



## Spacetime Curvature and the Higgs Stability During Inflation

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(Received 4 September 2014; published 17 November 2014)

It has been claimed that the electroweak vacuum may be unstable during inflation due to large fluctuations of the order  $H$  in the case of a high inflationary scale as suggested by BICEP2. We compute the standard model Higgs effective potential including UV-induced curvature corrections at one-loop level. We find that for a high inflationary scale a large curvature mass is generated due to renormalization group running of nonminimal coupling  $\xi$ , which either stabilizes the potential against fluctuations for  $\xi_{\text{EW}} \gtrsim 6 \times 10^{-2}$ , or destabilizes it for  $\xi_{\text{EW}} \lesssim 2 \times 10^{-2}$  when the generated curvature mass is negative. Only in the narrow intermediate region may the effect of the curvature mass be significantly smaller.

DOI: [10.1103/PhysRevLett.113.211102](https://doi.org/10.1103/PhysRevLett.113.211102)

PACS numbers: 98.80.Cq, 04.62.+v, 11.10.Hi

After the confirmed detection of the standard model (SM) Higgs boson, a substantial amount of work has been devoted to investigating its ramifications in the early Universe. The measured Higgs mass  $m_H \sim 125$  GeV lies in the range where no new physics between the electroweak scale and inflation is necessarily required by theoretical self-consistency [1–3]. However, if primordial gravitational waves possibly suggested by BICEP2 data [4] would be detected, the implied high inflationary scale  $H \gg 10^9$  GeV has been argued to be in tension with the pure SM Higgs [5,6].

According to the effective potential computed in Minkowski space, the SM vacuum is separated from the unstable false vacuum by a barrier of height  $V_{\text{max}}^{1/4} \sim 10^9$  GeV [2]. If  $H \gg 10^9$  GeV, the inflationary fluctuations of the effectively massless Higgs field immediately trigger a transition to the false vacuum as the probability density at the barrier scales as  $P \sim \exp(-8\pi^2 V_{\text{max}}/3H^4)$  [5,6]. Within the standard model it would therefore appear rather unlikely that our observable patch of the Universe would have survived in the SM vacuum for the observationally required  $N \sim 60$   $e$ -folds of inflation. Stabilizing the SM vacuum against inflationary fluctuations,  $V_{\text{max}}^{1/4} \gtrsim H$ , would require either a low top mass at least two sigma below the best fit value or new physics modifying the Higgs potential above the electroweak scale [5,6].

The conclusion however relies on the effective potential computed in Minkowski space and one should ask if the effects of curvature can be neglected during inflation. Indeed, treating the SM fields as test fields in a de Sitter background, the Higgs sector acquires a nonminimal coupling to gravity  $\xi(\mu)R\Phi^\dagger\Phi$  through loop corrections even if  $\xi = 0$  at tree level [7]. This effect and other curvature corrections could play a significant role under a high scale inflation where  $R = 12H^2$  would be much

larger than the (Minkowski) instability scale. In a tree-level analysis in [5] it was indeed found that the generated curvature mass stabilizes the SM vacuum during inflation for  $\xi \gtrsim 10^{-1}$ ; however, loop corrections in the curved background space and the renormalization group (RG) running of the  $\xi$ -coupling were not addressed.

Ultimately, the quantity of interest is the transition rate or probability from the EW vacuum to the unstable false vacuum. In order to compute it reliably, one should track the evolution of the Higgs fluctuations during inflation, which can be done for example by the stochastic Fokker-Planck (FP) equation (cf. Ref. [8]). Clearly, curvature-induced corrections by large IR fluctuations in de Sitter space are already accounted for by the FP equation, whereas the UV effects such as RG running of couplings are not reproduced, and hence must be incorporated directly in the input potential.

Therefore, it appears that a reliable approximation scheme would be to incorporate the UV-induced curvature corrections (arising from the UV part of the loop integrals) in the input effective potential, whereas the IR effects would be accounted for by the FP evolution equation itself.

