Statistical Diagnostics to Identify Galactic Foregrounds in B-Mode Maps

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Recent developments in the search for inflationary gravitational waves in the cosmic microwave background polarization motivate the search for new diagnostics to distinguish the Galactic foreground contribution to B modes from the cosmic signal. We show that B modes from these foregrounds should exhibit a local hexadecapolar departure in power from statistical isotropy (SI). We present a simple algorithm to search for a uniform SI violation of this sort, as may arise in a sufficiently small patch of sky. We then show how to search for these effects if the orientation of the SI violation varies across the survey region, as is more likely to occur in surveys with more sky coverage. If detected, these departures from Gaussianity would indicate some level of Galactic foreground contamination in the B-mode maps. Given uncertainties about foreground properties, though, caution should be exercised in attributing a null detection to an absence of foregrounds.

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The BICEP2 Collaboration recently reported [1] evidence for the signature [2] of inflationary gravitational waves [3] in the B-mode component [4,5] of the polarization of the cosmic microwave background (CMB). The extraordinary stream of papers [6] that have followed this announcement provides some indication of the significance of a B-mode detection. However, the remarkable implications of this measurement—the detection of a new relic from inflation—demand that the results receive the deepest possible scrutiny. Discussions that have taken place since the March 2014 announcement indicate that more work must be done to establish, with the type of confidence such an extraordinary result warrants, that the B-mode signal cannot be attributed fully to polarized emission from interstellar dust (see, e.g., Refs. [7,8]).

The gold standard to distinguish CMB from foregrounds (primarily synchrotron and dust emission from the Milky Way) has typically been to obtain high–signal-to-noise maps at multiple frequencies. Important steps in this direction should soon be taken for the BICEP2 B-mode signal with new data from the 100-GHz Keck Array [9] and from polarization measurements from Planck [10] at higher frequencies, and soon independently with other experiments (e.g., Refs. [11–14]). However, these measurements may, like any others, ultimately have limits. For example, extrapolation of measurements of the B-mode power from dust obtained with Planck's 353-GHz channel to BICEP2's 150-GHz channel may suffer from theoretical uncertainties in the frequency dependence of the dust polarization. Indeed, the frequency-dependent models 1 and 3 of Ref. [15] (see Fig. 8) predict an opposite trend with frequency than observed (see Fig. 13 in Ref. [16]), indicating our theoretical uncertainty. The use of spatial cross correlations between different frequency channels may be imperfect if the depths in the interstellar medium probed by those two frequencies differ. Even if the dust contribution turns out to be small enough that such subtleties do not prevent the confident establishment of a gravitational-wave signal, every detail about the early Universe that we can extract from detailed characterization of the B-mode signal will be priceless. It is thus imperative that we remain ever vigilant in our quest to find new ways to root out contaminants to the cosmic B-mode signal.

Here we propose two statistical tests that can be performed on an observed B-mode map—either a single-frequency map or one that has been cleaned with multi-frequency information—to help identify foreground contamination. In principle, one might just look at the data for a preferred orientation in the polarization map. Most CMB experiments, however, measure only differences in polarization and thus are not equipped to measure the average orientation. Moreover, much of what we discuss below for B modes also applies to E modes, but the additional information in E modes is likely to be swamped by cosmic variance from the dominant density-perturbation contribution to E modes. Still, higher-frequency E-mode maps may be useful for constructing dust orientation templates for cross correlation with B-mode maps. The idea for the proposed tests is simple: The departures in the inflationary gravitational-wave signal from Gaussianity and statistical isotropy (SI) are expected to be extremely small [17]. Any statistically significant departure from Gaussianity or SI would thus indicate some noncosmic contamination.

The question, though, is what *type* of non-Gaussianity or SI violation should we be seeking? Here we argue that the polarization due to foregrounds over a sufficiently small region of the sky induces a hexadecapolar anisotropy in the B-mode power, something that should be relatively simple to seek. We then show how to look for a spatially varying SI violation of this sort, something that is more likely to describe the foreground polarization pattern on larger patches of sky.

Let us begin by understanding how this SI violation arises, in particular, for the case of dust. Polarized emission from dust stems from the alignment of spinning dust grains with the Galactic magnetic field [15] (which also determines the synchrotron polarization). Galactic magnetic fields are known to have long-range correlations, implying an orientation angle that is fairly coherent on large regions of the sky [18], and perhaps larger than the patch covered by BICEP2. There may, of course, be significant changes in that orientation angle in small sky patches if there are regions of high-density plasma in the interstellar medium (ISM) in that patch. The BICEP2 patch, however, which lies in the "Southern Hole," was chosen for the expectation that it was relatively clean [19] and thus likely free from rapid variation in the orientation angle (as shown in Fig. 13 of Ref. [18], the typical angle dispersion of dust polarization is lowest in \the highest polarization-fraction regions of the sky, those cleanest and most suitable for B-mode measurements). Furthermore, measurements of polarized absorption of starlight (which is correlated with polarized dust emission [20]) in the BICEP2 region may provide some empirical indication that the orientation of the dust polarization in the BICEP2 patch is roughly uniform, as noted in Ref. [7]. However, as these data lie near the edges of the field, they cannot provide a robust constraint on the entire patch.

