Universal Postquench Prethermalization at a Quantum Critical Point

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We consider an open system near a quantum critical point that is suddenly moved towards the critical point. The bath-dominated diffusive nonequilibrium dynamics after the quench is shown to follow scaling behavior, governed by a critical exponent that emerges in addition to the known equilibrium critical exponents. We determine this exponent and show that it describes universal prethermalized coarsening dynamics of the order parameter in an intermediate time regime. Implications of this quantum critical prethermalization are: (i) a power law rise of order and correlations after an initial collapse of the equilibrium state and (ii) a crossover to thermalization that occurs arbitrarily late for sufficiently shallow quenches.

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Predicting the out-of-equilibrium dynamics of quantum many-body systems is a challenge of fundamental and practical importance. This research area has been boosted by recent experiments in cold-atom gases [1] and scaled-up quantum circuits [2], by ultrafast pump-probe measurements in correlated materials [3–5], and by performing heavy-ion collisions that explore the quark-gluon plasma [6]. In this context, the universality near a quantum critical point (QCP), well established in and near equilibrium, comes with the potential to make quantitative predictions for strongly interacting systems far from equilibrium. For example, the quantum version [7–11] of the Kibble-Zurek mechanism of defect formation [12,13] was developed for systems driven through a symmetry breaking QCP at a small, but finite rate. Similarly, near a QCP the long-time dynamics after a sudden change of Hamiltonian parameters is governed by equilibrium exponents [14]. These phenomena occur in the regime of longest time scales.

Recently, however, many physical systems away from equilibrium were identified which display novel dynamical behavior on intermediate time scales, a behavior often referred to as prethermalization [15–25]. The question arises whether one can expect universality during prethermalization if one drives a system towards a QCP. Even if this is done at a finite rate $1/\tau$, a system will fall out of equilibrium at some point, a behavior owed to the critical slowing down near the QCP. Then a scaling theory with characteristic time scale τ can be developed [10], where regions of the size of the freeze-out length $\propto \tau^{1/z}$ emerge that behave like in equilibrium. z is the dynamic critical exponent. In the case of a quantum quench, the time scale τ and the freeze-out length become comparable to microscopic time and length scales, respectively, and the system instantly falls out of equilibrium. The detailed recovery of this out-of-equilibrium dynamics, along with the time dependence of length scales, order-parameter correlations, and the potential for out-of-equilibrium universality are major theoretical and experimental challenges.

In this Letter, we show that the time evolution of observables in an open system that is suddenly moved to a QCP displays universal behavior [see Figs. 1(a)–1(b)]. Their nonequilibrium dynamics is governed by a critical exponent that describes the slow decay of postquench correlations and response soon after a quantum quench,

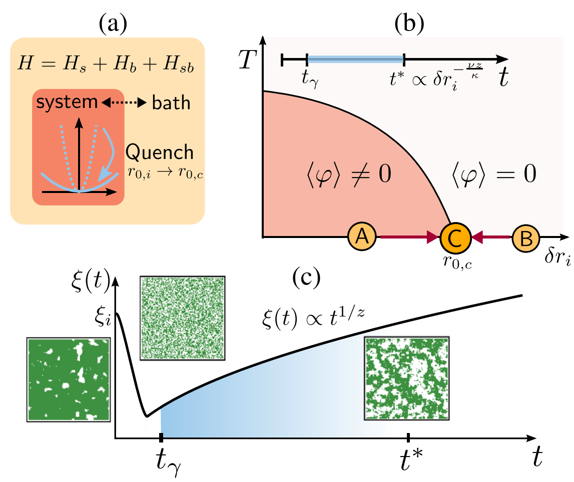

FIG. 1 (color online). (a) Schematic description of the setup and quench protocol. (b) Schematic phase diagram as a function of temperature T and mass $\delta r_i = r_{0,i} - r_{0,c}$. Red arrows describe the quench protocol. Dynamics exhibits three time regimes: $t < t_\gamma = \gamma^{-z/(2(z-1))}$ with nonuniversal dynamics, the universal prethermalized regime $t_\gamma < t < t^* \propto \delta r_i^{-\nu z/\kappa}$ which we study, and a quasiadiabatic regime $t > t^*$ described by equilibrium critical exponents. Here, γ is the system-bath coupling and κ/ν the scaling dimension of δr_i. (c) Correlation length collapse and light-cone-like revival following a quench with initial length ξ_i. Inset: sketches of order parameter configurations with domains of typical size $\xi(t)$.

