Geometric Scaling of Efimov States in a ^{6}Li-^{133}Cs Mixture

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In few-body physics, Efimov states are an infinite series of three-body bound states that obey universal discrete scaling symmetry when pairwise interactions are resonantly enhanced. Despite abundant reports of Efimov states in recent cold atom experiments, direct observation of the discrete scaling symmetry remains an elusive goal. Here we report the observation of three consecutive Efimov resonances in a heteronuclear Li-Cs mixture near a broad interspecies Feshbach resonance. The positions of the resonances closely follow a geometric series 1, λ, λ^2. The observed scaling constant $\lambda_{\mathrm{exp}} = 4.9(4)$ is in good agreement with the predicted value of 4.88.

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The emergence of scaling symmetry in physical phenomena suggests a universal description that is insensitive to microscopic details. Well-known examples are critical phenomena, which are universal and invariant under continuous scaling transformations [1]. Equally intriguing are systems with discrete scaling symmetry, which are invariant under scaling transformations with a specific scaling constant [2]; a classic example is the self-similar growth of crystals, as in snowflakes. Surprisingly, such discrete scaling symmetry also manifests in the infinite series of three-body bound states that Vitaly Efimov predicted in 1970 [3].

In the Efimov scenario, while pairs of particles with short-range resonant interactions cannot be bound, there exists an infinite series of three-particle bound states. These bound states have universal properties that are insensitive to the details of the molecular potential and display discrete scaling symmetry; the size R_n and binding energy E_n of the Efimov state with the nth lowest energy scale geometrically as $R_n = \lambda R_{n-1}$ and $E_n = \lambda^{-2} E_{n-1}$, where λ is the scaling constant. An alternative picture to understand discrete scaling symmetry is based on renormalization group limit cycles [4]. Away from the two-body scattering resonance, Efimov states couple to the scattering continuum and induce a series of three-body scattering resonances at scattering lengths $a_-^{(n)} < 0$, which also follow the scaling law $a_-^{(n)} = \lambda a_-^{(n-1)}$ [5] (Fig. 1).

Ultracold atom systems are ideal to test Efimov scaling symmetry given that their interatomic interactions can be tuned over several orders of magnitude using Feshbach resonances [6]. The first evidence of an Efimov state was reported in ultracold Cs atoms [7]; subsequent observations of Efimov resonances [8] in homonuclear systems were also reported in ^{7}Li [9,10], ^{39}K [11], ^{85}Rb [12], ^{133}Cs [13,14], and ^{6}Li [15,16]. Despite these numerous observations, experimental confirmation of discrete scaling symmetry remains a challenging goal.

The confirmation of universal discrete scaling symmetry requires the observation of multiple Efimov resonances. With two consecutive Efimov resonances, the scaling symmetry can be tested through a comparison between the ratio of the resonance positions and theory; with three or more resonances one can perform a model-independent test. The observation of multiple resonances is experimentally challenging because higher order Efimov resonances diminish when the scattering rate is unitarity limited [7,17,18]. This challenge is acute in homonuclear systems

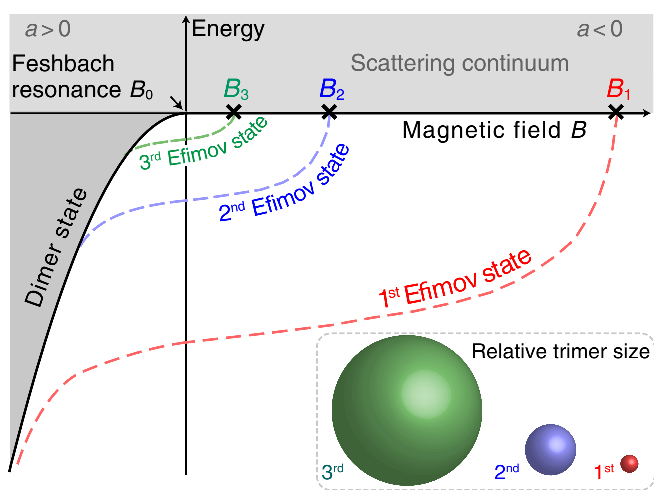

FIG. 1 (color online). Discrete scaling symmetry of Efimov states. A series of Efimov states (dashed curves) exists near a Feshbach resonance located at B_0. Away from the resonance, on the side with scattering length $a < 0$, they merge into the three-body scattering continuum (crosses). Physical observables in the Efimov scenario—including the molecular size R_n, the binding energies E_n, and the Efimov resonance positions at the scattering length a_n (associated with the magnetic field B_n)—show discrete scaling symmetry. The discrete scaling law is graphically represented by the location of the resonances on the B axis and the size of the spheres, respectively.

