Supersymmetric Black Holes with Lens-Space Topology

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(Received 3 September 2014; published 17 November 2014)

We present a new supersymmetric, asymptotically flat, black hole solution to five-dimensional supergravity. It is regular on and outside an event horizon of lens-space topology $L(2,1)$. It is the first example of an asymptotically flat black hole with lens-space topology. The solution is characterized by a charge, two angular momenta, and a magnetic flux through a noncontractible disk region ending on the horizon, with one constraint relating these.

DOI: 10.1103/PhysRevLett.113.211101 PACS numbers: 04.50.Gh, 04.50.−h, 04.60.Cf, 04.70.Bw

A fundamental result in the theory of black holes is Hawking's horizon topology theorem [1]. It shows that for asymptotically flat, stationary black holes satisfying the dominant energy condition, cross sections of the event horizon must be topologically S^2. It has been known for over a decade that black holes in higher dimensions are not so constrained. In five dimensions, an explicit example of an asymptotically flat black hole with horizon topology $S^1 \times S^2$—a black ring—was presented [2]. In conjunction with the S^3 topology Myers-Perry black hole, this explicitly demonstrated black hole nonuniqueness in five-dimensional vacuum gravity [3].

Hawking's horizon topology theorem was subsequently generalized to higher dimensions, revealing a weaker constraint on the topology, namely, cross sections of the horizon must have a positive Yamabe invariant [4]. However, it is unclear whether every topology allowed by this theorem is actually realized by a black hole solution. So far, the black ring is the only nonspherical example known with a connected horizon, although it is believed many other types exist [5,6].

In five dimensions, the positive Yamabe condition allows for $S^3, S^1 \times S^2$, quotients of S^3 by a discrete subgroup, and connected sums of these. In the context of stationary solutions with $U(1)^2$ rotational symmetry, it has been shown that the possible topologies are further constrained to be one of S^3, $S^1 \times S^2$, or $L(p,q)$ where $L(p,q) \cong S^3/\mathbb{Z}_p$ is a lens space [7]. The former two topologies are, of course, already realized by the Myers-Perry solutions and black rings. There have been various attempts at finding an asymptotically flat black hole solution with lens-space horizon topology—a black lens—to the vacuum Einstein equations, although they have all resulted in solutions with naked singularities [8,9].

In this note we show that black lenses do in fact exist, by writing down a simple supersymmetric, asymptotically flat, black lens solution to five-dimensional minimal supergravity. Specifically, we construct an example that is regular on and outside an event horizon with lens-space topology $L(2,1) \cong \mathbb{RP}^3 \cong S^3/\mathbb{Z}_2$.

The bosonic content of five-dimensional minimal supergravity is a metric g and a Maxwell field F. The general form for supersymmetric solution was found in [10],

$$ds^2 = -f^2(dt+\omega)^2 + f^{-1} ds_M^2, \tag{1}$$

where $V = \partial/\partial t$ is the supersymmetric Killing vector field, ds_M^2 is a hyper-Kähler base, and f, ω are a function and 1-form on the base M. We will choose the base to be a Gibbons-Hawking space,

$$ds_M^2 = H^{-1}(d\psi + \chi_i dx^i)^2 + H dx^i dx^i, \tag{2}$$

where $x^i, i = 1,2,3$, are Cartesian coordinates on $\mathbb{R}^3$, the function H is harmonic on $\mathbb{R}^3$, and χ is a 1-form on $\mathbb{R}^3$ satisfying $\star_3 d\chi = dH$. As is well known [10], such solutions are then specified by four harmonic functions H, K, L, M, in terms of which

$$\begin{aligned} f^{-1} &= H^{-1}K^2 + L, \\ \omega &= \omega_\psi(d\psi + \chi_i dx^i) + \hat{\omega}_i dx^i, \end{aligned} \tag{3}$$

where

$$\begin{aligned} \omega_\psi &= H^{-2}K^3 + \frac{3}{2}H^{-1}KL + M, \\ \star_3 d\hat{\omega} &= HdM - MdH + \frac{3}{2}(KdL - LdK). \end{aligned} \tag{4}$$

The Maxwell field is determined by

$$F = \frac{\sqrt{3}}{2} d\left[f(dt+\omega) - \frac{K}{H}(d\psi + \chi_i dx^i) - \xi_i dx^i\right], \quad (5)$$

where the 1-form ξ satisfies $\star_3 d\xi = -dK$.

