Coherent Quantum Dynamics in Steady-State Manifolds of Strongly Dissipative Systems

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Recently, it has been realized that dissipative processes can be harnessed and exploited to the end of coherent quantum control and information processing. In this spirit, we consider strongly dissipative quantum systems admitting a nontrivial manifold of steady states. We show how one can enact adiabatic coherent unitary manipulations, e.g., quantum logical gates, inside this steady-state manifold by adding a weak, time-rescaled, Hamiltonian term into the system's Liouvillian. The effective long-time dynamics is governed by a projected Hamiltonian which results from the interplay between the weak unitary control and the fast relaxation process. The leakage outside the steady-state manifold entailed by the Hamiltonian term is suppressed by an environment-induced symmetrization of the dynamics. We present applications to quantum-computation in decoherence-free subspaces and noiseless subsystems and numerical analysis of nonadiabatic errors.

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Introduction.—Weak coupling to the environmental degrees of freedom is often regarded as one of the essential prerequisites for realizing quantum-information processing. In fact, decoherence and dissipation generally spoil the unitary character of the quantum dynamics and induce errors into the computational process. In order to overcome such an obstacle, a variety of techniques have been devised including quantum error correction [1], decoherence-free subspaces (DFSs) [2,3], and noiseless subsystems [4,5]. However, recently, it has been realized that dissipation and decoherence may even play a positive role to the aim of coherent quantum manipulations. Indeed, it has been shown that, properly engineered, dissipative dynamics can, in principle, be tailored to enact quantum-information primitives such as quantum state preparation [6], quantum simulation [7,8], and computation [9].

In this Letter, we investigate the regime where the coupling of the system to the environment is very strong, and the open system dynamics admits a nontrivial steady-state manifold (SSM). We will show how, in the long time limit, unitary manipulations, e.g., quantum gates, inside the SSM can be enacted by adding a time-rescaled Hamiltonian acting on the system only. This coherent dynamics is governed by a sort of projected Hamiltonian which results from the nontrivial interplay between the weak unitary control term and the strong dissipative process. The latter effectively renormalizes the former by continuously projecting the system onto the steady-state manifold and adiabatically decoupling the nonsteady states. Several of the results of this Letter can be regarded as a rigorous formulation and significant extension of ideas first explored in [10] and [11]. We would also like to point out the relation to techniques relying on some type of quantum Zeno dynamics [8,12–14]. The latter can, in fact, be regarded as a special case of our general result (3).

This Letter is organized as follows: First, we set the stage of our analysis and describe the main theoretical ideas and results. Then, we discuss in detail, aided by numerical simulations, a few different models demonstrating dissipation-assisted computation over SSMs comprising decoherence-free subspaces and noiseless subsystems. For the reader's convenience, we have collected background technical material and all the mathematical proofs in [15].

Evolution of steady state manifolds.—In the following, $\mathcal{H}\,[\dim(\mathcal{H})<\infty]$ will denote the Hilbert space of the system and $L(\mathcal{H})$ the algebra of linear operators over it. A time-independent Liouvillian superoperator $\mathcal{L}_0$ acting on $L(\mathcal{H})$ is given. The SSM of $\mathcal{L}_0$, comprises all the quantum states ρ contained in the kernel $\mathrm{Ker}\mathcal{L}_0 := \{X/\mathcal{L}_0(X)=0\}$ of $\mathcal{L}_0$. We will denote by $\mathcal{P}_0$ ($\mathcal{Q}_0 := 1-\mathcal{P}_0$) the spectral projection over $\mathrm{Ker}\mathcal{L}_0$ (the complementary subspace of $\mathrm{Ker}\mathcal{L}_0$.). One has that $\mathcal{P}_0^2=\mathcal{P}_0$ and $\mathcal{P}_0\mathcal{L}_0=\mathcal{L}_0\mathcal{P}_0=0$; notice, also, that $\mathcal{P}_0$ may not be Hermitian. The Liouvillian $\mathcal{L}_0$ is also assumed to be such that: (a) the equation $\mathcal{E}_t^{(0)} := e^{t\mathcal{L}_0}$, $(t\geq 0)$ defines a semigroup of trace-preserving positive maps with $\|\mathcal{E}_t^{(0)}\|\leq 1$ [16]; (b) The nonzero eigenvalues λ_h, $(h>0)$ of $\mathcal{L}_0$ have negative real parts, i.e., the SSM is attractive. In this case, $\mathcal{P}_0=\lim_{t\to\infty}\mathcal{E}_t^{(0)}$.