with large scaling constant $\lambda \approx 22.7$ [3], such that the detection of an additional Efimov state demands a reduction of temperature by a factor of $\lambda^2 \approx 515$. Features from excited Efimov states in homonuclear systems have been observed in ^{6}Li [19], ^{39}K [11], ^{7}Li [20], and ^{133}Cs [14]. These results are consistent with the scaling prediction, but do not provide an independent test of the scaling symmetry.

Heteronuclear systems consisting of one light atom resonantly interacting with two heavy atoms can have a scaling constant significantly lower than 22.7 [4,21–23]; however, experiments in heteronuclear systems are considerably more challenging than those in homonuclear systems. Before our work, observations of Efimov resonances in heteronuclear systems were reported in K-Rb mixtures [24,25]. Recently, in a Li-Cs mixture [26], two Efimov resonances are found in the measurement of three-body loss coefficients, and the number loss data hint at the existence of a third Efimov resonance.

Here we report the observation of discrete scaling symmetry of Efimov states in a Fermi-Bose mixture of ^{6}Li and ^{133}Cs. Taking advantage of the large mass ratio between Li and Cs atoms, with a predicted scaling constant $\lambda = 4.88$ [21,23], we identify three consecutive Efimov resonances near a wide, isolated s-wave interspecies Feshbach resonance [27]. From the measured locations of the resonances, we provide a model-independent proof of the geometric scaling symmetry and determine a scaling constant $\lambda_{\mathrm{exp}} = 4.9(4)$.

Our experiment is based on a mixture of ^{6}Li and ^{133}Cs atoms near quantum degeneracy in an optical dipole trap. In our experiment, both species are prepared in their lowest states, $|F = 1/2, m_F = 1/2\rangle$ for Li and $|F = 3, m_F = 3\rangle$ for Cs, where F is the total angular momentum and m_F is its projection. We prepare mixtures with up to $N_{\mathrm{Li}} = 3.4 \times 10^4$ Li atoms, and $N_{\mathrm{Cs}} = 5.2 \times 10^4$ Cs atoms at temperatures in the range 190 nK $< T <$ 800 nK [28]. Efimov resonances manifest themselves as enhanced three-body recombination losses. In such collisions three atoms resonantly couple to an Efimov state and then decay into a deeply bound molecule and a free atom; the released binding energy allows them to escape the trap. We measure the Li and Cs atom numbers from which we infer atom loss and identify the Efimov resonances.

The mixture of ^{6}Li and ^{133}Cs has two primary inelastic collision pathways: three-body recombination of Cs-Cs-Cs and Li-Cs-Cs. At low temperatures, Li-Li-Cs as well as Li-Li-Li collisions are strongly suppressed by Fermi statistics. We investigate Efimov resonances near the broad Li-Cs Feshbach resonance located at 842.75 G with a width of 61.6 G and a strength parameter s_{res} of $\sim$0.7 [27]. The Efimov resonances reported in this work are away from p-wave Feshbach resonances [32], as well as Cs Feshbach and Efimov resonances [13,33].

Around the magnetic field region probed in this work, the Cs-Cs scattering length is large and negative, and Cs-Cs-Cs recombination is the major competing loss process, imposing a limitation on the lifetime of Cs. Away from the Li-Cs Feshbach resonance, Cs decay is dominated by Cs-Cs-Cs recombination collisions [Fig. 2(a)]; on the other hand, Li decays much faster in the presence of Cs, indicating the dominance of interspecies collisional loss. Near the Feshbach resonance [Fig. 2(b)] both decays of Li and Cs are enhanced by interspecies collisions.