In this work we compute the RG improved effective potential of the Standard Model Higgs in de Sitter space consistently accounting for the UV-induced curvature corrections and their RG running with the measured best fit values of SM parameters at the EW scale as the input. We treat the SM fields as test fields in a fixed inflationary background space assuming inflation is driven by new physics not directly coupled to SM. The renormalization procedure is therefore not hampered by ambiguities of the gravitational sector as for example in the Higgs inflation [9]. We find that for a high inflationary scale the curvature corrections are generally significant and either stabilize or

destabilize the potential against inflationary fluctuations, depending on the value of  $\xi$ -coupling at the EW scale.

Derivation of the effective action in curved background has been extensively studied in the literature [10]. Here we use the resummed heat kernel expansion method [11] which incorporates complete UV contributions from the loop integrals at one-loop level. Unlike typically in the literature [10], we do not treat the curvature scale  $R$  as a small expansion parameter, allowing us to consider the case where  $R$  is larger than the tree-level masses of the standard model particles. Moreover, we fully incorporate RG improvement on top of the one-loop effective potential to lift the dependence on the renormalization scale, which is crucially important considering the large hierarchy between the EW and inflationary scales. This provides a significant improvement over the RG improved tree-level potential, for which the renormalization scale dependence is not canceling up to one-loop level.

Using this method, we find for the standard model one-loop effective potential in de Sitter space improved by one-loop RG equations in the 't Hooft–Landau gauge and the  $\overline{MS}$  scheme [12]

$$V_{\text{eff}} = -\frac{1}{2}m^2(t)\phi^2(t) + \frac{1}{2}\xi(t)R\phi^2(t) + \frac{1}{4}\lambda(t)\phi^4(t) + \sum_{i=1}^9 \frac{n_i}{64\pi^2} M_i^4(\phi) \left[ \log \frac{|M_i^2(\phi)|}{\mu^2(t)} - c_i \right], \quad (1)$$

with

$$M_i^2(\phi) = \kappa_i \phi^2(t) - \kappa'_i + \theta_i R, \quad (2)$$

where the coefficients  $n_i$ ,  $\kappa_i$ ,  $\kappa'_i$ , and  $\theta_i$  for various contributions are given by Table I. The parameters  $\lambda(t)$  and  $m(t)$  are the SM quartic coupling and mass, whereas  $g(t)$ ,  $g'(t)$ , and  $y_t(t)$  are the SU(2), U(1), and top Yukawa couplings respectively, while  $\xi(t)$  is the Higgs nonminimal coupling to gravity [13]. All of them are running with renormalization group equations (RGE). The running of the Higgs field is given by

TABLE I. Contributions to the effective potential (1) from  $W^\pm$ ,  $Z^0$ , top quark  $t$ , Higgs  $\phi$ , and the Goldstone bosons  $\chi_{1,2,3}$ .

$\Phi$	$i$	$n_i$	$\kappa_i$	$\kappa'_i$	$\theta_i$	$c_i$
$W^\pm$	1	2	$g^2/4$	0	1/12	3/2
	2	6	$g^2/4$	0	1/12	5/6
	3	-2	$g^2/4$	0	-1/6	3/2
	4	1	$(g^2 + g'^2)/4$	0	1/12	3/2
$Z^0$	5	3	$(g^2 + g'^2)/4$	0	1/12	5/6
	6	-1	$(g^2 + g'^2)/4$	0	-1/6	3/2
	7	-12	$y_t^2/2$	0	1/12	3/2
$\phi$	8	1	$3\lambda$	$m^2$	$\xi - 1/6$	3/2
$\chi_i$	9	3	$\lambda$	$m^2$	$\xi - 1/6$	3/2

$$\phi(t) = Z(t)\phi_c, \quad Z(t) = \exp \left[ - \int_0^t dt' \gamma(t') \right], \quad (3)$$

where  $\phi_c$  is the classical field and  $\gamma(t)$  is the Higgs field anomalous dimension. The scale  $\mu(t)$  is related to the running parameter by

