Cherenkov Radiation from Short Relativistic Bunches: General Approach

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In recent years new interest in Cherenkov radiation has arisen based on progress in its new applications like biomedical imaging, photonic structures, metamaterials, and beam physics. These new applications require Cherenkov radiation theory of short bunches to be extended to rather more complicated media and structures than considered originally. We present a new general approach to the analysis of Cherenkov fields and loss factors for relativistic short bunches in arbitrary slow wave guiding systems. This new formalism is obtained by considering a general integral relation that allows calculation of the fields in the vicinity of the charge. The proposed approach dramatically simplifies simulations using analytical fields near the moving source of Cherenkov radiation.

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Introduction.—Cherenkov radiation (CR) from particle bunches is of considerable importance in many areas of fundamental and applied physics. Recent experiments and corresponding theoretical study were concentrated on CR sources ranging from low terahertz to visible light [1–3]. It was shown that an optical pulse can be used as a CR source [4], and that surface polaritons can be transformed into CR as well [5]. CR in metamaterials [6] and photonic crystals [7] was a subject of theoretical and experimental studies. Recently CR has been applied for biomedical imaging purposes [8]. The new research in the theory of CR [9] is also concentrated on radiation in a finite region of space, the Tamm problem and radiation of electric, magnetic, and toroidal dipoles [10].

The widest use of CR is in accelerator physics. Relativistic, high intensity, and small emittance electron bunches are the basis of linear collider [11,12] and free electron laser [13] projects. These bunches excite Cherenkov wakefields as long as electrons pass through the accelerating structures or other longitudinally extended components of a beam line (pipes, collimators, bellows) [1,14–22].

Theoretical analysis of CR commonly considers a "short bunch" approach. This is perfectly in agreement with Cherenkov generating systems, where the moving charge size is much less than the fundamental wavelength [9,10,16]. This includes Cherenkov imaging and capillary generation if the high frequency spectral range is not of interest [2–8]. This also holds for accelerating structures and other accelerator components, where the longitudinal size of electron bunches are significantly smaller than the lowest wavelength of the wakefields excited [1,14–16,19]: this defines a "short" bunch in this context.

We propose a new theoretical approach that can be used for obtaining direct analytical formulas for electromagnetic field components at the position of a pointlike Cherenkov radiation source that can be either a short electron bunch or laser pulse. The corresponding energy losses can be also calculated analytically. Here we define the loss factor as the modulus of the longitudinal electric field at the pointlike charge position divided by the absolute value of the charge [1,14,16]. For simplicity we consider a pointlike electron bunch passing through waveguides lined with arbitrary slowdown layers. It will be shown that the loss factor of the short bunch does not depend on the waveguide system material and is a constant for any given transverse dimensions and cross section of the waveguides. The equivalence and exact matching of the loss factor of beams passing through various waveguide configurations is analyzed. With the proposed approach one can use a relatively simple method for the calculation of the field components and loss factors using an integral relation, or "relativistic Gauss theorem" based on the cylindrical slow wave structure model. For various cross section geometries one can obtain the loss factor by using a conformal mapping from the solution for the cylindrical case.

Cherenkov wakefields and loss factors.—The equivalence of the loss factor of the beams passing through various types of waveguides with thin slowdown regions (features on the waveguide interior other than a smooth perfectly conducting surface) have been noted previously [14,15]. Indeed, the loss factor attains exactly the same value for all disk-loaded cylindrically symmetric structures [14,17,18,22], for a resistive pipe [1,16,23], a pipe with small periodic corrugations [19,20], and a cylindrical metal structure with a thin dielectric layer [19–21,24]. The same equivalence of the loss factor can be found for noncylindrical structures as well [14]. In a planar or rectangular all-metal waveguide with resistive walls [16,22], small corrugations [14,15], or a thin dielectric liner [24,25] the loss factor would be the same for structures with equal apertures: it is a constant that is dependent on transverse dimensions but independent of the material properties.