On top of the process described by $\mathcal{L}_0$, we now add a control Hamiltonian term $\mathcal{K} := -i[K,\cdot]$, where $K=K^\dagger=T^{-1}\tilde{K}$. The time T is a scaling parameter that, in the spirit of the adiabatic theorem, will be eventually sent to infinity. If $\|\tilde{K}\|=O(1)$, then $\|\mathcal{K}\|\leq 2\|K\|=O(1/T)$. The basic dynamical equation we are going to study is the following:

$$\frac{d\rho(t)}{dt} = (\mathcal{L}_0 + \mathcal{K})\rho(t) =: \mathcal{L}\rho(t). \tag{1}$$

Notice that, even if we are not assuming that $\mathcal{L}_0$ is of the Lindblad type [17], i.e., the $\mathcal{E}_t := e^{t\mathcal{L}}$ being completely positive (CP) maps, our basic equation (1) is set time-local and, in this sense, Markovian. The system is also strongly dissipative in the sense that, for large T, the dominant process is the one ruled by $\mathcal{L}_0$. If the system is initialized in one of its steady states, on general physical grounds one expects the system, for small $1/T$, to stay within the SSM with high probability. However, for $\mathcal{L}_0$ with a multidimensional SSM, a nontrivial internal dynamics may unfold.

In order to gain physical insight on this phenomenon, we would, first, like to provide a simple argument based on time-dependent perturbation theory. Equation (1) immediately leads to $\dot{\mathcal{E}}_t = (\mathcal{L}_0 + \mathcal{K})\mathcal{E}_t$ for the evolution semigroup. We can formally solve this equation by $\mathcal{E}_t = e^{t\mathcal{L}_0}(1 + \int_0^t d\tau e^{-\tau\mathcal{L}_0}\mathcal{K}\mathcal{E}_\tau)$ from which, by iteration, it follows the standard Dyson expansion with respect to the perturbation $\mathcal{K}$. Considering terms up to the first order applied to $\mathcal{P}_0$ and inserting the spectral resolution $\mathcal{P}_0 + \mathcal{Q}_0 = \mathbf{1}$, one obtains $\mathcal{P}_0 + t\mathcal{P}_0\mathcal{K}\mathcal{P}_0 + (e^{t\mathcal{L}_0} - 1)\mathcal{S}\mathcal{K}\mathcal{P}_0$, where $\mathcal{S} := -\int_0^\infty dt e^{t\mathcal{L}_0}\mathcal{Q}_0$ is a pseudoinverse of $\mathcal{L}_0$, i.e., $\mathcal{L}_0\mathcal{S} = \mathcal{S}\mathcal{L}_0 = \mathcal{Q}_0$. The norm of the third term is upper bounded by $O(||\mathcal{K}||||\mathcal{S}||)$ uniformly in $t \in [0, \infty)$. It follows that scaling $\mathcal{K}$ by T^{-1}, over a total evolution time $t = T$ the first and second term above are $O(1)$, while the third one—the only one involving transitions outside the steady state manifold—is $O(||\mathcal{S}||/T)$. This demonstrates that, at this order of the Dyson expansion, the dynamics is ruled by an effective generator $\mathcal{P}_0\mathcal{K}\mathcal{P}_0$ whose emergence is basically due to a Fermi golden rule mechanism. Moreover, by looking at the structure of the Liouvillian pseudoinverse $\mathcal{S}$ [15], we see that $||\mathcal{S}|| = O(\tau_R)$, where $\tau_R^{-1} := \min_{h>0}|\Re\lambda_h|$, and the λ_h's are the nonvanishing eigenvalues of $\mathcal{L}_0$ [15]. The meaning of this quantity is that the time scale τ_R sets a lower bound to the relaxation time of the irreversible process described by $\mathcal{L}_0$. Since $||\tilde{K}|| = O(1)$, if no nilpotent blocks are present in the spectral resolution of $\mathcal{L}_0$ [18], the leakage outside the SSM becomes negligible when