$$\mu(t) = m_t e^t, \quad (4)$$

where we have set the fixed scale at  $t = 0$  equal to physical top quark mass  $m_t$ . A direct comparison with the flat space results [14,15] shows that in this approximation spacetime curvature modifies the form of the effective potential only by shifting the effective masses by gravitational contributions proportional to  $R$ . We present more detailed derivation of the effective potential (1) elsewhere [16]. For example, a generic gauge field contribution in arbitrary curved spacetime is given by

$$V_{\text{eff}}^{(\text{gauge})} = \frac{1}{64\pi^2} \left\{ \text{tr} \left[ M_g^4 \left( \log \frac{|M_g^2|}{\tilde{\mu}^2} - \frac{3}{2} \right) \right] - M_s^4 \left( \log \frac{|M_s^2|}{\tilde{\mu}^2} - \frac{3}{2} \right) \right\}, \quad (5)$$

with  $(M_g^2)^\mu{}_\nu = M_s^2 \delta^\mu{}_\nu + R^\mu{}_\nu$ ,  $M_s^2 = m^2 - R/6$ , where  $m^2$  is the gauge boson mass term in the quadratic action and  $\log(\tilde{\mu}^2) \equiv \log(4\pi\mu^2) + 2/(4-d) - \gamma_E$  contains the dimensional pole in the limit  $d \rightarrow 4$ . The relevant terms in the potential (1) are then found by computing the trace over spacetime indices using de Sitter space expression for the Ricci tensor,

$$R = 12H^2, \quad (6)$$

with  $R^\mu{}_\nu = \delta^\mu{}_\nu R/4$ .

The RG running is determined by the one-loop  $\beta$ - and  $\gamma$ -functions and the boundary conditions at the EW scale. At one-loop level the nonminimal gravity coupling  $\xi$  does not couple into the  $\beta$ -functions of the SM couplings, which are therefore given by their usual expressions with next-to-leading order boundary conditions (cf. [3,14]), resulting in the standard one-loop running as shown in Fig. 1. The  $\beta$ -function for the nonminimal coupling  $\xi$  scales as  $\beta_{m^2}$  [17] and is given by [18]

$$16\pi^2 \beta_\xi = \left( \xi - \frac{1}{6} \right) \left( 12\lambda + 6y_t^2 - \frac{3}{2}g^2 - \frac{9}{2}g'^2 \right). \quad (7)$$

It can be directly integrated by using the solutions for the running SM couplings to get

$$\xi(t) = \frac{1}{6} + \left( \xi_{\text{EW}} - \frac{1}{6} \right) \Xi(t), \quad (8)$$

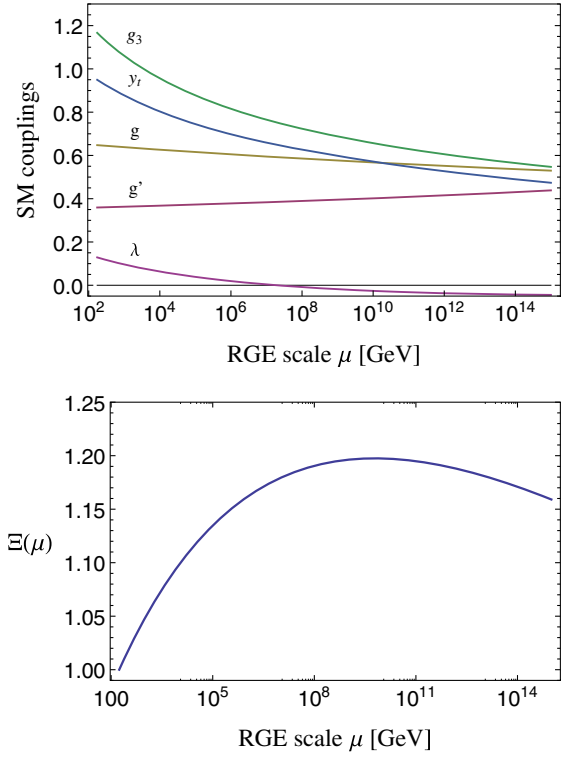


FIG. 1 (color online). RGE running of the SM couplings  $g_3$ ,  $y_t$ ,  $g$ ,  $g'$ ,  $\lambda$ , and the nonminimal gravity coupling  $\Xi \equiv (\xi - 1/6)/(\xi_{EW} - 1/6)$ .

