W^+W^- Production at Hadron Colliders in Next to Next to Leading Order QCD

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Charged gauge boson pair production at the Large Hadron Collider allows detailed probes of the fundamental structure of electroweak interactions. We present precise theoretical predictions for on-shell W^+W^- production that include, for the first time, QCD effects up to next to next to leading order in perturbation theory. As compared to next to leading order, the inclusive W^+W^- cross section is enhanced by 9% at 7 TeV and 12% at 14 TeV. The residual perturbative uncertainty is at the 3% level. The severe contamination of the W^+W^- cross section due to top-quark resonances is discussed in detail. Comparing different definitions of top-free W^+W^- production in the four and five flavor number schemes, we demonstrate that top-quark resonances can be separated from the inclusive W^+W^- cross section without a significant loss of theoretical precision.

Vector boson pair production is among the most important electroweak processes at hadron colliders. It allows for detailed studies of the gauge symmetry structure of electroweak interactions and of the mechanism of electroweak symmetry breaking. Any deviation from standard model expectations in measured production rates and kinematical distributions of vector boson pairs or their decay products could provide the first evidence for new-physics effects at the high-energy frontier. Vector boson pair production is, moreover, an important background in measurements of Higgs boson production [1,2] and in direct searches for new particles.

Among the massive vector boson pair production reactions, W^+W^- takes a special role, by having a larger cross section than $W^\pm Z$ and ZZ production, while at the same time producing the most challenging final state, with $W^+W^- \rightarrow l^+\nu l^-\bar{\nu}$. Because of the presence of two neutrinos, it does not allow us to reconstruct mass peaks, and its control requires a very thorough understanding of the W^+W^- signal and its background contamination. Measurements of W^+W^- hadroproduction have been carried out at the Tevatron [3,4] and the LHC [5–7]. The recent observation of the ATLAS Collaboration of a total W^+W^- cross section at 8 TeV in excess of theoretical expectations [8] has triggered intense discussion since the W^+W^- signature appears in many new-physics models [9]. In order to establish or refute this excess, it is necessary to have a solid theoretical prediction (with a reliable estimate of its residual uncertainty) for W^+W^- production. In this Letter, we bring this prediction to a new level of accuracy with the first-ever computation of next to next to leading order (NNLO) QCD corrections to the inclusive W^+W^- hadroproduction cross section.

Following the leading order (LO) estimate of the W^+W^- cross section [10], next to leading order (NLO) QCD corrections [11,12] were first evaluated by considering stable W bosons. The computation of the relevant one-loop helicity amplitudes [13] allowed complete NLO calculations [14,15], including spin correlations and off-shell effects. The loop-induced gluon fusion contribution, which is formally NNLO, has been computed in Refs. [16,17]. More recently, the corresponding leptonic decays have been included [18,19], and the interference [20] with the $gg \rightarrow H$ signal has been taken into account [19,21]. Since the gluon-induced contribution is enhanced by the gluon luminosity, it is often assumed to provide the bulk of the NNLO corrections. NLO predictions for W^+W^- production including the gluon-induced contribution, the leptonic decay with spin correlations, and off-shell effects have been presented in Ref. [22]. The NLO QCD corrections to $W^+W^- + $jet production have been discussed in Refs. [23–25], and even NLO results for $W^+W^- + 2$jets are available [26,27]. The effects of transverse-momentum [28–30], jet veto [31], and threshold [32] resummation for W^+W^- production have also been investigated. The electroweak corrections to this process have been computed in Refs. [33–35]. Detailed Monte Carlo simulations of $e^+\nu_e\mu^-\bar{\nu}_\mu$ production in association with up to one jet at NLO have been presented in Ref. [36].

In this Letter we report on the first calculation of the inclusive production of on-shell W-boson pairs at hadron colliders in NNLO QCD. The calculation parallels the one presented for Z-boson pairs in Ref. [37], but differs from it in one important respect. The higher-order QCD corrections to W^+W^- production include partonic channels with b quarks in the final state, which leads to a subtle interplay between W^+W^- and top production processes [23,36]. In the five flavor number scheme (FNS), where b quarks are included in the parton distribution functions and their mass is set to zero, the presence of real b-quark emission is crucial to canceling collinear singularities that arise from $g \rightarrow b\bar{b}$ splittings in the virtual corrections. At the same time, the occurrence of Wb pairs in real-emission matrix elements induces top-quark resonances that lead to a problematic contamination of W^+W^- production. The problem starts with the NLO cross section, which receives a contribution of about 30% (60%) at

7 (14) TeV from $pp \to W^{\pm}t \to W^+W^-b$, and at NNLO the appearance of doubly resonant $pp \to t\bar{t} \to W^+W^-b\bar{b}$ channels enhances the W^+W^- cross section by about a factor 4 (8).