where initial correlations are still important. It is therefore not related to equilibrium exponents. This behavior is astounding as universality is usually reserved for large time and length scales. From the value of the exponent we conclude that initial state correlations rapidly collapse after a quench and that the order parameter undergoes an intermediate coarsening, i.e., grows due to the growing light-cone length $\xi(t) \propto t^{1/z}$ [see Fig. 1(c)], before it decays quasiadiabatically at longer times. We also demonstrate that the duration of this intermediate prethermalization can be manipulated and tuned to be arbitrarily large. While there are important differences between classical and quantum quenches, the analysis of this Letter was motivated by the pioneering theory of classical dynamics in Ref. [[26]] (see also Ref. [27] for the case of colored noise).

The quench protocol that underlies our analysis is indicated in Figs. 1(a)–1(b). We consider a quantum many-body system that is coupled to an external bath of harmonic oscillators. Prior to the quench, the complete system is prepared in the ground state of the initial Hamiltonian $H_i = H_{s,i} + H_b + H_{sb}$. The initial Hamiltonian of the system, $H_{s,i}$, describes an N-component scalar quantum field $\boldsymbol{\varphi}(x,t)$ with components φ_a $(a = 1, \ldots, N)$:

$$H_{s,i} = \frac{1}{2} \int d^d x \left(\boldsymbol{\pi}^2 + r_{0,i} \boldsymbol{\varphi}^2 + (\nabla \boldsymbol{\varphi})^2 + \frac{u}{2} \boldsymbol{\varphi}^4 \right), \quad (1)$$

where $\boldsymbol{\pi}$ is the canonically conjugated momentum to $\boldsymbol{\varphi}$. $H_b = \frac{1}{2} \int d^d x \sum_l (\Omega_l^2 \boldsymbol{X}_l^2 + \boldsymbol{P}_l^2)$ describes the external bath of harmonic oscillators and $H_{sb} = \sum_l c_l \int d^d x \boldsymbol{X}_l \cdot \boldsymbol{\varphi}$ the coupling between the system and bath. Next, we suddenly change $H_{s,i} \to H_s$ by switching $r_{0,i} \to r_{0,c}$ to its value right at the QCP of system + bath in equilibrium [see Fig. 1(b)]. The time evolution after the quench is now governed by the new Hamiltonian $H = H_s + H_b + H_{sb}$. The bath ensures that the system eventually equilibrates at $T = 0$, which allows reaching the QCP for $t \to \infty$. A crucial variable is the distance to the critical point $\delta r_i = r_{0,i} - r_{0,c}$ before the quench. After the quench we consider $\delta r_f = r_{0,f} - r_{0,c} = 0$, while the same behavior is expected for generic quenches that move the system closer to the critical point $\delta r_f \ll \delta r_i$ or take place at a finite but small temperature $T \ll \delta r_i^{\nu z} / \gamma^{z/2}$ with system-bath coupling γ defined below.

The Hamiltonian $H_{s,i}$ of Eq. (1) describes a transverse-field Ising model for $N = 1$, systems near a superconducting-insulator quantum phase transition, Josephson junction arrays, and quantum antiferromagnets in an external magnetic field for $N = 2$, or quantum dimer systems for $N = 3$ [28]. Our theory for system + bath can then be applied to a range of systems such as dissipative superconducting nanowires [29], the superfluid-insulator transition in cold-atom gases coupled to other bath atoms [30], or low-dimensional Heisenberg spin dimers or transverse field Ising spins with strong quantum fluctuations and coupling to phonons. Another promising realization can be achieved by an ensemble of qubits in a photon cavity [2,31]. The effects of the bath are described in terms of $\eta(\omega) = -\sum_l c_l^2 / ((\omega + i0^+)^2 - \Omega_l^2)$. We consider for the spectral density of the bath,

$$\mathrm{Im}\eta(\omega) = \gamma \omega |\omega|^{\alpha - 1} e^{-|\omega|/\omega_c}, \quad (2)$$

with damping coefficient γ and cutoff energy ω_c. The exponent α determines the low-energy spectrum of the bath, where $\alpha = 1$ corresponds to Ohmic damping while $\alpha > (<) 1$ corresponds to super-Ohmic (sub-Ohmic) damping [32]. In the following, we consider the hierarchy of scales $\omega_c \gg t_\gamma^{-1} = \gamma^{1/(2-\alpha)}$ and analyze the regime $t > t_\gamma$ when the dynamics is dominated by the bath. For the one-loop RG analysis used in this Letter, it holds $z = 2/\alpha$.