Let us therefore consider a B-mode signal from a map in which the orientation angle of the polarization is constant. The Stokes parameters $Q(\vec{\theta})$ and $U(\vec{\theta})$, measured as a function of position $\vec{\theta} = (\theta_x, \theta_y)$ on a flat region of sky, are components of a polarization tensor,

$$P_{ab} = \frac{1}{\sqrt{2}} \begin{pmatrix} Q(\vec{\theta}) & U(\vec{\theta}) \\ U(\vec{\theta}) & -Q(\vec{\theta}) \end{pmatrix}. \quad (1)$$

The polarization map is then decomposed into scalar and pseudoscalar components $E(\vec{\theta})$ and $B(\vec{\theta})$ by

$$\nabla^2 E = \partial_a \partial_b P_{ab}, \qquad \nabla^2 B = \epsilon_{ab} \partial_a \partial_c P_{cb}, \quad (2)$$

where ϵ_{ab} is the antisymmetric tensor. The Fourier components of $E(\vec{\theta})$ and $B(\vec{\theta})$ are

$$\tilde{E}(\vec{l}) = 2^{-1/2} [\cos 2\varphi_{\vec{l}} \tilde{Q}(\vec{l}) + \sin 2\varphi_{\vec{l}} \tilde{U}(\vec{l})], \quad (3)$$

$$\tilde{B}(\vec{l}) = 2^{-1/2} [-\sin 2\varphi_{\vec{l}} \tilde{Q}(\vec{l}) + \cos 2\varphi_{\vec{l}} \tilde{U}(\vec{l})], \quad (4)$$

in terms of the Fourier transforms $\tilde{Q}(\vec{l})$ and $\tilde{U}(\vec{l})$ of the Stokes parameters and the angle $\varphi_{\vec{l}}$ that $\vec{l}$ makes with the θ_x axis.

If the polarization is constant across the map with orientation $\alpha = (1/2)\arctan(U/Q)$ with respect to the θ_x axis, then the Fourier modes for E and B will be

$$\tilde{E}(\vec{l}) = \frac{\tilde{P}(\vec{l})}{\sqrt{2}} \cos[2(\alpha - \varphi_{\vec{l}})], \quad (5)$$

$$\tilde{B}(\vec{l}) = \frac{\tilde{P}(\vec{l})}{\sqrt{2}} \sin[2(\alpha - \varphi_{\vec{l}})], \quad (6)$$

where $\tilde{P}(\vec{l})$ is the Fourier transform of the polarization amplitude $P(\vec{\theta}) \equiv (Q^2 + U^2)^{1/2}(\vec{\theta})$. We thus see that if the orientation angle of polarization is constant, the B modes that result are not statistically isotropic. They are, rather, modulated by $\sin[2(\alpha - \varphi_{\vec{l}})]$.

An estimator for this departure from statistical isotropy in the B-mode map can be obtained through a straightforward augmentation of the usual algorithm to determine the amplitude of the B-mode power. Equation (6)—which is what we expect if the observed B modes are due entirely to dust and if the dust polarization has uniform orientation—implies that the mean-square amplitude of each B-mode coefficient is

$$\langle |\tilde{B}(\vec{l})|^2 \rangle = A C_l^f [1 - \cos 4\alpha \cos 4\varphi_{\vec{l}} - \sin 4\alpha \sin 4\varphi_{\vec{l}}], \quad (7)$$

where C_l^f parametrizes an assumed fiducial l dependence (e.g., $C_l^f \propto l^{-2.22}$, as current measurements suggest [18]) and A an amplitude of the signal. Note that although the modulation of the Fourier amplitudes is quadrupolar ($\propto e^{2i\alpha}$), the departure from statistical isotropy in the power spectrum is a hexadecapole; it has an $e^{4i\alpha}$ dependence.