Now we write the $\mathbb{R}^3$ in polar coordinates,

$$dx^i dx^i = dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (6)$$

and consider the 2-centered solution

$$H = \frac{2}{r} - \frac{1}{r_1}, \qquad M = m + \frac{m_1}{r_1},$$
$$K = \frac{k_0}{r} + \frac{k_1}{r_1}, \qquad L = 1 + \frac{\ell_0}{r} + \frac{\ell_1}{r_1}, \quad (7)$$

where $r_1 = \sqrt{r^2 + a_1^2 - 2ra_1\cos\theta}$ is the distance from the origin to the "center" $(0, 0, a_1)$. We assume $a_1 > 0$. We used a shift freedom in the harmonic functions to remove any $1/r$ term in M, without any loss of generality [11]. To fully determine the solution, we must integrate to find the 1-forms $\chi, \hat{\omega}, \xi$. We find

$$\chi = \left[2\cos\theta - \frac{r\cos\theta - a_1}{r_1}\right] d\phi,$$
$$\hat{\omega} = \left[-\left(2m + \frac{3}{2}k_0\right)\cos\theta + \frac{(m - \frac{3}{2}k_1)(r\cos\theta - a_1)}{r_1} + \frac{(r - a_1\cos\theta)[2m_1 + \frac{3}{2}(\ell_1 k_0 - \ell_0 k_1)]}{a_1 r_1} + c\right] d\phi,$$
$$\xi = -\left[k_0\cos\theta + k_1\frac{r\cos\theta - a_1}{r_1} + c'\right] d\phi, \quad (8)$$

where c, c' are integration constants (we have set the one for χ to zero by suitably shifting ψ). Crucially, observe that $\chi \sim \cos\theta d\phi$ as $r \to \infty$, and $\chi \sim (1 + 2\cos\theta)d\phi$ as $r \to 0$; as we will show, this allows the spacetime to interpolate between S^3 at spatial infinity and $S^3/\mathbb{Z}_2$ near the horizon.

For a suitable choice of constants, the solution is asymptotically flat. Defining $r = \rho^2/4$, it is easy to check that the Gibbons-Hawking base for $\rho \to \infty$ looks like

$$ds_M^2 \sim d\rho^2 + \frac{1}{4}\rho^2(d\theta^2 + \sin^2\theta d\phi^2) + \frac{1}{4}\rho^2(d\psi + \cos\theta d\phi)^2, \quad (9)$$

with subleading terms of order $\mathcal{O}(\rho^{-2})$. Hence, the base is asymptotically $\mathbb{R}^4$ provided we fix the periods of the angles to be $\Delta\psi = 4\pi$, $\Delta\phi = 2\pi$, and $0 \leq \theta \leq \pi$. Now, it is also clear that $f = 1 + \mathcal{O}(\rho^{-2})$. Furthermore, $\omega_\psi = \mathcal{O}(\rho^{-2})$ and $\omega_\phi = \mathcal{O}(\rho^{-2})$, provided we fix the constants,

$$m = -\frac{3}{2}(k_0 + k_1), \qquad c = \frac{3\ell_0 k_1 - 3\ell_1 k_0 - 4m_1}{2a_1}, \quad (10)$$

respectively. We will assume these choices henceforth, so our solution is asymptotically flat $\mathbb{R}^{1,4}$.

Although the solution appears singular at centers $r = 0$ and $r_1 = 0$, we will show that by suitably choosing our constants, $r = 0$ corresponds to an event horizon and $r_1 = 0$ corresponds to a smooth timelike point.