$$T \gg \tau_R, \tag{2}$$

namely, when the timescale T is much longer that the relaxation time τ_R, i.e., dissipation is much faster than the coherent part of the dynamics. System specific examples of (2) will be given later when concrete applications are discussed.

Now, we present our main technical result on the projected dynamics over SSMs (see [15] for the proof's details)

$$||\mathcal{E}_T\mathcal{P}_0 - e^{\tilde{\mathcal{K}}_{\text{eff}}}\mathcal{P}_0|| = O(1/T), \tag{3}$$

where $\tilde{\mathcal{K}}_{\text{eff}} := \mathcal{P}_0\tilde{\mathcal{K}}\mathcal{P}_0$ and $\mathcal{E}_T$ denotes the evolution over [0, T] generated by $\mathcal{L}_0 + T^{-1}\tilde{\mathcal{K}}$. It should be stressed that (3) is based just on degenerate perturbation theory for general linear operators [18]. In particular, it does not rely on the assumption that $\mathcal{L}_0$ can be cast in Lindblad form [17] or on the SSM structure described in [19]. An immediate corollary of (3) is that $||\mathcal{Q}_0\mathcal{E}_T\mathcal{P}_0|| = O(1/T)$, namely, the probability of leaking outside of the SSM, induced by the unitary term $\mathcal{K}$, for large T, is smaller than cT^{-1}. The constant c controls the strength of the deviations from the ideal adiabatic behavior at finite T [the rhs of (3)], and it can be related to the spectral structure of $\mathcal{S}$. Roughly speaking, one expects c, and, therefore, violations of adiabaticity to increase when the dissipative gap τ_R^{-1} decreases. However, a subtler interplay between the gap with the matrix elements of $\mathcal{Q}_0\mathcal{K}\mathcal{P}_0$ may play an important role here as well in the information geometry of SSM [20].

Let us now turn to the structure of the effective generator $\tilde{\mathcal{K}}_{\text{eff}}$. Of course, it crucially depends on the projection $\mathcal{P}_0$ that, in turn, depends on the nature of $\mathcal{L}_0$. Here, we discuss two (nonmutually exclusive) cases. Their physical relevance relies on the importance, both theoretical and experimental, of the concepts of decoherence-free subspaces [2] and noiseless subsystems (NSs) [4] in quantum information.