Our measurements of atom loss and observation of Efimov resonances are summarized in Fig. 3. To eliminate the long-term drift in atom number, we scale the atom number so that it averages to unity over a fixed magnetic field range. Each panel shows resonant loss features in the scaled atom number. The main loss feature in both Li and Cs scans is associated with a broad Li-Cs Feshbach resonance [27,32]. Loss features associated with excited Efimov resonances on the negative scattering length side of the Feshbach resonance are only evident in low temperature scans, and indicated by arrows in Fig. 3. Efimov features are weaker in Cs data due to fast competing Cs-Cs-Cs recombination processes.

We determine the position of each Efimov resonance by using both Lorentzian and Gaussian fits with a linear background. Results from different fit functions and fit ranges are analyzed and combined to determine the final

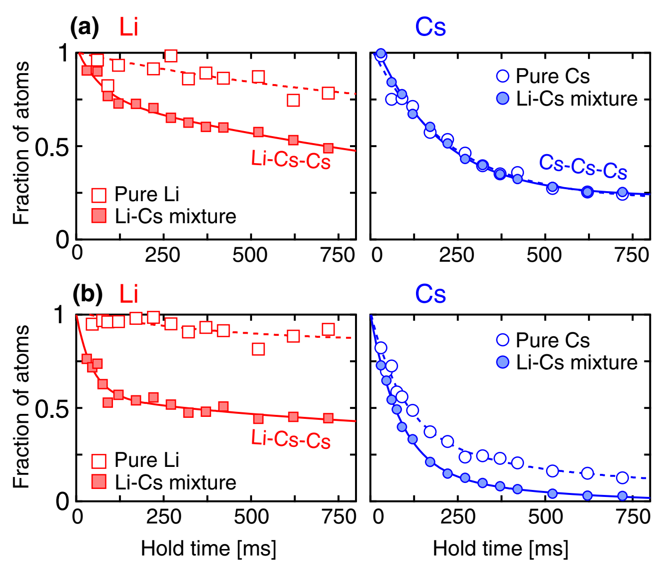

FIG. 2 (color online). Atom number decay of single-species and Li-Cs mixture samples. (a) At $B = 848.0$ G, $\sim$5 G away from the Li-Cs Feshbach resonance ($a_{\mathrm{LiCs}} = -354a_0$, $a_{\mathrm{CsCs}} = -1240a_0$) Li loss increases significantly when Cs is introduced (left panel). Cs loss is dominated by Cs-Cs-Cs recombination (right panel). Sample temperature is $T = 390$ nK. (b) Near the Li-Cs Feshbach resonance $B = 842.7$ G ($a_{\mathrm{CsCs}} = -1570a_0$), enhanced atom loss is evident in both the Li and Cs atom number evolution when both species are present. Sample temperature is $T = 340$ nK. Data in (a) and (b) are scaled to the initial atom numbers, $N_{\mathrm{Li}} = 2 \sim 3 \times 10^4$ and $N_{\mathrm{Cs}} = 4 \sim 5 \times 10^4$, obtained from double exponential fits (continuous and dotted lines), which also serve as guides to the eye.