The wakefield experienced by a pointlike charge (loss factor) in a waveguide of fixed transverse dimensions is independent of the detailed properties of the slowdown layer: this is a strong indication that a more general theory of loss factors can be obtained. We have developed a new approach to the loss factor analysis of relativistic pointlike charges with only the assumption that the phase velocity of the CR in the waveguide layers is less than the speed of light (the bunch is assumed relativistic, $V = c$).

Consider an ultrarelativistic point charge located on the plane $z = ct$ moving with the speed of light at a center of a vacuum channel with azimuthal symmetry and radius $r = a$ [Figs. 1(a) and 1(b)]. Other geometries besides cylindrical can be calculated using the conformal mapping technique that will be discussed below. Electric and magnetic fields of the moving charge are present inside the channel, but the field on the plane outside the cylindrical channel $r > a$, $z = ct$ is zero. Here we consider that the field of the pointlike charge on the plane vanishes because of (1) the presence of slowdown walls or layers outside the channel so that $V_{ph} < c$, and (2) the bunch is ultrarelativistic, $\gamma \to \infty$. The combination of these two factors inevitably delays the CR fields at radii $r > a$ away from the plane, $z = ct$, moving with the bunch. This allows the formulation of a general integral relation for the loss factor of a short relativistic bunch passing through an arbitrary waveguide, independent of the channel shape, the properties of the walls, or its materials [as in Figs. 1(a) and 1(b)] if $V_{ph} < c$ at $r > a$.

Moreover, a special conclusion of this approach is that the loss factor of waveguides with dielectric, corrugated, or resistive slowdown regions does not depend on the layer thickness and gives the same results as those for the loss factor of the bunch passing the channel in an infinite dielectric or any other media; see Fig. 1(c). Note that the CR of the bunch moving through an infinite medium is the same as for a particle passing along a channel inside an unbounded dielectric [Fig. 1(c)], if the channel transverse dimensions are close to the CR wavelength [9,26]. Finally, if the fields in the area outside the vacuum channel vanish for any reasons other than the ultrarelativistic limit (diffraction shadow, etc.), the same integral relations will hold.

Field-particle interactions in high energy physics are usually described in terms of wake and impedance formalism: more details can be found elsewhere [1,16,23]. Consider a point charge moving with the speed of light along the axis of a vacuum accelerating structure. A test charge also moving with the speed $V = c$ at a distance s behind this pointlike bunch will experience fields of the first charge if the bunch separation s is greater than the so-called "catch-up" distance [1,16,23]. To describe the interaction between the first and second particle, a function $W(s)$ called the wake potential was introduced [1,16,23]. Vanishing of the wake functions everywhere in front of a relativistic particle is a consequence of causality: the wake potential is equal to zero for $s < 0$. Wake potentials can be expressed using an eigenmode decomposition, where $w_n(s)$ is the wake function of the nth mode,

$$W(s) = \sum_n \kappa_n w_n(s), \qquad \kappa = W(0) = \sum_n \kappa_n. \quad (1)$$

Here κ_n is the loss factor for the nth eigenmode. The total loss factor κ is usually defined as (1). In the case of a thin corrugation layer [14,16–21] or dielectric [14,19,21,25] the total loss factor is equal to the loss factor of the fundamental mode of the structure. The expression for the loss factor of a conductive cylindrical pipe [Fig. 2(a)] can be found elsewhere [14,16,23]. The loss factor of a relativistic pointlike charge passing through cylindrical, κ_c, [14,16–19] and planar, κ_p, [15] structures [Figs. 2(a) and 2(c)] can be expressed as

$$\kappa_c = \frac{1}{2\pi a_c^2 \varepsilon_0}, \qquad \kappa_p = \frac{1}{2\pi a_p^2 \varepsilon_0} \frac{\pi^2}{16}, \quad (2)$$

where ε_0 is the dielectric permittivity of vacuum, a_c is the pipe radius, and a_p is the vacuum half gap. Using the simple integration over the structure cross section presented in the next section, we can prove that formulas (2) and (3) can be applied in the case of a thick layer and thus show that the material thickness and properties do not affect the total loss factor. Moreover, the calculation method is quite simple and is based on an analog of Gauss's law. We neglect for the moment frequency dispersion.

