy line indicates the Brueckner G matrix [33]. In the BBG expansion, an auxiliary single-particle potential $U(\mathbf{k})$ is introduced and calculated self-consistently according to the Brueckner prescription. However, the auxiliary potential is not unique. Two somehow opposite choices are possible. In the so-called "standard" or gap choice (GC), the potential is assumed to be zero above the Fermi momentum, while in the "continuous" choice (CC), the potential is calculated self-consistently also above the Fermi momentum. In principle, the final result should be independent of U , which is introduced in order to rearrange the perturbation series for a faster convergence. The comparison of the result obtained with the gap and the continuous choice can be used to estimate the degree of convergence of the expansion [2,3]. The rearrangement of the expansion is embodied in a series of U -insertion" diagrams. The first one is Fig. 1(d), while Fig. 1(c) is a self-energy insertion. Notice that the various self-energy or U -insertion diagrams must follow from the BBG expansion; otherwise, an arbitrary number of insertions would spoil the hole-line ordering of the diagrams. Figure 1(e) is generally indicated as a "ring diagram." It describes long-range correlations in the matter. Figure 1(f) describes the full scattering process of three particles that are virtually excited above the Fermi sphere, and its evaluation requires the solution of the Bethe-Faddeev equations [1,33,37]. The sum of Figs. 1(c)–1(f) gives the three hole-line contribution, which, according to the BBG hole-line expansion, is expected to be substantially smaller than the two hole-line (Brueckner) contribution.

Results.—The results for symmetric matter (SM) in the CC are reported in Table I for a set of values of Fermi momenta around saturation, with the breakdown of the contributions of each diagram.

The last column reports the final EOS obtained summing up the two hole- and three hole-line contributions. In the column before the last, the total contribution of the three

TABLE I. Three hole-line contributions to the symmetric matter EOS for different Fermi momenta k_F in fm^{-1} . E_3 is the total three hole-line contribution, B is the "bubble diagram" of Fig. 1(c), BU is the U -insertion diagram of Fig. 1(d), R is the "ring diagram" of Fig. 1(e), and H indicates the "higher order" diagrams, as defined in the text. Energies are in MeV.

k_F	$T + E_2$	B	BU	R	H	E_3	EOS
1.1	-17.090	-7.020	10.655	-0.750	0.177	3.072	-14.028
1.2	-19.680	-6.351	11.407	-1.270	0.157	3.943	-15.737
1.3	-22.154	-4.669	11.647	-1.761	0.144	5.361	-16.793
1.4	-24.393	-2.689	12.340	-2.030	-0.079	7.542	-16.851
1.5	-26.183	0.223	12.781	-2.122	0.050	11.022	-15.161
1.6	-27.498	4.162	13.759	-2.280	0.029	15.670	-11.828

TABLE II. The same as in Table I but in the gap choice for the single-particle potential.

k_F	$T + E_2$	B	R	H	E_3	EOS
1.1	-11.605	-0.556	-1.003	0.063	-1.496	-13.101
1.2	-13.525	-0.029	-1.119	0.040	-1.108	-14.633
1.3	-15.439	0.846	-1.251	0.033	-0.372	-15.721
1.4	-16.959	2.213	-1.301	0.021	0.933	-16.026
1.5	-18.212	4.234	-1.296	0.012	2.575	-15.272
1.6	-18.974	7.233	-1.328	0.006	5.911	-13.063

hole-line diagrams is reported. Table II is similar but for the gap choice. Notice that in this case, the U -insertion Fig. 1(d) vanishes. The comparison between the two sets of results for the EOS of SM is summarized in Fig. 2 for a wider range of density. One can see that at the Brueckner (two hole lines) level of approximation, the continuous and gap choices differ by a few MeV, being the continuous one more attractive. However, as the three hole-line contribution is added, the two EOSs are quite close. The discrepancy around saturation is not exceeding 1 MeV, and it is vanishing small just at saturation. We consider this result as a strong indication of the convergence of the BBG expansion. However, at high density the discrepancy tends to increase, which indicates a lower degree of convergence, but no divergence of the expansion is really apparent. It has to be noticed that for other NN interactions, in particular, the Argonne v_{18} potential and the corresponding simplified versions v_8 , v_6 , and v_4 , the three hole-line contribution is much smaller in the CC than in the GC [38]. For the present QM potential, it is the opposite; in the gap choice, the convergence of the energy looks faster. One can notice that around saturation, the ratio between the three hole-line correlation energy and the two hole-line one (BHF) is 0.15 for the CC and 0.02 for the GC. This is in line with the expectation of the hole-line expansion and supports the validity of the BBG expansion. Notice that the second columns of Tables I and II include the free kinetic energy T .