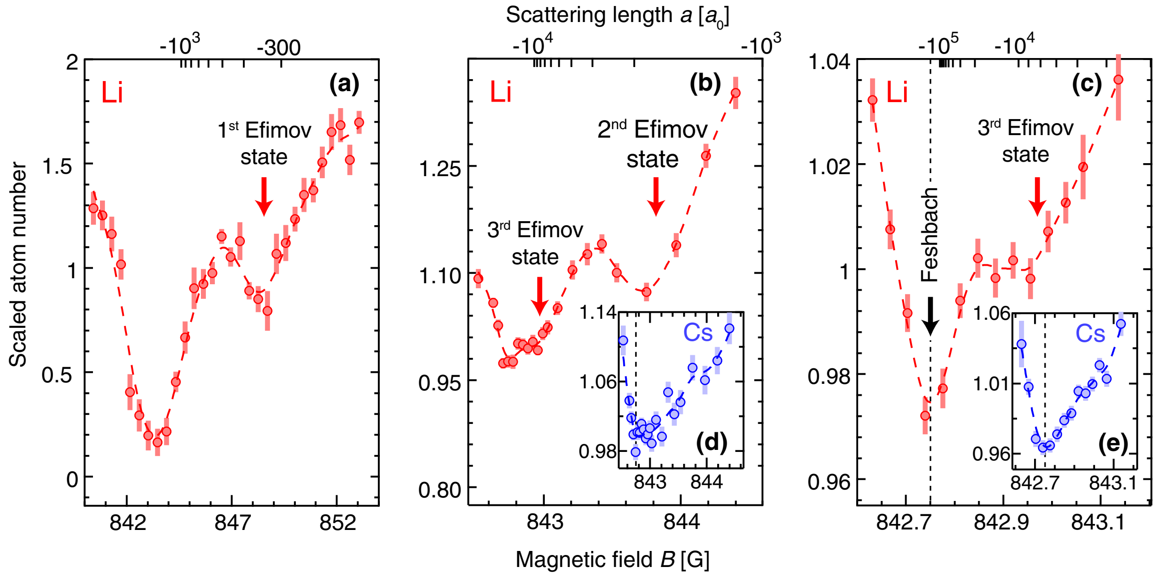

FIG. 3 (color online). Observation of three Li-Cs-Cs Efimov resonances. (a) Scaled Li number versus magnetic field showing the first Li-Cs-Cs Efimov resonance, from the average of 13 individual scans. Here, $N_{\mathrm{Li}} = 1.3 \times 10^4$ and $N_{\mathrm{Cs}} = 2.7 \times 10^4$ with typical temperature $T = 800$ nK and hold time 225 ms. (b) Scaled Li and Cs [inset, (d)] numbers near the second and third Li-Cs-Cs Efimov resonances, from the average of 68 scans with typical temperature $T = 360$ nK and hold time 115 ms. The mean atom numbers are $N_{\mathrm{Li}} = 1.4 \times 10^4$ and $N_{\mathrm{Cs}} = 2.1 \times 10^4$. (c) Scaled Li and Cs [inset, (e)] numbers close to the third Li-Cs-Cs Efimov resonance and the Li-Cs Feshbach resonance, from the average of 327 scans with typical temperature $T = 270$ nK and hold time 115 ms. The mean atom numbers are $N_{\mathrm{Li}} = 9.1 \times 10^3$ and $N_{\mathrm{Cs}} = 1.4 \times 10^4$. The scaled atom numbers come from the average of the individual scans divided by their respective mean values. The vertical dashed lines indicate the Feshbach resonance and arrows indicate the Efimov resonances. The dashed curves correspond to an interpolation of the data and serve as a guide to the eye.

resonance positions and uncertainties. Further details on the fit and the determination of the resonance positions and uncertainties are given in Ref. [28]. We determine the positions of the three Efimov resonances to be $B_1 = 848.55(12)_{\mathrm{stat}}(3)_{\mathrm{syst}}$, and $B_2 = 843.82(4)_{\mathrm{stat}}(3)_{\mathrm{syst}}$, and $B_3 = 842.97(3)_{\mathrm{stat}}(3)_{\mathrm{syst}}$ G, where $()_{\mathrm{stat}}$ denotes the statistical uncertainty and the systematic uncertainty of 30 mG arises from the daily magnetic field drift.

A precise determination of the Feshbach resonance position is crucial to check the scaling symmetry. Two independent methods are developed here. First, we observe that the strongest dip (Fig. 3) is ubiquitous in all measurements, even at high temperatures where Efimov features are indiscernible. This indicates that the strongest dip is associated with the Feshbach resonance. Fits to the lowest temperature data [Fig. 3(c)] locate the Feshbach resonance at $B_0 = 842.75(1)_{\mathrm{stat}}(3)_{\mathrm{syst}}$ G.

We convert our atom loss measurement into a spectrum of the recombination loss coefficient, see Fig. 4, based on a rate equation model [28]. The spectrum shows clearly three Efimov resonance features and can be compared with theoretical calculation. In addition, after comparing the extracted K_3 with a model that captures the steep rise of K_3 for $a > 0$ [28], we find the best agreement between the experiment and the model when $B_0 = 842.75(1)_{\mathrm{stat}}(3)_{\mathrm{syst}}$ G. The results from both our methods to determine B_0 agree with each other.