This huge contamination calls for a theoretical definition of W^+W^- production where top contributions are completely subtracted, similar to the experimental measurements of the W^+W^- cross section [3–8]. However, the need to cancel collinear $g \to b\bar{b}$ singularities does not allow for a trivial separation of W^+W^- and top production in the 5FNS. To address this issue, two different definitions of W^+W^- production will be adopted and compared in this Letter. The first definition is based on the 4FNS. In this case, since b quarks are massive and collinear divergences are not present, we define top-free W^+W^- production by simply omitting b-quark emissions. Alternatively, we will adopt a 5FNS definition of W^+W^- production where b-quark emissions are included. In this case, for a consistent separation of the tW and $t\bar{t}$ contributions we will introduce a top subtraction based on the scaling behavior of the (N)NLO cross section in the limit of vanishing top-quark width. The comparison of 4FNS and 5FNS predictions will permit us to quantify the theoretical ambiguities inherent in a top-free definition of the W^+W^- cross section at NNLO.

The computation of NNLO corrections requires the evaluation of the tree-level scattering amplitudes with two additional (unresolved) partons, of the one-loop amplitudes with one additional parton and the one-loop-squared and two-loop corrections to the Born subprocess $q\bar{q} \to W^+W^-$. In our calculation, all required tree and one-loop matrix elements are automatically generated with OPENLOOPS [38], which implements a fast numerical recursion for the calculation of NLO scattering amplitudes within the standard model. For the numerically stable evaluation of tensor integrals, we rely on the COLLIER library [39], which is based on the Denner-Dittmaier reduction techniques [40,41] and the scalar integrals of Ref. [42]. To check and further improve the numerical stability of exceptional phase space points the quadruple precision implementation of the integrand reduction method [43] in CUTTOOLS [44] is employed in combination with ONELOOP [45]. Following the recent computation of the relevant two-loop master integrals [46–50], the last missing contribution—the genuine two-loop correction to the W^+W^- amplitude—has been computed by some of us and will be reported elsewhere [51], thereby improving upon earlier results in the high-energy limit [52]. For the numerical evaluation of the multiple polylogarithms in the two-loop expressions, we employ the implementation [53] in the GINAC [54] library. Heavy quarks—i.e., top quarks and, in the 4FNS, b-quarks also—are consistently included in one-loop contributions to NLO and NNLO. For their contribution to the one-loop renormalization of α_S, we adopt the decoupling scheme used in Ref. [37]. At two-loop order, heavy quarks are neglected. Based on the size of two-loop contributions with a massless-quark loop, we estimate that their impact will be well below the per mille level.

The implementation of the various scattering amplitudes in a complete NNLO calculation is a nontrivial task due to the presence of infrared (IR) singularities at intermediate stages of the calculation that prevent a straightforward application of numerical techniques. To handle and cancel these singularities at NNLO, we employ the q_T subtraction method [55]. This approach determines the IR singular behavior of real radiation contributions from the resummation of logarithmically enhanced contributions to q_T distributions. In the case of the production of a colorless high-mass system, the q_T subtraction method is fully developed [56,57] thanks to the computation of the relevant hard-collinear coefficients [58,59], later confirmed with an independent calculation in the framework of soft-collinear effective theory [60,61]. The q_T subtraction method has been used for the computation of NNLO corrections to several hadronic processes [37,55,62–66].

We have performed our NNLO calculation for W^+W^- production starting with a computation of the $d\sigma_{\mathrm{NLO}}^{W^+W^-+\mathrm{jet}}$ cross section with the dipole-subtraction method [67,68]. The numerical calculation employs the generic Monte Carlo program that was developed for Refs. [37,65]. Although the q_T subtraction method and our implementation are suitable to perform a fully exclusive computation of W^+W^- production including the leptonic decays and the corresponding spin correlations, in this Letter we restrict ourselves to the inclusive production of on-shell W bosons.