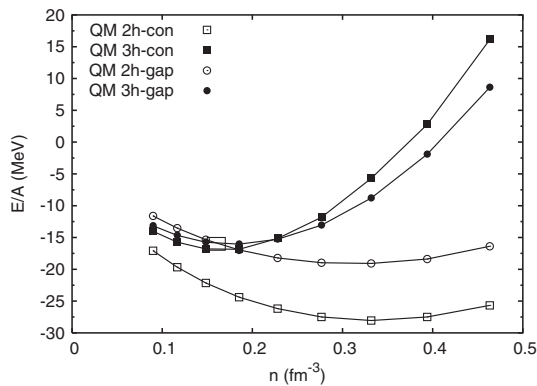


FIG. 2. EOS of symmetric nuclear matter at the two-hole level of approximation (open symbols) and at the three hole-line level (full symbols) for the continuous (squares) and the gap (circles) choices, respectively.

If the EOSs around saturation are fitted with a form of the type $E/A = a\rho + b\rho'$, the saturation point turns out to be $e_0 = -16.9$ MeV and $\rho_0 = 0.166$ fm $^{-3}$ for the CC and $e_0 = -16.06$ MeV and $\rho_0 = 0.177$ fm $^{-3}$ for the GC. This establishes the range of the uncertainty on the predicted saturation point and of the whole EOS in the considered density range. From the same fits, one can extract the compressibility at saturation, which turns out to be $K = 228$ MeV for the CC and $K = 192$ MeV for the GC. These values can be considered compatible with the range encompassed by the phenomenological constraints [39], the GC value being at the lower edge. A similar analysis can be performed for pure neutron matter (PNM). The corresponding EOSs for the CC and GC are reported in Fig. 3.

For comparison, two EOSs, which include three-body forces, are also reported, one from the BHF approach [5] and one from the variational method [9]. A blowup of the low density region is reported in Fig. 4, where, in addition, the EOS obtained from the chiral force approach of Ref. [40] is reported. The region enclosed inside the thick full line indicates the allowed area where the EOS should pass through according to the chiral approach of Ref. [41]. The relatively close agreement among the different approaches shows that the separation between two-body and three-body forces, as well as the relevance of higher order correlations, are substantially model dependent. The

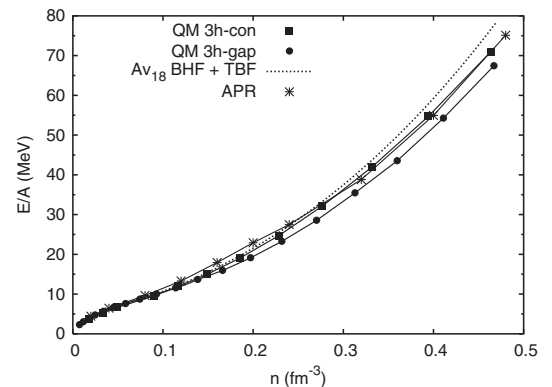


FIG. 3. The EOS of pure neutron matter at the three hole-line level of approximation with the QM potential in the continuous (full squares) and gap (full circles) choices. For comparison, one BHF EOS from Ref. [5] (dashed line) and the one of Ref. [9] (stars) are also reported. The latter two include three-body forces.

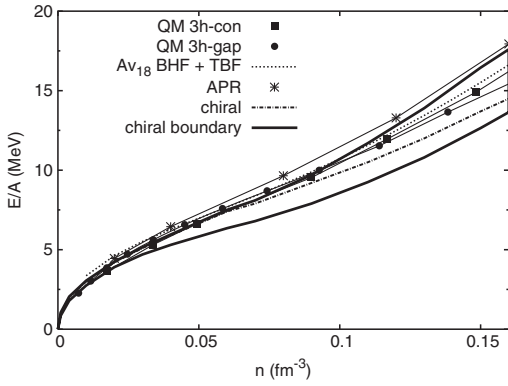


FIG. 4. Blowup of Fig. 3 in the low density region. For comparison, the EOS from the chiral expansion of Ref. [40] (dashed-dotted line) has been reported. The area delimited by the thick full line indicates the region where the EOS predicted from the chiral approach of Ref. [41] should be enclosed.

symmetry energy $S(\rho)$ can be then extracted as the difference between the PNM and SM, which is valid for a quadratic dependence of the EOS on asymmetry. The fits give also the derivative of asymmetry at saturation, as embodied in the parameter $L = 3\rho(\partial S/\partial\rho)$. One finds $S_0 = 34$ MeV and $L = 54$ MeV for the CC, and $S_0 = 33.7$ MeV and $L = 53$ MeV for the GC, again compatible with phenomenology [39].