(i) The most general dissipative generator $\mathcal{L}_0$ of a Markovian quantum dynamical semigroup $\mathcal{E}_t := e^{t\mathcal{L}_0}$ can be written as $\mathcal{L}_0(\rho) = \Phi(\rho) - \frac{1}{2}\{\Phi^*(\mathbb{1}), \rho\}$ where Φ is a CP map and Φ^* is the dual map, i.e., $\Phi(X) = \sum_i A_i\rho A_i^\dagger \Rightarrow \Phi^*(X) = \sum_i A_i^\dagger \rho A_i$ [17]. We now assume that Φ is trace preserving [$\Phi^*(\mathbb{1}) = \mathbb{1}$] and unital [$\Phi(\mathbb{1}) = \mathbb{1}$]. Under these assumptions, whence $\text{Ker}\mathcal{L}_0$ coincides with the set of fixed points of Φ, the latter is known to be the commutant $\mathcal{A}'$ [21] of the interaction algebra $\mathcal{A}$ generated by the Kraus operators A_i and their conjugates [22]. From [21], it follows that the SSM of $\mathcal{L}_0$ is $\sum_J n_J^2$ dimensional and is given by the convex hull of states of the form $\omega_J \otimes \mathbb{1}_{d_J}/d_J$ where ω_J is a state over the noiseless-subsystem factor $\mathbf{C}^{n_J}$. If, for some J, $d_J = 1$, one has that the corresponding $\mathbf{C}^{n_J}$ is a DFS and the SSM contains pure states. Conversely, if $d_J > 1$, $(\forall J)$, then no pure states are in the SSM. A characterization of the algebraic structure of SSMs for $\mathcal{L}_0$'s of the Lindblad form [17] is provided in [19].

Now $\mathcal{P}_0$ is the projection onto the commutant algebra [21] and one can check that $\tilde{\mathcal{K}}_{\text{eff}}|_{\text{Ker}\mathcal{L}_0} = -i[\tilde{K}_{\text{eff}}, \cdot]$ where $\tilde{K}_{\text{eff}} := \mathcal{P}_0(\tilde{K})$ [23]. By definition, $[\tilde{K}_{\text{eff}}, U] = 0$ for all the unitaries in $\mathcal{A}$, namely, the effective dynamics admits, as a symmetry group, the full-unitary group of the interaction algebra $\mathcal{A}$. This means that the renormalization process $\tilde{K} \mapsto \tilde{K}_{\text{eff}} \in \mathcal{A}'$ amounts to an environment-induced symmetrization of the dynamics [24]. From [21], it also follows that $\tilde{K}_{\text{eff}}$ has a nontrivial action just on the noiseless subsystems of $\mathcal{A}$; the symmetrization process dynamically decouples the system from the noise process driven by operators in $\mathcal{A}$ [11,24].

(ii) Suppose there exists a subspace $\mathcal{C} \subset \mathcal{H}$ such that $\text{Ker}\mathcal{L}_0 \supset \text{L}(\mathcal{C}) := \text{span}\{|\phi_i\rangle\langle\phi_j| / |\phi_i\rangle \in \mathcal{C}\}$. In particular,

$|\psi\rangle \in \mathcal{C} \Rightarrow \mathcal{L}_0(|\psi\rangle\langle\psi|) = 0$, i.e., $\mathcal{C}$ is a DFS [2] for the unperturbed $\mathcal{L}_0$. Also, if $\mathcal{P}_0(|\phi\rangle\langle\phi^\perp|) = \mathcal{P}_0(|\phi^\perp\rangle\langle\phi|) = 0$ holds for all $|\phi\rangle \in \mathcal{C}$ and $|\phi^\perp\rangle \in \mathcal{C}^\perp$, a simple calculation shows that $\mathcal{P}_0\tilde{\mathcal{K}}\mathcal{P}_0|_{L(\mathcal{C})} = -i[\Pi\tilde{K}\Pi, \cdot]$, where Π is the orthogonal projection over $\mathcal{C}$ [25].

Remarkably, in all cases (i)–(ii) above, we see that the induced SSM dynamics $e^{\tilde{K}_{\rm eff}}$ is unitary and governed by a dissipation-projected Hamiltonian. Qualitatively: this coherent dynamics results from the interplay between the weak (slow) Hamiltonian $K = T^{-1}\tilde{K}$ and the strong (fast) dissipative term $\mathcal{L}_0$. The former induces transitions out of the SSM, while the latter projects the system back into it on a much faster timescale. As a result, nonsteady states of the Liouvillian are adiabatically decoupled from the dynamics up to contributions $O(1/T)$. Now, we would like to make a few important remarks.