In the following, we present LO, NLO, and NNLO predictions for $pp \to W^+W^- + X$ with $\sqrt{s}$ ranging from 7 to 14 TeV. We use the MSTW2008 sets of parton distribution functions (PDFs) with four [69] or five [70] active flavors. Parton densities and the related α_S values are evaluated at each corresponding order; i.e., we use $(n + 1)$-loop α_S at N^{n}LO, with $n = 0, 1, 2$. The default renormalization (μ_R) and factorization (μ_F) scales are set to $\mu_R = \mu_F = m_W$, and to assess scale uncertainties they are varied in the range $0.5m_W < \mu_{R,F} < 2m_W$, with $0.5 < \mu_F/\mu_R < 2$. In the 4FNS we use the pole mass $m_b = 4.75$ GeV, while in the 5FNS b quarks are massless. The electroweak parameters are defined in the G_μ scheme, with $G_F = 1.16639 \times 10^{-5}$ GeV^{-2}, $m_W = 80.399$ GeV, and $m_Z = 91.1876$ GeV. Our NLO and NNLO predictions involve resonant top quarks and off-shell Higgs bosons, and for the respective mass and width parameters, we use $m_t = 173.2$ GeV, $\Gamma_t = 1.443$ GeV, $m_H = 125$ GeV, and $\Gamma_H = 4.09$ MeV. Higgs contributions are included via squared one-loop amplitudes in the $gg \to H^* \to W^+W^-$ channel but are strongly suppressed by the off-shellness of the Higgs boson.

In Table I we present LO, NLO, and NNLO predictions for inclusive W^+W^- production in the 4FNS, where top

TABLE I.   LO, NLO, and NNLO cross sections (in picobarn) for on-shell W^+W^- production in the 4FNS and reference results for $gg \to H \to WW^*$ from Ref. [71].

$\sqrt{s}/\mathrm{TeV}$	σ_{LO}	σ_{NLO}	σ_{NNLO}	$\sigma_{gg\to H\to WW^*}$
7	$29.52^{+1.6\%}_{-2.5\%}$	$45.16^{+3.7\%}_{-2.9\%}$	$49.04^{+2.1\%}_{-1.8\%}$	$3.25^{+7.1\%}_{-7.8\%}$
8	$35.50^{+2.4\%}_{-3.5\%}$	$54.77^{+3.7\%}_{-2.9\%}$	$59.84^{+2.2\%}_{-1.9\%}$	$4.14^{+7.2\%}_{-7.8\%}$
13	$67.16^{+5.5\%}_{-6.7\%}$	$106.0^{+4.1\%}_{-3.2\%}$	$118.7^{+2.5\%}_{-2.2\%}$	$9.44^{+7.4\%}_{-7.9\%}$
14	$73.74^{+5.9\%}_{-7.2\%}$	$116.7^{+4.1\%}_{-3.3\%}$	$131.3^{+2.6\%}_{-2.2\%}$	$10.64^{+7.5\%}_{-8.0\%}$

contributions are removed by omitting b-quark emissions. We see that at 7 (14) TeV the LO predictions receive a positive NLO shift of 53% (58%), and the NNLO corrections induce a further enhancement of 9% (12%). The decent perturbative convergence is contrasted by the observation that the scale uncertainty does not significantly decrease when moving from LO to NLO and NNLO. Moreover, the NNLO (NLO) corrections turn out to exceed the scale uncertainty of the NLO (LO) predictions by up to a factor 3 (34). The fact that LO and NLO scale variations underestimate higher-order effects can be attributed to the fact that the gluon-quark- and gluon-gluon-induced partonic channels, which yield a sizable contribution to the W^+W^- cross section, appear only beyond LO and NLO, respectively. The NNLO is the first order at which all partonic channels are contributing. The NNLO scale dependence, which amounts to about 3%, can thus be considered a realistic estimate of the theoretical uncertainty due to missing higher-order effects.

In Fig. 1, theoretical predictions in the 4FNS are compared to CMS and ATLAS measurements at 7 and 8 TeV [5–8]. For a consistent comparison, our results for on-shell W^+W^- production are combined with the $gg \to H \to WW^*$ cross sections reported in Table I. It turns out that the inclusion of the NNLO corrections leads to an excellent description of the data at 7 TeV and decreases the significance of the observed excess at 8 TeV. In the lower frame of Fig. 1, predictions and scale variations at NNLO are compared to NLO ones, and the individual contribution of the $gg \to W^+W^-$ channel is also shown. Using NNLO PDFs throughout, the loop-induced gluon fusion contribution is only about 35% of the total NNLO correction.
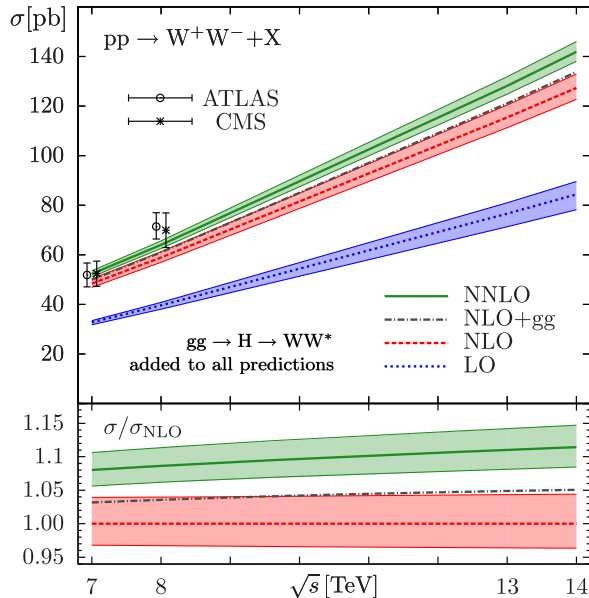