More generally, if the orientation of the dust polarization is not perfectly uniform, but is rather spread over some small range $\delta\alpha$, then the modulation in Eq. (7) will be reduced by a factor $\sim(\delta\alpha)/\alpha$. Thus, to test for dust, we should aim to measure the parameters in the angle-dependent power spectrum,

$$\langle |\tilde{B}(\vec{l})|^2 \rangle = A C_l^f [1 - f_c \cos 4\varphi_{\vec{l}} - f_s \sin 4\varphi_{\vec{l}}], \quad (8)$$

where f_s, $f_c < 1$ measure the departure from statistical isotropy, and the dust-polarization orientation, if these parameters are found to be nonzero, is $\alpha = (1/4)\arctan(f_s/f_c)$.

The minimum-variance estimator for the isotropic amplitude A is the usual one,

$$\hat{A} = \frac{\sum_{\vec{l}} |\tilde{B}_{\vec{l}}|^2 C_l^f / \sigma_l^2}{\sum_{\vec{l}} (C_l^f)^2 / \sigma_l^2}, \quad (9)$$

where the sum is over all Fourier modes $\vec{l}$ with amplitudes $\tilde{B}(\vec{l})$ each measured with variance σ_l^2 (which may receive contributions from detector noise and from lensing-induced B modes [21,22]). The minimum-variance estimators for the amplitudes of the SI-violating terms are likewise,

$$\widehat{Af}_c = \frac{\sum_{\vec{l}} |\tilde{B}_{\vec{l}}|^2 C_l^f \cos 4\varphi_{\vec{l}}/\sigma_l^2}{\sum_{\vec{l}} (C_l^f \cos 4\varphi_{\vec{l}})^2/\sigma_l^2}, \tag{10}$$

and similarly for the f_s term with the replacement cos → sin. If there is no prior information about the orientation of the dust polarization, then the parameters f_s and f_c are both obtained simultaneously and independently from the data. If, however, there is some prior information about the expected orientation—e.g., from starlight polarization—then the ratio f_s/f_c can be fixed and the sensitivity to dust-induced SI violation thus accordingly improved. Either way, any statistically significant detection of nonzero f_s and/or f_c indicates at least some contamination of the cosmic signal. If, moreover, either of the inferred values f_c or f_s differs significantly from zero, then there is good evidence that the signal is predominantly noncosmic. If there is a strong reason to believe that the foreground-polarization orientation is indeed uniform across the survey, then a strong null result may imply that the observed signal is *not* foreground dominated. If, though, that orientation is uncertain, then a null result in this SI test cannot be used to rule out foreground contamination.

The variances and covariances with which the parameters A, f_s, and f_c can be measured are easily derived. However, they will depend considerably on the details of any given experiment and perhaps a bit on the fact that the lensing-induced B-mode map is not precisely Gaussian. We thus leave these covariances to simulations of the complete analysis pipelines. Heuristically, though, the estimator measures the difference in the B-mode power for modes oriented perpendicular or parallel to some axis versus those oriented at 45°. If there is a $\gtrsim 5\sigma$ detection of power, and if that power is due entirely to uniformly oriented dust, then the violation of statistical isotropy should appear with high statistical significance. Indeed, a crude estimate for the minimum amplitude A that can be measured at 1σ is given by

$$\begin{aligned}\sigma_{\hat{A}}^{-2} &= \sum_{\vec{l}} (C_l^f)^2/\sigma_l^2 \sim \Omega \int \frac{d^2 l}{(2\pi)^2} (C_l^f)^2/\sigma_l^2 \\ &= 4\pi f_{\rm sky} \int \frac{d^2 l}{(2\pi)^2} (C_l^f)^2/\sigma_l^2 \\ &= 2 f_{\rm sky} \int l dl (C_l^f)^2/\sigma_l^2 \end{aligned} \tag{11}$$

and $\sigma_{\widehat{Af_c}}^2 = 2\sigma_{\hat{A}}^2$ (as $\int_0^{2\pi} d\varphi = 2\int_0^{2\pi} \cos^2(4\varphi) d\varphi = 2\pi$). The signal-to-noise ratio in a particular experiment is governed by the sensitivity per Fourier mode,

$$\sigma_l = \sqrt{\frac{2}{f_{\rm sky}(2l+1)}} (C_l^{\rm lens} + f_{\rm sky} w^{-1}(T) e^{l^2\sigma_b^2}), \tag{12}$$

where the pixel noise $\sigma_{\rm pix} = s/\sqrt{t_{\rm pix}}$ is determined by the detector sensitivity s and the observation time $t_{\rm pix} = T/N_{\rm pix}$ dedicated to each pixel, and where we used the definition $w^{-1}(T) \equiv 4\pi s^2/T$.