(1) By defining $\tilde{\rho}(t) := \mathcal{U}_t^\dagger[\rho(t)]$, Eq. (1) gives rise to a dynamical equation of the form $d\tilde{\rho}(t)/dt = \mathcal{L}_t[\tilde{\rho}(t)]$, where $\mathcal{L}_t := \mathcal{U}_t^\dagger \circ \mathcal{L}_0 \circ \mathcal{U}_t$ and $\mathcal{U}_t(X) := e^{-itK}Xe^{itK}$. Namely, in this rotated frame, $\tilde{\rho}(t)$ evolves in a time-dependent bath described by $\mathcal{L}_t$. This establishes a connection of the present approach to the one with time-dependent baths in [10] and [11]. Smallness of K in the picture (1) translates into slowness of the bath time dependence in the rotated frame.

(2) The environment-induced renormalization $\tilde{K} \mapsto \tilde{K}_{\rm eff} = \mathcal{P}_0 K \mathcal{P}_0$ is not an algebra homomorphism; this implies that the algebraic structure of a set of projected Hamiltonians may differ radically from the algebraic structure of the original (unprojected) ones. In particular, commuting (noncommuting) $\tilde{K}$'s may be mapped onto noncommuting (commuting) $\tilde{K}_{\rm eff}$, this implying a potential increase (decrease) of their ability to enact quantum control [11,14]. Also, notice that the Hamiltonian locality structure may be affected by the projection, e.g., a 1-local K may give rise to a 3-local $K_{\rm eff}$. The dissipative technique discussed here might then be exploited to effectively generate nonlocal interactions out of simpler ones in a fashion similar to perturbative gadgets [26] (see also [8]).

(3) Any extra term $\mathcal{V}$, either Hamiltonian or dissipative, in the Liouvillian such that $\mathcal{P}_0\mathcal{V}\mathcal{P}_0 = 0$ will not contribute to the effective dynamics (3) in the limit in which $\mathcal{L}_0$ dominates. For example, in case (ii) discussed in the above, the projected dynamics does not change by perturbing K with any extra Hamiltonian term K' such that $||K'|| = O(1/T)$ and $\mathcal{P}_0(K') = \sum_J {\rm Tr}_{d_J}(\Pi_J K' \Pi_J) \otimes \mathbb{1}_{d_J}/d_J = 0$ [here Π_J is the projector $\mathbb{1}_{n_J} \otimes \mathbb{1}_{d_J}$ of the Jth summand in [21]]. The projected dynamics has a degree of resilience against perturbations that are eliminated by the environment-induced symmetrization.

(4) If the interaction algebra $\mathcal{A}$ in (ii) is Abelian, then, from [21], one finds $\mathcal{P}_0(K) = \sum_J \Pi_J K \Pi_J$. This shows that quantum Zeno dynamics and the associated control and computation techniques of Refs. [12–14] can be regarded as a special case of the projection phenomenon described by Eq. (3).

(5) When $\mathcal{P}_0\mathcal{K}\mathcal{P}_0 = 0$, the Dyson series for $\mathcal{E}_t$ shows that, $\mathcal{E}_t\mathcal{P}_0 = 1 - t\mathcal{P}_0\mathcal{K}\mathcal{S}\mathcal{K}\mathcal{P}_0 + O(||\mathcal{K}||||\mathcal{S}||)$. This means that the dynamics inside the SSM is now ruled by the second-order effective generator $\mathcal{L}_{\rm eff} := -\mathcal{P}_0\mathcal{K}\mathcal{S}\mathcal{K}\mathcal{P}_0$, [up to errors $O(\tau_R||K||)$]. This dynamics is, in general, nonunitary, and its effective relaxation time can be roughly estimated by $\tau_R^{\rm eff} = O(||\mathcal{L}_{\rm eff}||^{-1}) = O(\tau_R^{-1}||\mathcal{K}||^{-2}) = \tau_R O((\tau_R||\mathcal{K}||)^{-2}) \gg \tau_R$. Notice the counterintuitive fact that the stronger the dissipation outside the SSM, the weaker the effective one inside [10].