where  $\Xi(t)$  is shown in Fig. 1 and  $\xi_{EW} \equiv \xi(m_t)$  is the initial value at the electroweak scale.

The potential (1) is renormalization scale  $\mu(t)$  invariant, if the derivative  $dV_{\text{eff}}/dt$  vanishes. By direct computation we get

$$\begin{aligned} \frac{dV_{\text{eff}}}{dt} = & \phi^4 \left( \frac{1}{4} \beta_\lambda - \gamma_\lambda - \sum_i \frac{n_i \kappa_i^2}{32\pi^2} \right) \\ & - \phi^2 \left( \frac{1}{2} \beta_{m^2} - \gamma_{m^2} - \sum_i \frac{n_i \kappa_i \kappa'_i}{16\pi^2} \right) \\ & + \phi^2 R \left( \frac{1}{2} \beta_\xi - \gamma_\xi - \sum_i \frac{n_i \kappa_i \theta_i}{16\pi^2} \right) + \dots, \quad (9) \end{aligned}$$

where the higher order contributions are neglected. Using the one-loop  $\beta$ - and  $\gamma$ -functions for the SM couplings (cf. [3]) and Eq. (7) we explicitly find that each parenthesis in (9) vanishes and therefore the potential (1) is indeed renormalization scale invariant up to higher loop corrections.

The optimum scale  $\mu_*$ , where the neglected higher order corrections have the smallest impact on the observables, turns out to be a certain average of the masses  $M_i^2(\phi)$  such that the logarithms in (1) do not result in large corrections. In the case of flat spacetime,  $R = 0$ , it can be shown [14] that  $\mu = \phi$  is a good choice resulting in small corrections to

the optimal choice. Based on this consideration, we now make a choice of the scale  $\mu$  in the presence of curvature corrections as

$$\mu^2 = \phi^2 + R, \quad (10)$$

for which the corrections compared to the optimal choice are expected to be small [20].

Once the running couplings are solved, the potential can be plotted by choosing the renormalization scale as in Eq. (10). The Minkowski potential, corresponding to  $R = 0$ , is shown in the upper panel of Fig. 2, where the scale of the maximum is given by

$$\bar{\Lambda}_{\text{max}} \approx 6 \times 10^7 \text{ GeV}, \quad \bar{V}_{\text{max}}^{1/4} \approx 9 \times 10^6 \text{ GeV}, \quad (11)$$

where the bar on top of the symbol indicates quantities calculated from the Minkowski potential. With these values, inflationary fluctuations would be able to overcome the potential barrier if  $H \gtrsim 10^7$  GeV, rendering the physical vacuum unstable. At two-loop level the barrier is higher [2,3],  $\bar{V}_{\text{max}}^{1/4} \sim 10^9$  GeV, but the instability remains, although it requires a higher inflationary scale  $H$ .

The full effective potential (1) for a particular choice  $H = 10^{10}$  GeV and  $\xi_{EW} = 0.1$  of the free parameters, for which we find  $\Lambda_{\text{max}} \approx 6 \times 10^{10}$  GeV and  $V_{\text{max}}^{1/4} \approx 2 \times 10^{10}$  GeV, is shown in the lower panel of Fig. 2.