Integral transformation.—Let us consider the circulation of the magnetic field on the metal boundary of a waveguide using the Maxwell-Ampère law; it could be written as

$$\int_{l_\perp} \mathbf{H} \cdot d\mathbf{l} = \iint_{S_\perp} \left(\frac{\partial D_z}{\partial t} + \rho V \right) dS, \quad (3)$$

where the integral on the right-hand side is calculated over the cross section $S_\perp$ of the waveguide, and the left-hand

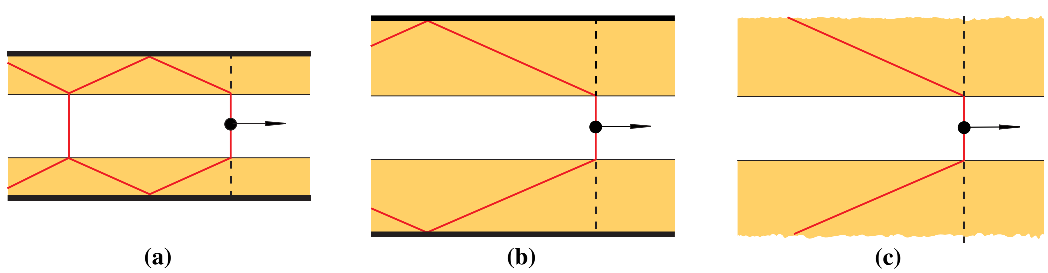

FIG. 1 (color online). Cherenkov wakefield cones of a pointlike charge moving along (a) a waveguide with a thin arbitrary slowdown layer on a metal surface; (b) waveguide with a thick layer; (c) infinite medium.

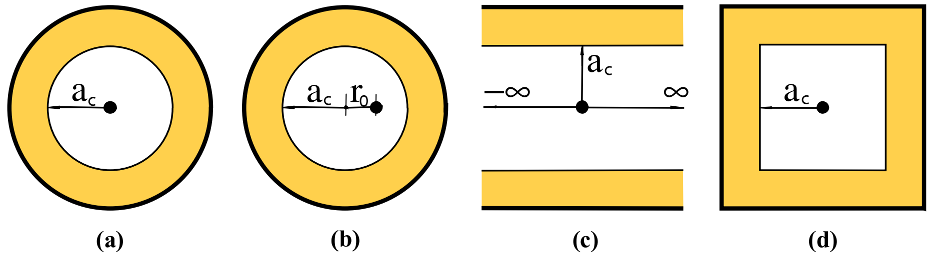

FIG. 2 (color online). Cross section of the considered metal waveguides with slowdown layers (yellow): (a) cylindrical, (b) cylindrical with displaced charge, (c) planar, and (d) square.

side integral is taken along the metal sleeve of the waveguide. Here $\rho = q\delta(z - Vt)\delta(x - x_0)\delta(y - y_0)$ is the charge density and $V = \mathbf{V}\cdot\mathbf{e}_z$ component of the charge velocity vector codirectional with the z axis. Now consider the waveguide cross sectional area $S_\perp^q$ that includes the charge. If we assume that our particle is moving with the speed of light $V \approx c$, and using the fact that in the medium the phase speed of light is lower than the speed of the moving charge, we can conclude that in the cross section $S_\perp^q$ the nonzero field is localized only in the vacuum gap. From this position in the limit $V \to c$, substituting $\zeta = Vt - z$ and rewriting (3) for $S_\perp^q$ we arrive at

$$\iint\limits_{S_\perp^q} \frac{\partial D_z}{\partial \zeta} dS = -q\delta(\zeta) + \frac{1}{c}\int\limits_{l_1} \mathbf{H}\cdot d\mathbf{l}. \qquad (4)$$

Here the integral on the right side is calculated along the metal surfaces of the waveguide l_1 that are not covered by the material, and D_z is the z component of the electric displacement vector that corresponds to the Cherenkov field. In the case where uncovered metal walls are not present, one can see that as long as the nonzero field is localized only in the vacuum gap, the integral on the right side is equal to zero. Now decompose the integral on the left side into an integral over the vacuum channel S_V and an integral over S_D, the cross section of the medium,

$$\varepsilon_0 \iint\limits_{S_V} \frac{\partial E_z^V}{\partial \zeta} dS = -q\delta(\zeta) + \varepsilon_0 \varepsilon \iint\limits_{S_D} \frac{\partial E_z^D}{\partial \zeta} dS. \qquad (5)$$