Unitaries over a DFS.—Here, we show how to perform coherent manipulations on a logical qubit built upon the SSM of four qubits which comprises a DFS [2]. Consider the following unperturbed Liouvillian:

$$\mathcal{L}_0(\rho) = \sum_{\alpha=x,y,z} \gamma_\alpha \left(S^\alpha \rho S^{\alpha\dagger} - \frac{1}{2}\{S^{\alpha\dagger}S^\alpha, \rho\} \right), \qquad (4)$$

where $S^\alpha = \sum_{j=1}^N S_j^\alpha$ are collective spin operators and γ_α decoherence rates. The interaction algebra $\mathcal{A}$ generated by the S^α's is the algebra of permutation invariant operators [4]. Therefore, from (i), it follows that ${\rm Ker}\mathcal{L}_0$ has the structure [21] where J is now a total angular momentum label, $d_J = 2J + 1$ and the n_J's are the dimensions of irreducible representations (irreps) of the permutation group $\mathcal{S}_N$ [27]. For $N = 4$, the one-dimensional $J = 0$ representation shows with multiplicity two. If we denote by $\mathcal{C}$ this two-dimensional subspace, the conditions in (ii) are met.

Let us denote with Π the projector onto $\mathcal{C}$. It is known that one can construct a universal set of gates in similar DFSs (see, e.g., [28]) when the dynamics is entirely contained in the DFS. Here, we show that coherent manipulation is also possible when the dynamics leaks out of the DFS. Consider, for example, the following Hamiltonian perturbations $H^x = \frac{3}{2}(\sigma_1^z\sigma_2^z + \sigma_2^z\sigma_3^z) + \mathbb{1}$ and $H^z = -(\sqrt{3}/2)(\sigma_1^z\sigma_2^z - \sigma_2^z\sigma_3^z) + \sigma_1^z$. One can check that, in the logical space $\mathcal{C}$, such Hamiltonians reduce to elementary Pauli operations, i.e., $\Pi H^\alpha \Pi = \sigma^\alpha$. We now build the perturbed Liouvillians $\mathcal{L}^\alpha = \mathcal{L}_0 - i\vartheta/T[H^\alpha, \cdot]$, $\alpha = x, z$, let us also denote $\tilde{\mathcal{K}}_{\rm eff}^\alpha = -i\vartheta\mathcal{P}_0[H^\alpha, \cdot]\mathcal{P}_0$ with ϑ free parameter. In Fig. 1, we show a numerical experiment confirming our general theorem Eq. (3) for such $\mathcal{L}^\alpha$. In the logical qubit space, the effective evolution $e^{\tilde{\mathcal{K}}_{\rm eff}^\alpha}$ is a unitary evolution $e^{\tilde{\mathcal{K}}_{\rm eff}^\alpha}(X) \simeq u^\alpha X u^{\alpha\dagger}$ with $u^\alpha = \exp(-i\vartheta\sigma^\alpha)$, and one can easily generate any unitary in SU(2) by concatenating such gates. Moreover, the bound in Eq. (3) implies that, for any vectors $|i\rangle$, $|j\rangle$ in the logical space $\mathcal{C}$, $||(\mathcal{E}_T - e^{\tilde{\mathcal{K}}_{\rm eff}})(|i\rangle\langle j|)|| \leq ||(\mathcal{E}_T - e^{\tilde{\mathcal{K}}_{\rm eff}})\mathcal{P}_0|| = O(1/T)$, showing that, effectively, one can generate unitary gates on the logical qubit space $\mathcal{C}$ up to an error $1/T$. In view of remark (3), one is allowed to add to $\mathcal{L}^\alpha$ any perturbation $\mathcal{V}$ satisfying $\mathcal{P}_0\mathcal{V}\mathcal{P}_0 = 0$ and still obtain the same unitary gates u^α within an error c/T albeit with a possibly different