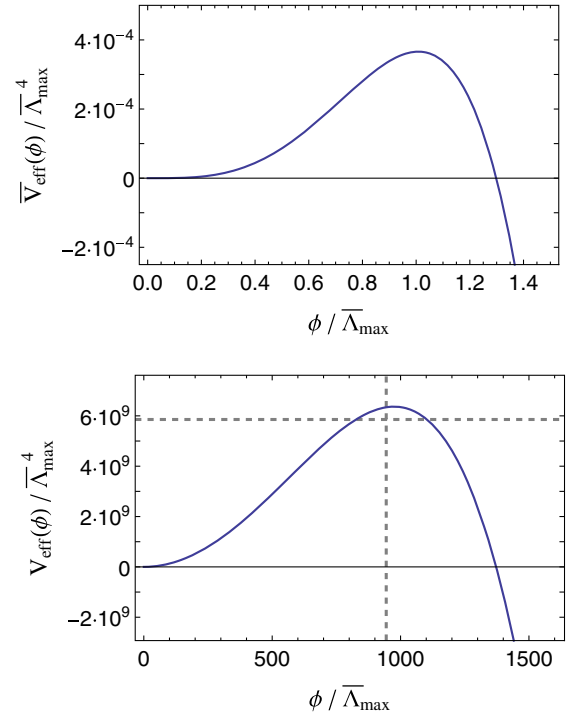


FIG. 2 (color online). The Higgs effective potential without (top) and with (bottom) curvature corrections for  $H = 10^{10}$  GeV and  $\xi_{EW} = 0.1$ . The dashed lines correspond to the approximation in Eqs. (12–13) for the maximum.

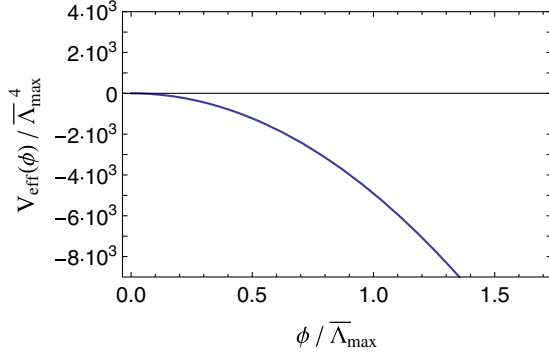


FIG. 3 (color online). The Higgs effective potential including curvature corrections with  $H = 10^{10}$  GeV and  $\xi_{EW} = 0$ .

We find that the scale of the maximum is orders of magnitude higher than the prediction of the Minkowski potential due to large effective curvature mass.

A reasonable order of magnitude estimate for the maximum of the potential at scales  $H \gg \bar{\Lambda}_{\max} \sim 10^8$  GeV can be obtained by fixing  $\mu = R^{1/2} = 12^{1/2}H$ , since the running couplings evolve mildly for  $\mu \gtrsim 10^8$  GeV,

$$\Lambda_{\max} \simeq \left( \frac{12\xi_R}{|\lambda_R|} \right)^{1/2} H \gtrsim (10^3 \xi_R)^{1/2} H, \quad (12)$$

$$V_{\max}^{1/4} \simeq \frac{(6\xi_R)^{1/2}}{|\lambda_R|^{1/4}} H \gtrsim 10\xi_R^{1/2} H, \quad (13)$$

where we denote  $\xi_R \equiv \xi(R^{1/2})$  and similarly for  $\lambda_R$  and we have used  $|\lambda_R| \lesssim 10^{-2}$ . Using Eqs. (13) and (8) we then find that the criterion for stability,

$$V_{\max}^{1/4} \gtrsim H, \quad (14)$$

can be solved for  $\xi_{EW}$  to get

$$\xi_{EW} \gtrsim \frac{1}{6\xi_R} (|\lambda_R|^{1/2} + \Xi_R - 1) \sim 10^{-2}, \quad (15)$$

where  $\Xi_R \equiv \Xi(R^{1/2}) = 1.15-1.20$  for  $H \gg 10^8$  GeV. We show the stability region given by Eq. (15) as blue in Fig. 4.