Discussion.—The microscopic EOS obtained from the QM interaction compares well with phenomenological constraints, at variance with modern NN meson-exchange interaction models. Since the interactions are phase equivalent, the reason of the discrepancy must be due to the different off-shell behavior of the QM interaction. In particular, this can be related to the characteristic non-locality of the repulsive core [32] as produced by the quark-exchange processes. Notice that the presence of nonlocality not necessarily improves the saturation point [11]. We also find that at the BHF level, the contribution of the ${}^3S_1 - {}^3D_1$ channel is much larger than for the other interactions, e.g., the Av_{18} . This large contribution is responsible almost completely for the too-attractive EOS in BHF, whose saturation point is well outside the Coester band. The larger three hole-line contribution for QM than for Av_{18} is qualitatively in line with the trend of the corresponding values of the wound parameters at saturation, 0.072 and 0.037, respectively. However, a large contribution (0.019) to the wound parameter comes from the ${}^3S_1 - {}^3D_1$ channel in Av_{18} , while this is quite small for QM (0.009), probably due to the softer short-range repulsion. A comparison with Av_{18} shows that the saturation mechanism with the QM is the steeper increase with density of the three hole-line contribution. This is due to two reasons: (i) The larger value of the U -insertion diagrams and its increase with density, while for Av_{18} it is decreasing, and (ii) the very small values of the higher order diagrams, which in Av_{18} are relevant and negative.

The need of three-body forces with the QM interaction seems to be reduced to a minimum. The consistency of this conclusion could be checked by deriving explicitly the three-body forces from the same quark model, which is left to a future long-term project.

Partial support from NewCompStar, COST Action MP1304 is gratefully acknowledged.

-
- [1] B. D. Day, *Phys. Rev. C* **24**, 1203 (1981); *Phys. Rev. Lett.* **47**, 226 (1981).
 - [2] H. Q. Song, M. Baldo, G. Giansiracusa, and U. Lombardo, *Phys. Rev. Lett.* **81**, 1584 (1998).
 - [3] M. Baldo, G. Giansiracusa, U. Lombardo, and H. Q. Song, *Phys. Lett. B* **473**, 1 (2000); M. Baldo, A. Fiasconaro, H. Q. Song, G. Giansiracusa, and U. Lombardo, *Phys. Rev. C* **65**, 017303 (2001).
 - [4] J. Carlson, V. J. Pandharipande, and R. B. Wiringa, *Nucl. Phys.* **A401**, 59 (1983); R. Schiavilla, V. J. Pandharipande, and R. B. Wiringa, *Nucl. Phys.* **A449**, 219 (1986).
 - [5] G. Taranto, M. Baldo, and G. F. Burgio, *Phys. Rev. C* **87**, 045803 (2013).
 - [6] P. Grange, A. Lejeune, M. Martzloff, and J.-F. Mathiot, *Phys. Rev. C* **40**, 1040 (1989); W. Zuo, A. Lejeune, U. Lombardo, and J.-F. Mathiot, *Nucl. Phys.* **A706**, 418 (2002); *Eur. Phys. J. A* **14**, 469 (2002).
 - [7] Z. H. Li, U. Lombardo, H.-J. Schulze, W. Zuo, L. Chen, and H. Ma, *Phys. Rev. C* **74**, 047304 (2006).
 - [8] S. C. Pieper and R. B. Wiringa, *Annu. Rev. Nucl. Part. Sci.* **51**, 53 (2001).
 - [9] A. Akmal, V. R. Pandharipande, and D. G. Ravenhall, *Phys. Rev. C* **58**, 1804 (1998).
 - [10] P. Doleschall and I. Borbely, *Phys. Rev. C* **62**, 054004 (2000); P. Doleschall, I. Borbely, Z. Papp, and W. Plessas, *Phys. Rev. C* **67**, 064005 (2003); P. Doleschall, *Phys. Rev. C* **69**, 054001 (2004); R. Lazauskas and J. Carbonell, *Phys. Rev. C* **70**, 044002 (2004).
 - [11] M. Baldo and C. Maieron, *Phys. Rev. C* **72**, 034005 (2005).
 - [12] T. Gross-Boelting, C. Fuchs, and A. Faessler, *Nucl. Phys.* **A648**, 105 (1999).
 - [13] G. E. Brown, W. Weise, G. Baym, and J. Speth, *Comments Nucl. Part. Phys.* **17**, 39 (1987).
 - [14] S. Weinberg, *Phys. Lett. B* **251**, 288 (1990); *Nucl. Phys.* **B363**, 3 (1991); *Phys. Lett. B* **295**, 114 (1992); *Phys. Rev.* **166**, 1568 (1968).
 - [15] D. R. Entem and R. Machleidt, *Phys. Rev. C* **68**, 041001 (2003).
 - [16] M. P. Valderrama and D. R. Phillips, arXiv:1407.0437.
 - [17] H. Leutwyler, *Ann. Phys. (N.Y.)* **235**, 165 (1994).
 - [18] Ulf-G. Meissner, *Nucl. Phys.* **A751**, 149 (2005).
 - [19] E. Epelbaum, H. W. Hammer, and Ulf-G. Meissner, *Rev. Mod. Phys.* **81**, 1773 (2009).
 - [20] T. Otsuka, T. Suzuki, J. D. Holt, A. Schwenk, and Y. Akaishi, *Phys. Rev. Lett.* **105**, 032501 (2010); J. D. Holt, J. Menendez, and A. Schwenk, *Phys. Rev. Lett.* **110**, 022502 (2013).
 - [21] K. Hebeler, S. K. Bogner, R. J. Furnstahl, A. Nogga, and A. Schwenk, *Phys. Rev. C* **83**, 031301 (2011).