The separations between the Efimov resonances and the Feshbach resonance $\Delta B_n = B_n - B_0$ are $\Delta B_1 = 5.80(12)$, $\Delta B_2 = 1.07(4)$, and $\Delta B_3 = 0.22(3)$ G; the uncertainties include both statistical and systematic errors. Remarkably, they closely follow a geometric progression $\Delta B_1 : \Delta B_2 : \Delta B_3 \approx 1 : 1/5 : 1/5^2$ and provide direct evidence of the discrete scaling symmetry. (Note that $\Delta B_n \propto -1/a_-^{(n)}$ near the Feshbach resonance.) More precisely, using an updated scattering model for the Li-Cs Feshbach resonance [28], we determine the Efimov resonances in scattering length to be $a_-^{(1)} = -323(8)a_0$, $a_-^{(2)} = -1635(60)a_0$, and $a_-^{(3)} = -7850(1100)a_0$, where a_0 is the Bohr radius. Two scaling constants are extracted: $\lambda_{21} = a_-^{(2)}/a_-^{(1)} = 5.1(2)$ and $\lambda_{32} = a_-^{(3)}/a_-^{(2)} = 4.8(7)$, which mutually agree within uncertainty. The averaged scaling constant $\lambda_{\mathrm{exp}} = 4.9(4)$ is in good agreement with the predicted value $\lambda = 4.88$ for $\mathrm{LiCs_2}$ Efimov states [21,23].

Even though the observed scaling ratios are consistent with the predicted value, we would like to point out the practical factors that could contribute to differences between experiment and theory. The first Efimov resonance can be shifted by finite-range corrections given that it occurs at a scattering length near the van der Waals length of Cs-Cs ($r_{\mathrm{CsCs}} = 101a_0$) and Li-Cs ($r_{\mathrm{LiCs}} = 45a_0$). The location of the Efimov resonance can also be shifted by finite temperature and finite trap size effects, which are

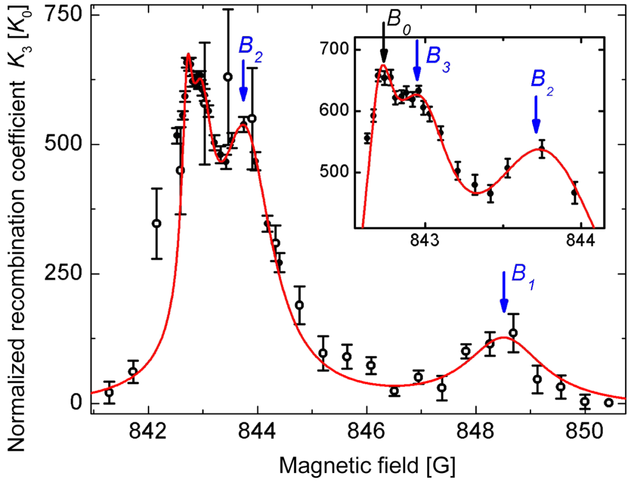

FIG. 4 (color online). Feshbach and Efimov resonance structure in the recombination coefficient. We extract and normalize the recombination coefficient K_3 from the atom number measurement based on a rate equation model [28]. Using the data in Fig. 3(a) ($T = 800$ nK, open circles) and Fig. 3(b) ($T = 360$ nK, solid circles), we show that the extracted K_3 displays four peaks. The three peaks at magnetic fields B_1, B_2, and B_3 are associated with Efimov resonances, and the global maximum at the lower field B_0 represents the Feshbach resonance. $K_0 = 10^{-25}$ cm^6/s is the loss coefficient we obtain at ~852 G. The inset shows the zoom-in view of the resonance structure from the lower temperature measurement, which offers higher resolution to the higher order Efimov resonances. The solid line represents a four-Lorentzian fit to the data, and serves as a guide to the eye.

stronger for excited Efimov states [34,35]. Based on a closer inspection, our data do not detect a clear position shift of the third Efimov resonance due to finite temperature effect. Finite size effects are estimated to be negligible in our experiment [28].

In conclusion, we observed three Efimov resonances in a Li-Cs mixture and extracted two scaling constants. From their mutual agreement and that with theoretical calculations, our results provide experimental evidence of discrete scaling symmetry of Efimov states. Based on our observations, an intriguing question is whether the discrete scaling symmetry can be tested in the original Efimov scenario of three identical bosons, which requires extremely low temperature. In addition, our result also hints at the persistence of discrete scaling symmetry when the scattering length diverges (unitary Bose gas) [36], in contrast with the continuous scaling symmetry of a unitary Fermi gas [37].

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