Integration of (5) with respect to ζ, with $E_z^V(-\infty) = 0$ and $E_z^D(-\infty) = 0$ leads to

$$\iint\limits_{S_V} E_z^V(\zeta) dS = -\frac{q}{\varepsilon_0}\int\limits_{-\infty}^{\zeta} \delta(x)dx + \varepsilon \iint\limits_{S_D} E_z^D(\zeta) dS. \qquad (6)$$

If now we set $\zeta = 0$ because the flux through S_D is zero, it immediately gives

$$\iint\limits_{S_V} E_z^V(0) dS = -\frac{q}{2\varepsilon_0}, \qquad (7)$$

and in the case $\zeta = 0^+$ a factor of 2 has to be applied. Formula (7) gives a simple connection between the longitudinal electric field in the cross section of the bunch and the total bunch charge, which looks like a classical Gauss's law. Using this expression we will show that radiation losses and transverse distribution of the electric field can be found using the well-known technique of conformal mapping.

Cylindrical waveguide, longitudinal loss factor.—Consider a round cross section of the metal waveguide with an arbitrary nonuniformity along the metal walls and a vacuum channel along the axis; see Fig. 2(a).

It is easy to show that in the case of a cylindrical structure, if $V \to c$ then $E_z^V(\zeta)$ does not depend on transverse coordinates when the particle is moving along the z axis of a cylinder. Thus for a cylinder from (7) we have

$$E_z^c(0) = -\frac{q}{2\pi a_c^2 \varepsilon_0}, \qquad E_z^c(0^+) = -\frac{q}{\pi a_c^2 \varepsilon_0}. \qquad (8)$$

Here a_c is the radius of the vacuum gap. It should be noticed that no assumptions on the thickness of the loading configuration (dielectric, corrugation, resistivity, etc.) were used while obtaining expressions (7) and (8). Typically only the thin layer approximation is considered in the loss factor analysis [14,16,20,21,24]. Based on the proposed approach one can conclude that the expressions (7) and (8) are true for any layer thickness including unbounded media as in Fig. 1(c).

Cylindrical waveguide: Kick factor.—If the beam traverses the waveguide off axis [Fig. 2(b)], a deflecting field will affect the beam [14–16]; if the offset distance of the beam is relatively small, only the dipole mode of almost the same frequency as the fundamental mode will be excited. If the beam is deflected with a larger offset, additional multipoles will contribute to the dipole deflection force [16,21].

The deflecting force factor or the "kick" factor for the cylindrical waveguide with an arbitrary slowdown layer will be presented in this section. Consider a conformal transformation of a circle $|\omega| \le a_c$ on a circle $|\psi| \le a_c$ such as that the point r_0 ($\mathrm{Arg}[r_0] = 0$) of the first circle transforms into the center $\psi = 0$ of a second circle. The corresponding mapping is then given by

$$\psi = a_c^2 \frac{\omega - r_0}{a_c^2 - \omega r_0}. \qquad (9)$$

We consider now an integral over the vacuum gap cross section along the ψ plane and rewrite it for the ω plane,

$$\int E_z^c dS_\psi = \int E_z^c J dS_\omega. \quad (10)$$

Here J is the determinant of the Jacobi matrix, dS_ψ is the elementary square of the ψ plane, and dS_ω is the surface square element on the ω plane. Using the fact that a conformal transformation is an analytic function, one can write

$$J = \left|\frac{dF(\omega)}{d\omega}\right|^2 = \left|\frac{d}{d\omega} a_c^2 \frac{\omega - r_0}{a_c^2 - \omega r_0}\right|^2. \quad (11)$$

Assuming $|\omega| = r$ and $\mathrm{Arg}[\omega] = \varphi$ one can obtain

$$J(r,\varphi) = \frac{a_c^4(a_c^2 - r_0^2)^2}{[a_c^4 + r^2 r_0^2 - 2 r r_0 a_c^2 \cos(\varphi)]^2}. \quad (12)$$