We start with general scaling arguments for the nonequilibrium dynamics after a quench towards the QCP. The scaling behavior will be confirmed using a perturbative renormalization group (RG) analysis later in the Letter. In equilibrium, the order parameter behaves as a function of the distance δr to the QCP according to $\langle \varphi_a(\delta r) \rangle_{\mathrm{eq}} = b^{-\beta/\nu} \langle \varphi_a(b^{1/\nu} \delta r) \rangle_{\mathrm{eq}}$, with scaling parameter $b > 1$, which leads to the well-known behavior $\langle \varphi_a(\delta r) \rangle_{\mathrm{eq}} \propto \delta r^{\beta}$. In our case δr rapidly changes as a function of time from δr_i to δr_f, leading to a t dependence of the order parameter. The generalization of the equilibrium scaling relation can be performed in analogy to boundary layer scaling theory as it occurs near surfaces and interfaces [33]. Here, a new healing length scale associated with surface fields appears. In our problem, the boundary layer incorporating the initial value problem corresponds to a "surface in time" [34–36] and exhibits an associated new healing time scale t^*. Following Ref. [33] we obtain for the order parameter $\langle \varphi_a(\delta r_i, \delta r_f, t) \rangle = b^{-\beta/\nu} \langle \varphi_a(b^{\kappa/\nu} \delta r_i, b^{1/\nu} \delta r_f, b^{-z} t) \rangle$. While δr_f scales as in equilibrium, reflecting the fact that the system approaches equilibrium for $t \to \infty$, the initial mass δr_i has a nontrivial scaling exponent κ/ν. For $\delta r_f = 0$, i.e., a quench right to the QCP, it follows with $b = t^{1/z}$,

$$\langle \varphi_a(t, \delta r_i) \rangle = t^{-\beta/(\nu z)} \Phi(t^{\kappa/(\nu z)} \delta r_i), \quad (3)$$

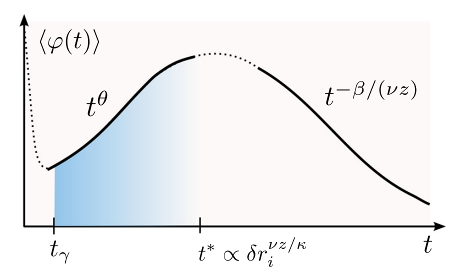

FIG. 2 (color online). Schematic order parameter dynamics $\langle \varphi(t) \rangle$. In the prethermalized regime $t_\gamma < t < t^*$ (blue) it is governed by a new universal critical exponent θ. At longer times, $\langle \varphi(t) \rangle$ decays to zero quasiadiabatically as described by equilibrium exponents.

with universal function $\Phi(y)$. As shown in Fig. 2, in the long time limit $t \gg t^*$ for $\Phi(y \gg 1) \to \text{const}$, the order parameter decays quasiadiabatically ($\langle \varphi_a(t) \rangle \propto t^{-\beta/(\nu z)} \propto \xi(t)^{-\beta/\nu}$) to zero with time scale

$$t^* \propto \delta r_i^{-\nu z/\kappa}. \tag{4}$$

In the opposite limit, $t \ll t^*$, the situation is qualitatively different. Assuming in analogy to Ref. [33] that the susceptibility with respect to a temporal boundary-layer term $\langle \varphi_{a,i} \rangle \propto \delta r_i^{\beta}$ is finite, it follows $\Phi(y \ll 1) \propto y^{\beta}$, such that

$$\langle \varphi_a(t) \rangle \propto t^{\theta} \quad \text{with} \quad \theta = \frac{(\kappa - 1)\beta}{\nu z}. \tag{5}$$

Thus, a new universal time dependence of the order parameter emerges at short times. The value of the exponent θ is determined by the scaling dimension κ of δr_i. The time scale t^* separates the regime governed by the initial quench and concomitant fall out of equilibrium from the quasiadiabatic long time behavior. Thus, in analogy to spatial boundary layer problems it describes the dynamic healing after the quench.