- [22] C. Drischler, V. Soma, and A. Schwenk, *Phys. Rev. C* **89**, 025806 (2014).
- [23] K. Hebeler and A. Schwenk, *Phys. Rev. C* **82**, 014314 (2010).
- [24] A. Carbone, A. Polls, and A. Rios, *Phys. Rev. C* **88**, 044302 (2013).
- [25] A. Ekström *et al.*, *Phys. Rev. Lett.* **110**, 192502 (2013).
- [26] L. Coraggio, J. W. Holt, N. Itaco, R. Machleidt, L. E. Marcucci, and F. Sammarruca, *Phys. Rev. C* **89**, 044321 (2014).
- [27] M. Oka and K. Yazaki, *Phys. Lett. B* **90**, 41 (1980); *Prog. Theor. Phys.* **66**, 556 (1981); *Prog. Theor. Phys.* **66**, 572 (1981); in *Quarks and Nuclei*, edited by W. Weise (World Scientific, Singapore, 1984), p. 489.
- [28] Y. Fujiwara, Y. Suzuki, and C. Nakamoto, *Prog. Part. Nucl. Phys.* **58**, 439 (2007), and references therein.
- [29] Y. Fujiwara, M. Kohno, C. Nakamoto, and Y. Suzuki, *Phys. Rev. C* **64**, 054001 (2001); Y. Fujiwara, T. Fujita, M. Kohno, C. Nakamoto, and Y. Suzuki, *Phys. Rev. C* **65**, 014002 (2001).
- [30] Y. Fujiwara, Y. Suzuki, M. Kohno, and K. Miyagawa, *Phys. Rev. C* **77**, 027001 (2008).
- [31] Y. Fujiwara, *Few-Body Syst.* **55**, 993 (2014).
- [32] Y. Fujiwara and K. Fukukawa, *Few-Body Syst.* **54**, 2357 (2013).
- [33] M. Baldo, in *Nuclear Methods and the Nuclear Equation of State*, edited by M. Baldo, International Review of Nuclear Physics Vol. 8 (World Scientific, Singapore, 1999), p. 1.
- [34] Y. Suzuki, H. Matsumura, M. Orabi, Y. Fujiwara, P. Descouvemont, M. Theeten, and D. Baye, *Phys. Lett. B* **659**, 160 (2008).
- [35] Y. Fujiwara and K. Fukukawa, *Prog. Theor. Phys.* **124**, 433 (2010).
- [36] K. Fukukawa, Y. Fujiwara, and Y. Suzuki, *Mod. Phys. Lett. A* **24**, 1035 (2009).
- [37] R. Rajaraman and H. Bethe, *Rev. Mod. Phys.* **39**, 745 (1967).
- [38] M. Baldo, A. Polls, A. Rios, H.-J. Schulze, and I. Vidaña, *Phys. Rev. C* **86**, 064001 (2012).
- [39] M. Dutra, O. Lourenço, J. S. Sá Martins, A. Delfino, J. R. Stone, and P. D. Stevenson, *Phys. Rev. C* **85**, 035201 (2012).
- [40] L. Coraggio, J. W. Holt, N. Itaco, R. Machleidt, and F. Sammarruca, *Phys. Rev. C* **87**, 014322 (2013).
- [41] I. Tews, T. Krüger, K. Hebeler, and A. Schwenk, *Phys. Rev. Lett.* **110**, 032504 (2013).