First consider the center $r_1 = 0$. Near this center the Gibbons-Hawking base approaches $-\mathbb{R}^4$ smoothly, provided the angles are identified in the same manner as required by asymptotic flatness [11,12]. To see this, change to $\mathbb{R}^3$ polar coordinates with respect to this center (r_1, θ_1), then set $\rho = 2\sqrt{r_1}$. One finds that ds_M^2 as $\rho \to 0$ approaches minus (9) with (θ, ψ, ϕ) replaced by $(\theta_1, \psi + 2\phi, -\phi)$. Introducing $\mathbb{R}^2$ polar coordinates (X, Φ), (Y, Ψ) on the orthogonal 2-planes, $X = \rho\cos\frac{1}{2}\theta_1, Y = \rho\sin\frac{1}{2}\theta_1$, $\Phi = \frac{1}{2}(\psi + \phi)$, and $\Psi = \frac{1}{2}(\psi + 3\phi)$, one can then demonstrate smoothness at the center [12]. Further, imposing that the center is a timelike point requires $f|_{\mathbf{x}=\mathbf{x}_1} \neq 0$, which implies $\ell_1 = k_1^2$. In fact, to get the correct spacetime signature we require $f|_{\mathbf{x}=\mathbf{x}_1} < 0$. One can then check the function f is smooth at the center $r_1 = 0$. Since ∂_ψ degenerates at the center, smoothness also requires that $V \cdot \partial_\psi = -f^2\omega_\psi$ vanishes at that point. In fact, ω_ψ is singular at the center unless $m_1 = \frac{1}{2}k_1^3$. Then further imposing ω_ψ vanishes at the center also implies the constraint

$$3a_1(2k_1 + k_0) + 3\ell_0 k_1 - 3k_0 k_1^2 - 2k_1^3 = 0. \quad (11)$$

These conditions imply $\omega = \mathcal{O}(X^2)d\Phi + \mathcal{O}(Y^2)d\Psi$ ensuring the 1-form ω—and hence the spacetime metric—is smooth at the center $r_1 = 0$. The Maxwell field is then also smooth at the center. Thus, our solutions are parametrized by (ℓ_0, k_0, k_1, a_1), subject to the constraint (11), resulting in a three-parameter family. Observe that if $k_1 = 0$, then $k_0 = 0$; we will show this is incompatible with smoothness of the axis of rotation [see Eqs. (21), (22)]. Thus, $k_1 \neq 0$ which allows us to solve (11) for ℓ_0.

Now consider the center $r = 0$. We will show that this corresponds to a regular event horizon if

$$R_1^2 \equiv 2\ell_0 + k_0^2 > 0, \qquad R_2^2 \equiv \frac{\ell_0^2(8\ell_0 + 3k_0^2)}{(2\ell_0 + k_0^2)^2} > 0. \quad (12)$$

To this end, transform to new coordinates $(v, r, \psi', \theta, \phi)$,

$$dt = dv + \left(\frac{A_0}{r^2} + \frac{A_1}{r}\right) dr,$$
$$d\psi + d\phi = d\psi' + \frac{B_0}{r} dr, \quad (13)$$

where A_0, A_1, B_0 are constants to be determined. Then,

$$g_{vv} = -\frac{4r^2}{R_1^4} + \mathcal{O}(r^3), \qquad g_{\psi'\psi'} = \frac{1}{4}R_2^2 + \mathcal{O}(r),$$
$$g_{v\psi'} = -\frac{(3\ell_0 + k_0^2)k_0 r}{R_1^4} + \mathcal{O}(r^2). \tag{14}$$

In general, g_{rr} contains $1/r^2$ and $1/r$ singular terms, whereas $g_{r\psi'}$ contains $1/r$ singular terms. Demanding that the $1/r$ term in $g_{r\psi'}$ and the $1/r^2$ term in g_{rr} vanish, corresponds to fixing the constants

$$B_0 = \frac{4k_0(3\ell_0 + k_0^2)A_0}{\ell_0^2(8\ell_0 + 3k_0^2)}, \qquad A_0^2 = \frac{1}{4}\ell_0^2(8\ell_0 + 3k_0^2). \tag{15}$$

This then gives $g_{vr} = \pm(2/R_2) + \mathcal{O}(r), g_{r\psi'} = \mathcal{O}(1)$, where the sign corresponds to that of A_0 (positive if $A_0 < 0$ and vice versa). Finally, demanding that the $1/r$ term in g_{rr} also vanishes, fixes A_1 to be a complicated constant. Then $g_{rr} = \mathcal{O}(1)$. Furthermore, $\chi = [1 + 2\cos\theta + \mathcal{O}(r^2)]d\phi$ and $\hat{\omega} = \mathcal{O}(r)d\phi$ [to show the latter one needs (11)].