The same exponent θ also determines the time dependence of correlation and response functions. To analyze the nonequilibrium dynamics we employ the Keldysh formalism of many-body theory [37] and use the specific form of the Keldysh contour of Ref. [38], appropriate for our quench protocol. The key quantities are the retarded response function G^R and the Keldysh correlation function G^K:

$$G^R(k, t, t') = -i\theta(t - t')\langle [\varphi_a(k, t), \varphi_a(-k, t')]_- \rangle,$$
$$G^K(k, t, t') = -i\langle [\varphi_a(k, t), \varphi_a(-k, t')]_+ \rangle, \tag{6}$$

with momentum k. They are no longer related by the fluctuation-dissipation theorem. We expect from dimensional arguments

$$iG^{R(K)}(k, t, t') = \left(\frac{t}{t'}\right)^{\theta(\theta')} \frac{f^{R(K)}(k^z t/\gamma^{z/2}, t'/t)}{k^{2-\eta-z}\gamma^{z/2}}. \tag{7}$$

In an out-of-equilibrium state the correlation and response functions depend on both time variables. This gives rise to an additional dimensionless ratio t/t' compared to scaling in equilibrium. The singular dependence on this ratio in G^R and G^K is characterized by exponents θ and θ', respectively. Thus, the scaling functions f^R and f^K depend only weakly on t'/t if $t \gg t'$. The exponents θ and θ' are not independent. Relating G^R and G^K in the Dyson equation yields $\theta = \theta' + \frac{2-z-\eta}{z}$.

Let us now demonstrate that θ in Eqs. (5) and (7) is indeed the same. We consider an initial state characterized by a finite order parameter $\langle \boldsymbol{\varphi}_i \rangle$ [path $A \to C$ in Fig. 1(b)]. A region of volume $\xi(t)^d$ is correlated at time t after the quench and Eq. (7) yields for the local, i.e., momentum averaged, correlation function $G^K_{\text{loc}}(t, t') \propto (t/t')^{\theta'} t^{-(d-\eta-z+2)/z}$. The initial order parameter $\langle \boldsymbol{\varphi}_i \rangle$ polarizes the system for a certain time. The magnetization at time t is then $\langle \boldsymbol{\varphi}_i \rangle$ multiplied by the local correlation function up to t and the size of the correlation volume: $\langle \boldsymbol{\varphi}(t) \rangle \simeq \langle \boldsymbol{\varphi}_i \rangle i G^K_{\text{loc}}(t, t') \times t^{d/z}$. We obtain the power law behavior of the order parameter of Eq. (5). The time dependence of the order parameter is therefore a balance between the decay of local correlations encoded in $G^K_{\text{loc}}(t, t')$ and the growth of the volume encompassed by light-cone propagation, i.e., $\xi(t)$.

Next, we demonstrate this behavior in an explicit analysis and determine the value of the exponent θ. We start using simple perturbation theory and perform a more rigorous renormalization group analysis in the second step. At time t after the quench correlations are limited by the light cone. This gives rise to a time dependent mass $r(t) = \gamma a/t^{2/z}$ in the propagator, where a is a dimensionless coefficient. Scattering events caused by collisions of excitations in regions of t-dependent size turn out to be highly singular. A perturbation theory in a that includes such scattering events yields to leading order and for $t' \ll t$:

$$G^R(k, t, t') = G_0^R(k, t)[1 + \theta \log(t/t') + \cdots], \tag{8}$$

where the omitted terms are nonsingular for $t' \to 0$ and

$$\theta = -\frac{a \sin(\pi/z)}{\Gamma(2/z)}. \tag{9}$$

G_0^R is the bare retarded Green's function given in the Supplemental Material [39]. Exponentiation of the logarithm leads to Eq. (7).

We now perform a momentum-shell RG approach to sum up these logarithms in a controlled fashion and determine the exponent θ. In full analogy to the equilibrium case we integrate out states in a shell with momenta $\Lambda/b < k < \Lambda$ with $b > 1$ and rescale fields, momenta, and time variables. The small parameter controlling the calculation is the deviation from the upper critical dimension $\epsilon = 4 - d - z$. The mass δr_i in the initial Hamiltonian is a strongly relevant perturbation and rapidly flows to large values. The nonequilibrium dynamics of the system is therefore governed by the deep-quench fixed point $(\hat{u}^*, \delta r_i^*, \delta r_f^*) = (\hat{u}^*, \infty, 0)$. Here, $\hat{u}^* = c_z \epsilon/(N + 8)$ is the equilibrium value of the dimensionless coupling constant $\hat{u} = u K_d \Lambda^{-\epsilon}/\gamma^{z/2}$ with $K_d = \Gamma(d/2)/(2\pi^{d/2}(2\pi)^d)$ and coefficient $c_z = 4\sin(\pi z/2)/(z(2-z)\sin^{z/2}(\pi/z))$. The scaling dimension of δr_i is relative to the fixed point $\delta r_i^* = \infty$, i.e., $1/\delta r_i \propto b^{-\kappa/\nu}$ is a dangerously irrelevant variable at the deep quench fixed point.