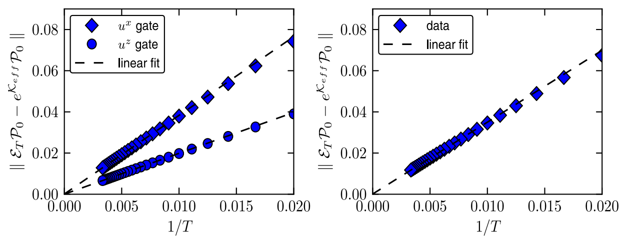

FIG. 1 (color online). Distance of the effective evolution from the exact evolution as a function of $1/T$. Left panel: DFS example with $\mathcal{L}_0$ given by Eq. (4). We used $\gamma_\alpha = 1$ and $\vartheta = 1$ (see text for details). Right panel: example for noiseless subsystem with parameters $\phi_j = \vartheta = 1$ (see text). The norm used is the maximum singular value of the maps realized as matrices over $\mathcal{H}^{\otimes 2}$. The linear fits are obtained using the four most significant points.

c [29]. In [15], we also show the stability of this dynamics against certain dissipative perturbations of $\mathcal{L}_0$. Figure 1 shows that the whole 14 dimensional SSM is evolving unitarily in the long time limit.

To illustrate our results, let us consider the experimental DFS system, studied in [3], consisting of a couple of trapped $^9Be^+$ ions subject to collective dephasing [$\gamma_{x,y} = 0$ in (4)]. In this case, $\tau_R \sim 5\ \mu$s, and (assuming a similar relaxation time for a four qubits system) Eq. (2) and Fig. 1 show that, for $T \sim 500\ \mu$s, one should observe small deviations of the effective dynamics from unitarity.

Unitaries over noiseless subsystem.—Next, we discuss dissipation-assisted computation over noiseless subsystems [4]. The Liouvillian is in the class previously discussed, $\mathcal{L}_0(\rho) = \Phi(\rho) - \rho$, taking $\Phi(\rho) = \frac{1}{3}\sum_{\alpha=1}^{3} U_\alpha \rho U_\alpha^\dagger$, $U_\alpha = e^{i\phi_\alpha S^\alpha}$, where S^α are again collective spin operators. For generic ϕ_α's, the SSM coincides with one of the former examples, i.e., rotationally invariant state. The latter, for an odd number N of spins, contains only mixed states. As perturbation, we use the following Hamiltonian $H = \sigma_1^x \sigma_2^x$ and the full Liouvillian reads $\mathcal{L} = \mathcal{L}_0 - i\vartheta/T[H, \cdot]$. Again, one observes an effective unitary evolution, up to an error $O(1/T)$, (see Fig. 1 right panel) over the full five-dimensional SSM; in particular, this construction can be seen as a scheme to enact dissipation-assisted control over the noiseless subsystem $\mathbf{C}^2$ factor [11].

In [5], noiseless subsystems have been realized in a NMR system comprising three nuclear spins subject to collective (artificial) noise; for a relaxation time $\tau_R < 1/30$ s the noiseless encoding provides an advantage. Figure 1 shows that setting the operation time, say at $T = 100\tau_R$, then effective dynamics over the NSs becomes very close to a unitary one.

Finally, we would like to stress that the Markovian form (1) is just sufficient (and mathematically convenient) enough to prove the existence of an effective projected dynamics, but not necessary. The spin-boson Hamiltonian discussed in [15] indicates that the relevant dynamical mechanism is the existence of a strong system-bath coupling that adiabatically decouples nonsteady states from the dynamics.