As long as E_z^c is a constant, we can conclude from (10) that the field distribution E_z^{dp} over the ω plane can be found as $E_z^{dp}(r, r_0, \varphi) = J(r,\varphi) E_z^c$. Thus at the origin ($r = r_0$, $\varphi = 0, \zeta = 0$) the longitudinal field can be written as

$$E_z^{dp}(r_0, r_0, 0) = -\frac{q}{2\pi a_c^2 \varepsilon_0} \frac{1}{[1 - (r_0/a_c)^2]^2}, \quad (13)$$

which corresponds to [16]. The radial part of the Lorentz force can be calculated using the Panofsky-Wenzel theorem [16,23]. The force derivative at the origin, also known as the kick factor, can be found if $r = r_0$ and $\varphi = 0$. Using (13) we obtain

$$\kappa_\perp^c = \frac{1}{q^2 r_0} \frac{\partial F_r(r,0)}{\partial \zeta}\bigg|_{\substack{r=r_0\\ \zeta=0}} = \frac{1}{q r_0} \frac{\partial E_z(r,0)}{\partial r}\bigg|_{\substack{r=r_0\\ \zeta=0}}$$
$$= \frac{1}{2\pi a_c^4 \varepsilon_0} \frac{4}{[1 - (r_0/a_c)^2]^3}, \quad (14)$$

$$\kappa_\perp^c \approx \frac{2}{\pi a_c^4 \varepsilon_0}[1 + 3(r_0/a_c)^2]; \qquad r_0/a_c \ll 1. \quad (15)$$

One can see that for small offsets $r_0/a_c \ll 1$ the first term of the kick factor (15) is equal to the well-known result for the kick factor of a pipe with small corrugations or resistive walls as expected [14,16,19–21,24]. Note the divergence of (14) at $r_0/a_c \to 1$ that corresponds to the dispersionless model of the slowdown layer. The same kick factor divergence is observed with the mode decomposition simulations [16,21].

Square waveguide: Longitudinal loss factor.—For a pointlike charge moving along the symmetry axis in between two infinitely long plates, [Fig. 2(c)] the conformal transformation of a strip onto the interior of a circle allows us to obtain the longitudinal loss factor corresponding to the right part of formula (2) [14,15]:

$$E_z^p(0) = -\frac{q}{2a_c^2\varepsilon_0}\frac{\pi}{16}; \qquad E_z^p(0^+) = -\frac{q}{a_c^2\varepsilon_0}\frac{\pi}{16}. \quad (16)$$

At the same time, the loss factor for a loaded square cross section metal structure [Fig. 2(d)] has not been previously calculated and it can be obtained using the Christoffel-Schwarz integral that gives a conformal mapping of the inner part of a circle $|\psi| < a_c$ to a square with each side equal to $2a_c$,

$$\omega = \frac{1}{\sqrt{f}} \int_0^\psi \frac{dt}{\sqrt{1-t^4}}, \quad \text{where } f = \frac{\pi}{2}\left[\frac{\Gamma(\frac{5}{4})}{\Gamma(\frac{3}{4})}\right]^2 \approx 0.86. \quad (17)$$

Here $\Gamma(x)$ is the Euler Gamma function. Taking into account that $\psi = r\exp(i\varphi)$, one can write the determinant of a Jacobi matrix for transformation of an elementary square as

$$J(r,\varphi) = \frac{1}{f}\frac{1}{\sqrt{1 + (r/a_c)^8 - 2(r/a_c)^4\cos(4\varphi)}}. \quad (18)$$

Taking into account (10) we have

$$E_z^{\mathrm{rec}}(r,\varphi) = \frac{E_z^{\mathrm{cyl}}}{J(r,\varphi)}, \quad E_z^{\mathrm{rec}}(0,\varphi) = \frac{E_z^{\mathrm{cyl}}}{J(0,\varphi)} = f E_z^{\mathrm{cyl}}, \quad (19)$$

where the right part of (18) is taken at the position of a pointlike charge ($r = 0$), and we have

$$E_z^{\mathrm{rec}}(0) = -f\frac{q}{2\pi a_c^2 \varepsilon_0}; \qquad E_z^{\mathrm{rec}}(0^+) = -f\frac{q}{\pi a_c^2\varepsilon_0}. \quad (20)$$