We work with $\delta r_i > 0$ corresponding to a quench out of the unbroken phase and assume that θ is the same for the two paths $A \to C$ and $B \to C$. For the mass renormalization after the quench follows at one loop,

$$r_f'(t) = b^2 r_f(b^z t) + u\frac{N+2}{2}\int^{>}\frac{d^d k}{(2\pi)^d} iG_0^K(k,t,t), \quad (10)$$

where $>$ refers to momenta inside the shell. In equilibrium $r_f(t)$ is t independent and we recover the usual one-loop result for the mass renormalization. The quench mixes $r_f(t)$ at different times during the flow. For a similar analysis of classical surface criticality, see Ref. [40]. We replace δr_i, that enters G_0^K, and $\hat{u}$ by their deep-quench fixed-point values. From Eq. (10) we then obtain a differential equation for the corresponding time-dependent fixed-point mass $r_f^*(t)$:

$$2r_f^* + zt\frac{dr_f^*}{dt} + \frac{(N+2)\hat{u}^*\Lambda^2}{2} f_0^K(\Lambda^z t/\gamma^{z/2}, 1) = 0. \quad (11)$$

The scaling function f_0^K characterizes G_0^K according to Eq. (7). The solution of Eq. (11) is

$$r_f^*(t) = \frac{\gamma a}{t^{2/z}} - \frac{(N+2)\hat{u}^*\Lambda^2}{2zt^{2/z}}\int^t dt' f_0^K\left(\frac{\Lambda^z t'}{\gamma^{z/2}}, 1\right) t'^{(2-z)/z}, \quad (12)$$

where a denotes the integration constant of Eq. (11). We find $f_0^K(\Lambda^z t/\gamma^{z/2} \to \infty, 1) \to f_{\mathrm{eq},0}^K$, where $f_{\mathrm{eq},0}^K$ describes the equal-time Keldysh function in equilibrium after the quench. For a perturbative RG analysis a long range decay of the mass parameter cannot emerge. We can therefore fix the integration constant a from the condition that $r_f^*(t)$ rapidly approaches its equilibrium value, i.e., that $\delta r_f^*(t) = r_f^*(t) - r_{\mathrm{eq}}^* \to 0$ for $t \gg \gamma^{z/2}\Lambda^{-z}$:

$$a = \frac{(N+2)\hat{u}^*}{2z}\int_0^\infty dx[f_0^K(x,1) - f_{\mathrm{eq}}^K]x^{(2-z)/z}. \quad (13)$$

The derivation of the free nonequilibrium Keldysh function G_0^K and thus of $f_0^K(x,1)$ is given in the Supplemental Material [39]. Once we determine the coefficient a, the exponent θ follows from Eq. (9). For an Ohmic bath with $\alpha = 1$, i.e., $z = 2$, we find analytically $a_{z=2} = -(N+2)/(N+8)(\epsilon/4)$, which yields with Eq. (9) the exponent [39]

$$\theta_{z=2} = \frac{N+2}{N+8}\frac{1}{4}\epsilon > 0. \quad (14)$$

For a bath with colored noise, we determine the exponent numerically. Our results for $\mathcal{C}_z = \theta(N+8)/(N+2)(1/\epsilon)$ are shown in Fig. 3(a). We find a maximal value for $\mathcal{C}_z$ (and thus θ) in the slightly sub-Ohmic regime, while $\theta(z \to 4) \to 0$ since $\epsilon > 0$ requires at least $z < 4$. For $z < 2$ the exponent decreases and changes sign for $z \approx 1.8$. From Eq. (13) it follows that the coefficient a and thus θ can only change sign if equal-time correlations decay nonmonotonically. In Fig. 3(b) we show the scaling function $f_0^K(x,1)$ which proves that this is indeed the case for a super-Ohmic bath. Note, in our analysis the limit $z \to 1$ does not correspond to the closed system with ballistic time evolution as we always consider the limit of bath-dominated dynamics.