It is now easily checked that the metric and its inverse are analytic at $r = 0$ and therefore can be extended to a new region $r < 0$. The surface $r = 0$ is a degenerate Killing horizon with respect to the supersymmetric Killing field $V = \partial/\partial v$, with the upper (lower) sign corresponding to a future (past) horizon. It is also easily checked that the Maxwell field is regular on the horizon. The near-horizon geometry (NH) may be extracted by scaling $(v, r) \to (v/\epsilon, \epsilon r)$ and letting $\epsilon \to 0$ [13]. We find

$$ds^2_{\rm NH} = -\frac{4r^2dv^2}{R_1^2R_2^2} \pm \frac{4dvdr}{R_2} + R_1^2(d\theta^2 + \sin^2\theta d\phi^2)$$
$$+ \frac{R_2^2}{4}\left(d\psi' + 2\cos\theta d\phi - \frac{4(3\ell_0 + k_0^2)k_0 r dv}{R_2^2R_1^4}\right)^2$$
$$F_{\rm NH} = \frac{\sqrt{3}}{2}d\left[\frac{2rdv}{R_1^2} + \frac{(3\ell_0 + k_0^2)k_0}{2R_1^2}(d\psi' + 2\cos\theta d\phi)\right], \tag{16}$$

where we have used $k_0^2(3\ell_0 + k_0^2)^2 = R_1^4(R_1^2 - R_2^2)$. This near-horizon geometry is *locally* isometric to that of the Breckenridge-Myers-Peet-Vafa (BMPV) black hole [14], as guaranteed by [15] (cf. [12]). However, the period $\Delta\psi' = 4\pi$ has already been fixed by asymptotic flatness and regularity at the other center. Therefore, cross sections of the horizon $v = \text{const}, r = 0$ are of topology $L(2, 1) \cong \mathbb{RP}^3 \cong S^3/\mathbb{Z}_2$, as claimed. The area of the horizon is

$$A = 8\pi^2R_1^2R_2. \tag{17}$$

The above black hole solution has $U(1)^2$-rotational symmetry. The z axis of the $\mathbb{R}^3$ base in the Gibbons-Hawking space corresponds to the axes where the $U(1)^2$ Killing fields vanish. We will now examine the geometry on these various axes. Because of our choice of harmonic functions, the z axis splits naturally into three intervals: $I_+ = \{z > a_1\}, I_D = \{0 < z < a_1\}, I_- = \{z < 0\}$. The semi-infinite intervals $I_\pm$ correspond to the two axes of rotation that extend out to infinity. As we will see, the finite interval I_D corresponds to a noncontractible disk topology surface that ends on the horizon.

The 1-form $\chi = \pm d\phi$ on $I_\pm$ and $\chi = 3d\phi$ on I_D. Remarkably, it can also be verified that $\hat{\omega} = 0$ on the whole z axis [on I_D one needs to use (11)]. Thus, the geometry and Maxwell field induced on the axis are

$$ds^2_{\rm axis} = -f^2dt^2 - \frac{\Omega_I(z)dtd\psi_I}{P_I(z)^2} + \frac{P_I(z)dz^2}{z^2|z - a_1|} + \frac{Q_I(z)}{P_I(z)^2}(d\psi_I)^2,$$
$$F_{\rm axis} = \frac{\sqrt{3}}{2}d\left[fdt + \frac{R_I(z)}{P_I(z)}d\psi_I\right], \tag{18}$$

where P_I, Q_I, Ω_I, R_I are polynomials and (ψ_I, ϕ_I) are angles that depend on the interval. In particular, we have $(\psi_\pm, \phi_\pm) = (\psi \pm \phi, \phi)$, $(\psi_D, \phi_D) = (\psi + 3\phi, \phi)$, and

$$f = \begin{cases} \frac{z(z-2a_1)}{P_\pm(z)}, & z \in I_\pm \\ \frac{z(2a_1-3z)}{P_D(z)}, & z \in I_D. \end{cases} \tag{19}$$

The explicit polynomials are

$$P_\pm(z) = z^2 \pm [k_1^2 + (k_0 + k_1)^2 + \ell_0 \mp 2a_1]z \mp a_1(2\ell_0 + k_0^2),$$
$$P_D(z) = -3z^2 + (2a_1 - 3\ell_0 - k_0^2 + 2k_0k_1 + 2k_1^2)z$$
$$+ a_1(2\ell_0 + k_0^2), \tag{20}$$

whereas the Q_I are quintics such that $Q_+ \sim a_1^{-2}(z - a_1)P_+(a_1)^3, Q_D \sim a_1^{-2}(a_1 - z)P_D(a_1)^3$ and $\Omega_+ = \mathcal{O}(z - a_1)$, $\Omega_D = \mathcal{O}(a_1 - z)$, as $z \to a_1$.