FIG. 1 (color online).   The on-shell W^+W^- cross section in the 4FNS at LO (dotted line), NLO (dashed line), NLO + gg (dot-dashed line), and NNLO (solid line) combined with $gg \to H \to WW^*$ is compared to recent ATLAS and CMS measurements [5–8]. In the lower panel, NNLO and NLO + gg results are normalized to NLO predictions. The bands describe scale variations.

In light of the small scale dependence of the 4FNS NNLO cross section, the ambiguities associated with the definition of a top-free W^+W^- cross section and its sensitivity to the choice of the FNS might represent a significant source of theoretical uncertainty at NNLO. In particular, the omission of b-quark emissions in our 4FNS definition of the W^+W^- cross section implies potentially large logarithms of m_b in the transition from the 4FNS to the 5FNS. To quantify this kind of uncertainty, we study the NNLO W^+W^- cross section in the 5FNS and introduce a subtraction of its top contamination that allows for a consistent comparison between the two FNSs. An optimal definition of W^+W^- production in the 5FNS requires maximal suppression of the top resonances in the $pp \to W^+W^-b$ and $pp \to W^+W^-b\bar{b}$ channels. At the same time, the cancellation of collinear singularities associated with massless $g \to b\bar{b}$ splittings requires a sufficient level of inclusiveness. The difficulty of fulfilling both requirements is clearly illustrated in Fig. 2 (left panel), where 5FNS predictions are plotted versus a b-jet veto that rejects b jets with $p_{T,\text{bjet}} > p_{T,\text{bjet}}^{\text{veto}}$ over the whole rapidity range and are compared to 4FNS results. In the inclusive limit, $p_{T,\text{bjet}}^{\text{veto}} \to \infty$, the higher-order corrections in the 5FNS suffer from a huge top contamination. At 7 (14) TeV the resulting relative enhancement with respect to the 4FNS amounts to about 30% (60%) at NLO and a factor 4 (8) at NNLO. In principle, it can be suppressed through the b-jet veto. However, for natural jet veto values around 30 GeV the top contamination remains larger than 10% of the W^+W^- cross section, and a complete suppression of the top contributions requires a veto of the order of 1 GeV. Moreover, as $p_{T,\text{bjet}}^{\text{veto}} \to 0$, the (N)NLO cross section does not approach a constant, but, starting from $p_{T,\text{bjet}}^{\text{veto}} \sim 10$ GeV, it displays a logarithmic slope due to singularities associated with initial state $g \to b\bar{b}$ splittings. This sensitivity to the jet veto parameters represents a theoretical ambiguity at the several percent level, which is inherent in the definition of top-free W^+W^- production based on a b-jet veto.