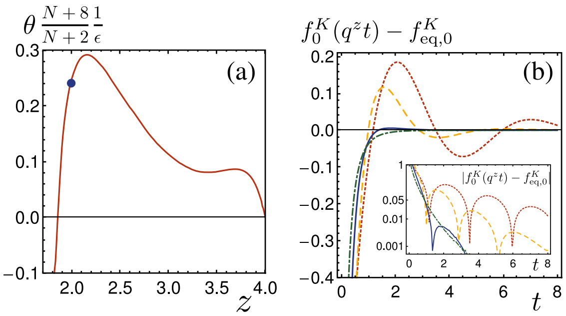

FIG. 3 (color online). (a) Prethermalization exponent θ as a function of dynamic critical exponent z. Plot shows $\mathcal{C}_z = \theta(N+8)/(N+2)(1/\epsilon)$, where N is the number of components of $\boldsymbol{\varphi}$ and $\epsilon = 4 - d - z$. Blue dot indicates the analytical result of Eq. (14). (b) Free Keldysh scaling function $f_0^K(q^z t) - f_{\mathrm{eq},0}$ after the quantum quench for different dynamic critical exponents $z = 1.2$ (red dotted), 1.4 (yellow dashed), 2 (blue dot-dashed), and 2.5 (green). Inset shows the exponential decay of the envelope towards the equilibrium distribution, which becomes algebraic in the presence of interactions.

For $z = 2$ the value of $\mathcal{C}_{z=2}$ turns out to be the same as for a classical phase transition [26,27]. Identical coefficients for classical and quantum phase transitions might suggest that quantum effects are not important for the quench dynamics. However, considering generic values of z the exponents (for given ϵ) of a classical and quantum quench are distinct, demonstrating the quantum quench dynamics is in a different universality class as the classical one.

Let us discuss the physical implications of these results. (i) Collapse of the correlation length: We compare the correlation length prior to the quench $\xi_i \propto \delta r_i^{-\nu}$ with its value at the crossover between the prethermalized regime and equilibration $\xi(t^*) \propto \delta r_i^{-\nu/\kappa}$. $\theta > 0$ implies with Eq. (5) that $\kappa > 1$, such that $\xi(t^*) < \xi_i$ for small δr_i. Right after the quench the system falls out of equilibrium and breaks up into many small uncorrelated regions. The correlation length collapses and does not reach its prequench value during prethermalization. It takes until after the time scale t^* that the system recovers its initial correlations [see spin configurations in Fig. 1(c)]. (ii) Order parameter dynamics: From Eq. (5) follows for $\theta > 0$ that the order parameter grows as function of time. The physical explanation for this behavior follows from our discussion of the path $A \to C$. $\theta > 0$ leads to a slowing down of the temporal decay of local correlations. On the other hand, the size of correlated regions increases according to the light-cone scale $\xi(t)$. The order parameter grows because of the coarsening that takes place at intermediate time scales, where the growth in $\xi(t)$ outweighs the decay of correlations. Thus, the growth of the order parameter $\propto t^{\theta}$ is caused by the recovery of locally ordered regions after the collapse of the correlation length. The long-time, quasiadiabatic order-parameter dynamics $\langle \varphi_a(t) \rangle \propto \xi(t)^{-\beta/\nu}$ only sets in when initial correlations are recovered. (iii) Equal time correlations: a straightforward extension of our RG analysis to the scaling function f^K in Eq. (7) yields, instead of the exponential decay of the bare correlation function shown in Fig. 3(b), a power law decay $f^K(x,1) = f^K_{\mathrm{eq}} - [2\theta\Gamma(2/z)/(c_z \sin(\pi/z))]x^{-2/z}$ with universal coefficient proportional to θ. (iv) The regime with $\theta < 0$: In this case no coarsening growth of the order parameter occurs, yet its decay is slowed down if compared to the quasiadiabatic regime. In addition, the correlation length recovers before the crossover time t^* is reached. (v) Duration of prethermalization: Since the crossover time t^* diverges for weak quenches, an almost critical system, subject to a sudden change of its parameters, undergoes universal out-of-equilibrium dynamics for arbitrarily long periods of time.

In conclusion, we determined universal behavior that governs quantum critical prethermalization. The intermediate time dynamics of a system that is suddenly moved to a nearby QCP is characterized by a new exponent θ. Owed to the quench, the system instantly falls out of equilibrium and breaks up into small correlated regions. The quantum critical prethermalization describes the recovery after this collapse and extends over long times, depending on the initial distance from the critical point. A quench close to a quantum critical point opens the possibility to quantitatively analyze the universal far-from-equilibrium dynamics of a many-body system and to manipulate the crossover between prethermalization and thermalization regimes.

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