Discussion of the results.—Using (8), (16), and (20), one can obtain an expression for the loss factors of the pointlike charges for the cylindrical κ_c, planar κ_p, and square κ_{sq} metal waveguides with any kind of nonuniformity or slowdown material along the metal walls. Corresponding formulas are presented in Table I including the kick factor (15) for the cylindrical waveguide $\kappa_\perp^c$. Meanwhile the wake potentials at $\zeta = 0^+$ differ from the loss factors by a factor of 2 [1,16,23].

Formulas (2) and (3) for cylindrical and planar waveguides and those of Table I look identical, but we emphasize here that (2) and (3) as derived in Refs. [14–16,18–24] used assumptions on the particular mechanism (corrugation, dielectric) used to form a slow wave structure while our formulas presented in Table I were obtained in the general case formula (7). The kick factor formula (15), Table I, was obtained for the full solution including all multipoles, not only dipoles. Also, at first the loss factor formula (20), Table I, was derived here for a waveguide with the square cross section metal wall completely lined with the slowdown layer. This formula can be used for dielectric wakefield acceleration or THz generation devices [2,25,27,28]. In addition to the resistance and roughness, the waveguide wall may have an oxide layer, which is usually a dielectric. This effect for very short bunches was previously studied only in a round pipe [20,24].

Loss factor determination often becomes a complicated problem and involves massive numerical mode summations. With the new approach shown above, one can use relatively simple and yet powerful tools for the calculation of the asymptotical loss factors. Using the integral relation

TABLE I. Longitudinal loss factors for various cross sections of the waveguides [see Figs. 2(a), 2(c), 2(d)], and the kick for cylindrical waveguides (small offsets) [Fig. 2(b)].

Cylindrical	Planar	Square	Cylindrical; kick factor
$\kappa_c = (2\pi a_c^2 \varepsilon_0)^{-1}$	$\kappa_p = (2\pi a_c^2 \varepsilon_0)^{-1}(\pi^2/16)$	$\kappa_{sq} = 0.86(2\pi a_c^2 \varepsilon_0)^{-1}$	$\kappa_\perp^c = 2(\pi a_c^4 \varepsilon_0)^{-1}[1 + 3(r_0/a_c)^2]$

on the basis of the cylindrical slowdown waveguide model, the full loss factor of the structure can be calculated. For other cross section geometries one can obtain the loss factor by use of a conformal mapping that allows finding the ratio of the known loss factor for a cylindrical structure to that of the other structure of interest. The loss factor in this case is simply the value of the Jacobi matrix determinant at the origin, and the Jacobi determinant away from the origin gives the transverse structure of the loss factor.

For many practical applications, impedance boundary conditions (IBC) or Leontovich conditions are commonly used [1,14–16,23]. The question had arisen [29] whether the integral relation method, formula (7), can be applied for IBC. In [30], it was demonstrated that the IBC can be also applied if integral relation (7) is used. It was first shown that the theorem (7) can be derived directly from the Maxwell equations even if (instead of the standard boundary conditions) only the IBC approximation is applied. The same approach was then extended to the solution for a dispersive medium. Finally, wakefield calculations for the Leontovich conditions were carried out for an arbitrary slowdown waveguide using both the standard mode decomposition method and the proposed integral theorem formula (7), and were found to give identical results [30].

In conclusion, we considered the Cherenkov fields and loss factors of a pointlike electron bunch passing through waveguides lined with arbitrary slowdown layers. It was shown that the Cherenkov loss factor of the short bunch does not depend on the waveguide system material and is a constant for any given transverse dimensions and cross sections of the waveguides. The exact matching of the loss factor of the beams passing through various types of waveguides was analyzed. It was shown that with the proposed approach one can use a relatively simple method for the calculation of the total loss factor using an integral relation based on the cylindrical slowdown waveguide model.

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