In order for the axis geometry to be a smooth Lorentzian metric we require $P_I > 0$ and $Q_I > 0$ on each of their corresponding intervals. Thus, on I_+ we must have $P_+(z) > 0$, which in fact is equivalent to $P_+(a_1) > 0$ and $P'_+(a_1) > 0$ (since $R_1^2 > 0$, P_+ has positive discriminant). Explicitly, these inequalities read

$$2k_0k_1 + 2k_1^2 - a_1 - \ell_0 > 0, \tag{21}$$

$$(k_0 + k_1)^2 + k_1^2 + \ell_0 > 0. \tag{22}$$

It is easily seen that these conditions also guarantee that $P_D(z) > 0, P_-(z) > 0$ on their respective intervals, since $P_D(0) = a_1R_1^2 > 0, P_D(a_1) = P_+(a_1) > 0$ and $P_-(0) = a_1R_1^2 > 0, -P'_-(0) = P'_+(a_1) + 2a_1 > 0$. Furthermore, we have verified numerically that in the domain (21) and (22) the polynomials $Q_\pm, Q_D$ are positive on $I_\pm, I_D$, so this places no further constraints. Observe that $P_I > 0$ also guarantees the Maxwell field is smooth.

Now, on I_+ the Killing field $v_+ = \partial_{\phi_+} = \partial_\phi - \partial_\psi$ vanishes, whereas $\partial_{\psi_+} = \partial_\psi$ is nonvanishing everywhere and degenerates smoothly at the end point $z = a_1$ (one can check the conical singularity at $z = a_1$ in (18) is absent since $\Delta\psi_+ = 4\pi$). Next, on I_D the Killing field $v_D = \partial_{\phi_D} = \partial_\phi - 3\partial_\psi$ vanishes, whereas $\partial_{\psi_D} = \partial_\psi$ is nonvanishing everywhere and vanishes at the end point $z = a_1$ smoothly (again since $\Delta\psi_D = 4\pi$ the conical singularity is absent). On the other hand, ∂_{ψ_D} does not vanish at the end point $z \to 0$ which ends on the horizon, so the finite interval I_D is a disk topology surface D. On the final interval I_- the Killing field $v_- = \partial_{\phi^-} = \partial_\phi + \partial_\psi$ vanishes, whereas $\partial_{\psi_-} = \partial_\psi$ is nonvanishing everywhere including on the horizon $z \to 0$. It is worth noting that in the 2π-normalized basis $(\partial_{\phi_+}, 2\partial_{\psi_+})$, the vanishing Killing fields on I_+ and I_D are $v_+ = (1,0)$ and $v_D = (1,-1)$, respectively, so the compatibility condition for adjacent intervals is satisfied [7],

$$\det(v_D^T v_+^T) = 1. \tag{23}$$

Finally, observe that on the axis, $f = 0$ at $z = 2a_1$ and $z = \frac{2}{3}a_1$, so the supersymmetric Killing field is null on these circles. In fact, $P_+(2a_1) = a_1(k_0 + 2k_1)^2$ and $P_D(\frac{2}{3}a_1) = \frac{1}{3}a_1(k_0 + 2k_1)^2$, so we must have $k_0 + 2k_1 \neq 0$. It can be shown this implies that $\Omega_+(2a_1) \neq 0$ and $\Omega_D(\frac{2}{3}a_1) \neq 0$, which ensures the metric on the axis (18) is smooth and invertible even where $f = 0$. It is worth emphasizing this is guaranteed by our above conditions. To see this, suppose $k_0 = -2k_1$, so then (11) may be solved to get $\ell_0 = -\frac{4}{3}k_1^2$; in this case (21) is violated so we deduce $k_0 \neq -2k_1$. To summarize, we have shown that the metric on the whole z axis is smooth and invertible if and only if $R_1^2 > 0$, (21) and (22) are satisfied.