To circumvent this problem we will adopt an alternative definition of the W^+W^- cross section in the 5FNS, where resonant top contributions are subtracted along the lines of Refs. [73,74] by exploiting their characteristic scaling behavior in the limit of vanishing top-quark width. The idea is that doubly (singly) resonant contributions feature a quadratic (linear) dependence on $1/\Gamma_t$, while top-free W^+W^- contributions are not enhanced at small Γ_t. Using this scaling property, the $t\bar{t}$, $tW^\pm$, and (top-free) W^+W^- components in the 5FNS are determined from high-statistics evaluations of the 5FNS cross section at different values of Γ_t. The 5FNS top-free W^+W^- cross section σ_{WW}^{5F}, defined in this way, is presented in Fig. 2 (right panel) for $\sqrt{s} = 8$ TeV. Its dependence on the b-jet veto demonstrates the consistency of the employed top subtraction: at $p_{T,\text{bjet}}^{\text{veto}} \to 0$ we clearly observe the above-mentioned QCD singularity from initial-state $g \to b\bar{b}$, while for $p_{T,\text{bjet}}^{\text{veto}} \gtrsim 10$ GeV, consistent with the absence of top contamination, σ_{WW}^{5F} is almost insensitive to the veto. Thus the inclusive limit of σ_{WW}^{5F} can be used as a precise theoretical definition of W^+W^- production in the 5FNS, and compared
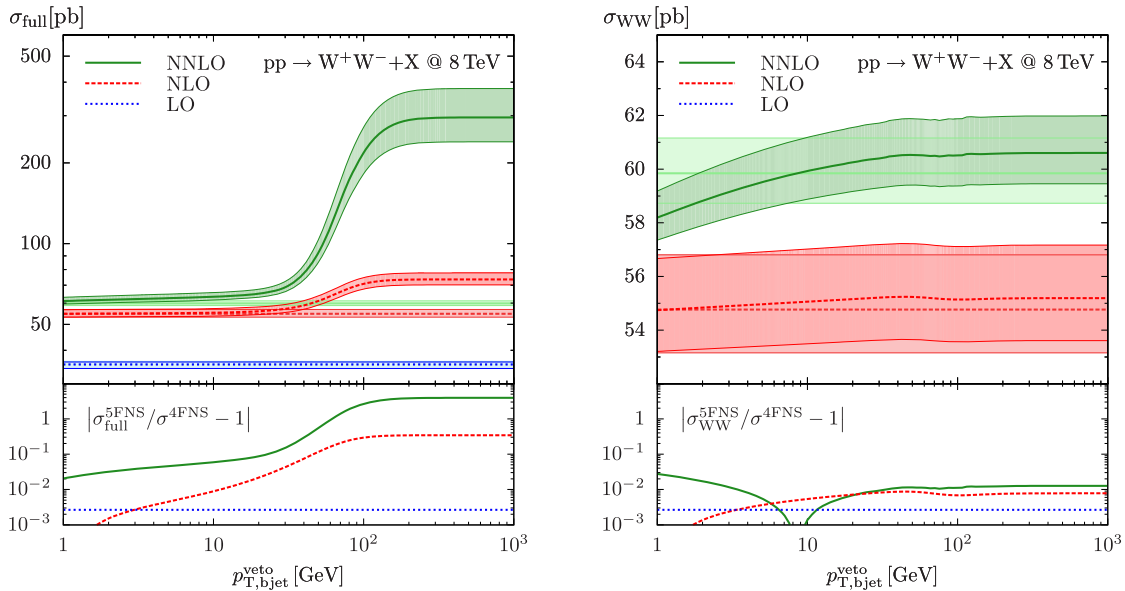
FIG. 2 (color online).   The $pp \to W^+W^-$ cross section in the 5FNS at $\sqrt{s} = 8$ TeV is plotted versus a b-jet veto, $p_{T,\text{bjet}} < p_{T,\text{bjet}}^{\text{veto}}$, and compared to results in the 4FNS (which are $p_{T,\text{bjet}}^{\text{veto}}$ independent). Full 5FNS results (left panel) are contrasted with top-subtracted 5FNS predictions (right panel). The relative agreement between 5FNS and 4FNS results is displayed in the lower frames. Jets are defined using the anti-k_T algorithm [72] with $R = 0.4$, and in order to guarantee the cancellations of final-state collinear singularities, $b\bar{b}$ pairs that are recombined by the jet algorithm are not vetoed.

to the 4FNS. The agreement between the two schemes turns out to be at the level of 1% (2%) at 7 (14) TeV, and this finding puts our NNLO results and their estimated uncertainty on a firm theoretical ground. Additional uncertainties from PDFs are at the 1%–2% level at NLO if one considers CT10, MSTW2008, and NNPDF variations in the 5FNS [8]. At NNLO, repeating our calculation in the 4FNS with NNPDFS [75] we found a shift well below 1%. A thorough NNLO analysis of PDF uncertainties will be presented elsewhere.

In summary, we have presented the first NNLO calculation of the total W^+W^- production cross section at the LHC. The W^+W^- signature is of crucial importance to precision tests of the fundamental structure of electroweak interactions and provides an important background in Higgs boson studies and searches for new physics. Introducing consistent theoretical definitions of W^+W^- production in the four and five flavor number schemes, we have demonstrated that the huge top contamination of the W^+W^- signal can be subtracted without significant loss of theoretical precision. The NNLO corrections to W^+W^- production increase from 9% at 7 TeV to 12% at 14 TeV, with an estimated 3% residual uncertainty from missing contributions beyond NNLO. Gluon fusion amounts to about 35% of the total NNLO contribution. The inclusion of the newly computed NNLO corrections provides an excellent description of recent measurements of the W^+W^- cross section at 7 TeV and diminishes the significance of an observed excess at 8 TeV. In the near future more differential studies at NNLO, including leptonic decays and off-shell effects, will open the door to high-precision phenomenology with W^+W^- final states.

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