On the other hand, for small enough  $\xi_{EW}$  we find that  $\xi(\mu)$  is running negative such that the negative curvature mass term  $\xi R \phi^2/2$  dominates the potential (1) at high scales if  $H \gg \bar{\Lambda}_{\max}$ . For example, for  $\xi_{EW} = 0$  the unstable potential is shown in Fig. 3. The condition for  $\xi_{EW}$  to yield unstable potential during inflation for  $H \gg \bar{\Lambda}_{\max}$  is given by

$$\xi_{EW} \lesssim \frac{1}{6\xi_I} \left( \Xi_I - 1 - \frac{|\lambda_I| \bar{\Lambda}_{\max}^2}{4H^2} \right) \sim 10^{-2}, \quad (16)$$

where we denote  $\Xi_I \equiv \Xi(\bar{\Lambda}_{\max})$  and similarly for  $\lambda_I$ . The corresponding region where the EW vacuum is unstable from the onset of inflation is shown as red in Fig. 4.

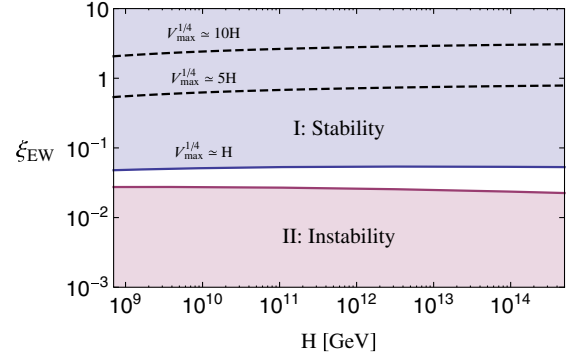


FIG. 4 (color online). The parametric regions I (blue, top), where  $V_{\max}^{1/4} \gtrsim H$  and the transition probability to unstable vacuum is suppressed, and II (red, bottom), where the EW vacuum is unstable for all  $\phi$  due to large negative curvature mass.

The intermediate region between I and II in Fig. 4 is relatively narrow and requires fine-tuning for the non-minimal coupling  $\xi$  at the EW scale. This is because  $\xi$  is running away from the conformal point  $\xi_c = 1/6$  by a factor of 1.15–1.20 (see Fig. 1), and therefore without fine-tuning  $|\xi(\mu)|$  is typically of order  $10^{-2}$  or larger at high scales. The curvature corrections may be important in this region as well; however, further investigation is required for a conclusive survey.

In conclusion, we find that for a high inflationary scale  $H \gg \bar{\Lambda}_{\max} \sim 10^8$  GeV the UV-induced (subhorizon) curvature corrections alter the SM Higgs effective potential significantly during inflation. In particular, for  $\xi_{EW} \gtrsim 6 \times 10^{-2}$  a large curvature mass stabilizes the potential against fluctuations of order  $H$ , while for  $\xi_{EW} \lesssim 2 \times 10^{-2}$  the resulting curvature mass is negative such that the EW vacuum is unstable from the onset of inflation. These results are in agreement with the tree-level analysis in [5] where the stability bound was found to be  $\xi \gtrsim 10^{-1}$ . We will examine the vacuum transition rate and the implications on cosmology in more detail elsewhere [16]. We stress however that the exponential suppression of any fluctuation probability  $P \sim \exp(-V_{\max}/H^4)$  makes the stability of the regime  $V_{\max}^{1/4} \gtrsim H$  a robust statement.

Finally, we note that higher loop corrections may alter these quantitative estimates considerably. For example, the flat space instability scale  $\bar{\Lambda}_{\max} \sim 10^{11}$  GeV from the next-to-next-to-leading order calculation [2,3] is roughly three orders of magnitude higher than in the present one-loop calculation. However, we expect that our qualitative results persist.

M. H. and T. M. would like to thank Anders Tranberg for useful and illuminating discussions and the University of Stavanger for hospitality. M. H. is supported by the Villum Foundation Grant No. YIP/VKR022599, T. M. and S. N. are supported by the Academy of Finland Grants No. 1134018 and 257532, respectively, and A. R. is supported by STFC Grant No. ST/J000353/1.

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