We now address regularity and causality in the domain of outer communication $r > 0$. It is easy to prove that $R_1^2 > 0$ and (21) imply that $K^2 + HL > 0$ away from the centers, ensuring f is smooth everywhere. Remarkably, this also guarantees that the full spacetime metric is smooth and invertible, and the gauge field is smooth, everywhere away from the centers (even where $H = 0$). We also require stable causality with respect to the time function t, thus,

$$g^{tt} = -f^{-2} + fH\omega_\psi^2 + fH^{-1}\hat{\omega}_i\hat{\omega}_i < 0. \tag{24}$$

Asymptotically $r \to \infty$, it is clear that this is satisfied since $g^{tt} \to -1$. Also, $g^{tt} \sim -\frac{1}{4}R_1^2R_2^2r^{-2}$ as $r \to 0$, so the solution is stably causal near the horizon. On the axes of symmetry, the condition reduces to $-f^{-2} + fH\omega_\psi^2 < 0$ and hence away from the center $z = a_1$ it is equivalent to positivity of the polynomials $Q_D, Q_\pm$ discussed above. Away from the axis we have performed extensive numerical checks and found no violation of (24), provided that (12), (21), and (22) are satisfied. Therefore, we believe our solution is stably causal if and only if (12), (21), and (22) are satisfied. This ensures there are no closed timelike curves in the domain of outer communication.

We will now briefly discuss some of the physical properties of our black lens solution. We find the Maxwell charge and Komar angular momenta are

$$\begin{aligned} Q &= 2\pi\sqrt{3}[\ell_0 + k_1^2 + (k_0 + k_1)^2], \\ J_\psi &= \pi\left\{\frac{1}{2}k_1^3 + (k_0 + k_1)\left[(k_0 + k_1)^2 + \frac{3}{2}(\ell_0 + k_1^2)\right]\right\}, \\ J_\phi &= \frac{3}{2}\pi a_1(k_0 + 2k_1). \end{aligned} \tag{25}$$

The mass is given by the Bogomol'nyi-Prasad-Sommerfield (BPS) relation $M = (\sqrt{3}/2)Q$. Our solution also carries a magnetic flux through the disk topology surface D discussed above,

$$q[D] = \frac{1}{4\pi}\int_D F = \frac{\sqrt{3}}{4}(k_0 + 2k_1). \tag{26}$$

Since our solution is a three-parameter family there must be one constraint between these four physical quantities. As for any BPS black hole, the surface gravity and angular velocity must vanish and the electric potential $\Phi_H = (\sqrt{3}/2)$. Furthermore, the electric flux $\mathcal{Q}[D]$ which appears in the first law of black hole mechanics [16] also vanishes [12], so the Smarr relation and first law reduce to the BPS bound.

The magnetic flux $q[D]$ for our solution is necessarily nonvanishing, since, as shown above, smoothness of the axes of rotation requires $k_0 \neq -2k_1$. One might be tempted to interpret the magnetic flux as "supporting" the black lens, since the disk D is required for a lens-space horizon topology. However, this need not be the case. Black rings also possess a disk topology region ending on the horizon, which shows that rotation may be sufficient for supporting nontrivial topology.

One might wonder if our solution may possess the same conserved charges as the BMPV black hole. Equal angular momenta with respect to the orthogonal $U(1)^2$ Killing field at infinity requires $J_\phi = 0$ or $J_\psi = 0$. In fact, $J_\phi \neq 0$ since, as shown above, $k_0 \neq -2k_1$. It also turns out the solution with $J_\psi = 0$ is not compatible with our regularity constraints, although this is less straightforward to show. Hence, there are no regular black lenses in our family of solutions, with the same charges as BMPV. On the other hand, the supersymmetric black ring possesses nonequal angular momenta [17], so we may expect there are black lenses with the same conserved charges.

In conclusion, the black lens we have presented, together with the recently found spherical black hole with an exterior 2-cycle [12], demonstrate that black hole uniqueness in five dimensions is violated much more drastically than previously thought, even for supersymmetric black holes. It would be interesting to explore the implications of this for the microscopic entropy calculations in string theory. We also expect nonextremal versions of our solutions to exist. In particular, we do not expect magnetic flux is required to support lens-space topology, so a regular vacuum black lens may also exist.

H. K. K. is supported by a Discovery Grant from the National Sciences and Engineering Research Council of Canada (NSERC). J. L. is supported by a Career Acceleration Fellowship from the Engineering and Physical Sciences Research Council (EPSRC).

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