Conclusions.—In this Letter, we have shown how an effective unitary dynamics can be enacted over the manifold of steady states of a strongly dissipative system. The strategy is to introduce a small time-rescaled Hamiltonian term in the system's Liouvillian largely dominated by the dissipative processes. In the long time limit, the dynamics leaves the steady state manifold invariant and becomes unitary up to a small error whose strength is connected to the Liouvillian relaxation time and total operation time. The effective Hamiltonian ruling the long time dynamics is shaped by the continuous interplay of the weak Hamiltonian control with the fast relaxation process that adiabatically decouples nonsteady states. This effective projected Hamiltonian, in some cases, can be seen as a symmetrized form of the bare one, and it is robust against all perturbations, dissipative or Hamiltonian, that are filtered out by this environment-induced symmetrization.

To illustrate these ideas, we have shown how to realize quantum gates on steady-state manifolds comprising decoherence-free subspaces [2] as well as noiseless subsystems [4]. In all these cases, we have also provided a numerical estimate of the deviations from the ideal long-time unitary behavior and the actual, finite time one. Agreement with the theoretical prediction (3) is found in all cases.

The results of this Letter seem to suggest the intriguing possibility of fighting quantum decoherence by introducing even more quantum decoherence.

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[16] In this Letter, unless otherwise stated, the norms for superoperators $\mathcal{M}$ will be $||\mathcal{M}|| := \sup_{||X||_1=1}||\mathcal{M}(X)||_1$. For semigroups $\{\mathcal{E}_t\}_{t\geq 0}$ of CP maps, one has $||\mathcal{E}_t|| \leq 1$. $||X||$ will denote the standard operator norm for $X \in \mathrm{L}(\mathcal{H})$.

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[21] By definition $\mathcal{A}' =: \{X/[X, O] = 0, \forall\ O \in \mathcal{A}\}$. Standard structure theorems for C^* algebras imply that $\mathcal{A}' \cong \bigoplus_J \mathrm{L}(\mathbf{C}^{n_J}) \otimes \mathbb{1}_{d_J}$, where J labels the irreducible representations of $\mathcal{A}$ with dimension d_J and multiplicity n_J [4]. In this case, $\mathcal{P}_0(X) = \int dUUXU^\dagger$, where the Haar-measure integral is performed over the unitary group of the algebra $\mathcal{A}$.

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[23] If $X=\mathcal{P}_0(X)$, one has : $\mathcal{P}_0\tilde{\mathcal{K}}\mathcal{P}_0(X) = -i\mathcal{P}_0([\tilde{K}, X]) = -i\mathcal{P}_0(\tilde{K}X) + i\mathcal{P}_0(X\tilde{K}) = -i\mathcal{P}_0(\tilde{K})X + iX\mathcal{P}_0(\tilde{K}) = -i[\mathcal{P}_0(\tilde{K}), X]$. Where we used, e.g., $\mathcal{P}_0(\tilde{K}X) = \mathcal{P}_0(\tilde{K})X$ valid as X is in $\mathcal{A}'$ [21].

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[25] Let $X \in \mathrm{L}(\mathcal{C})$, and consider $\mathcal{P}_0\tilde{\mathcal{K}}\mathcal{P}_0(X) = -i\mathcal{P}_0([\tilde{K}, X])$. One can write $\tilde{K} = \Pi\tilde{K} + Q\tilde{K}$, where $Q := 1 - \Pi$. Therefore, $\mathcal{P}_0(\tilde{K}X) = \mathcal{P}_0(\Pi\tilde{K}X + Q\tilde{K}X) = \Pi\tilde{K}X = (\Pi\tilde{K}\Pi)X$, where we have used the properties of $\mathcal{P}_0$ assumed in the main text and $X = \Pi X$. Considering now the $X\tilde{K}$ term in the commutator in the same way one finds $\mathcal{P}_0([\tilde{K}, X]) = [\Pi\tilde{K}\Pi, X]$ for all $X \in \mathrm{L}(\mathcal{C})$.

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