It should be noted that some of the dust-polarization templates used by BICEP2 and investigated in subsequent work were constructed assuming a uniform dust-polarization orientation. The departures from SI considered above are then effectively incorporated into the data-template cross-correlation analyses done already. Those cross correlations, though, may still vanish if either (1) the assumed orientation angle is incorrect, or (2) the spatial variation of the polarization amplitude is not correctly represented, as can be seen in Ref. [7]. The SI-violation analysis suggested above, though, does not rely on prior knowledge of the spatial variation of the amplitude nor the assumed orientation angle.

So far we have supposed that the sky patch is small enough that a uniform dust-polarization orientation may be reasonably hypothesized. However, future experiments will cover larger regions of the sky (e.g., Ref. [11–13]), and it is increasingly likely that the foreground-polarization orientation will meander across the survey region as the size of that region increases. The foreground polarization may thus be modeled in terms of an amplitude that has rapid small-scale variation with an orientation that has longer-range correlations. This can be sought in a straightforward fashion by simply measuring the correlations in the polarization amplitude and in the orientation angle. If the signal is cosmic, the correlations in both should be similar. Evidence that those two correlation lengths differ could indicate a noncosmic source of contamination. Such an analysis, though, will likely be limited by cosmic variance from the dominant density-perturbation–induced polarization.

Instead, we now spell out a diagnostic for spatial variations of the type of SI violation above that parallels algorithms developed to search for spatially varying cosmic birefringence [23], optical depth ("patchy screening") [24], and cosmological parameters [25], and before those, weak lensing [26] (which has now been detected [22,27]). For clarity, we work here in the flat-sky limit; the generalization to the full sky is straightforward and follows other previous analogous work.

We suppose that there are variations of the orientation angle that vary slowly across the sky with small-scale fluctuations in the polarization amplitude. We thus assume the polarization can be written

$$P_{ab}(\vec{\theta}) = P_{ab}^o(\vec{\theta})\phi(\vec{\theta}) \tag{13}$$

in terms of a smooth "orientation field" $P_{ab}^o(\theta)$ with Stokes parameters $Q_o(\vec{\theta})$ and $U_o(\vec{\theta})$ and a more rapidly varying polarization-amplitude field $\phi(\vec{\theta})$ (which for dust should be correlated with the dust-intensity field, although we do not use any such information here). The orientation field can be decomposed in the usual manner into E and B modes, $E_o(\vec{\theta})$ and $B_o(\vec{\theta})$. There is an ambiguity in the definitions of $P^o_{ab}(\vec{\theta})$ and $\phi(\vec{\theta})$—one can be increased while the other is reduced without changing P_{ab}—that can be removed by demanding, e.g., that the polarization amplitude field have unit variance or some specific maximum value.

Consider a spatial variation of the orientation that consists of a single Fourier mode of wave vector $\vec{L}$ of either the E type or the B type. The orientation pattern in the first case always has only nonzero Q (measured with respect to axes aligned with $\vec{L}$) and, in the latter case, only nonzero U. Thus, in the first case (E-mode orientation), the polarization is always aligned or perpendicular to $\vec{L}$, and in the second (B-mode orientation), the polarization is always aligned at axes rotated by 45° from $\vec{L}$. Therefore, in either case—a pure-E orientation or a pure-B orientation—the orientation of the SI violation in the polarization B modes are the same everywhere, even though the orientation angle is changing. Thus, in either of these two cases, there will be SI violation in the observed B modes that is uniform across the sky, and the simple SI-violation test above will capture the effect in its entirety and have a positive result.

To make things a bit more interesting, consider an orientation that rotates clockwise as we move in the θ_x direction, completing a full revolution after a distance $\theta_x = 2\pi/L$. In other words,

$$\begin{pmatrix} Q_o \\ U_o \end{pmatrix}(\vec{\theta}) = R_{\vec{L}} \begin{pmatrix} \cos L\theta_x \\ \sin L\theta_x \end{pmatrix}. \tag{14}$$

This is a linear combination of an E mode and a B mode, both of the same $\vec{L}$, added out of phase—i.e., $E + iB$—and $R_{\vec{L}}$ is the amplitude of this Fourier mode. More precisely,

$$\begin{pmatrix} Q_o \\ U_o \end{pmatrix}(\vec{\theta}) = \left[\frac{R_{\vec{L}}}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} e^{i\vec{L}\cdot\vec{\theta}} + \text{c.c.}\right], \tag{15}$$

where now we have made $R_{\vec{L}}$ complex to allow a phase different from that in Eq. (14). We then suppose that the observed polarization is obtained by multiplying this slowly varying orientation field with a rapidly varying amplitude $\phi(\vec{\theta})$; i.e.,

$$\begin{pmatrix} Q \\ U \end{pmatrix}(\vec{\theta}) = \left[\frac{R_{\vec{L}}}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} e^{i\vec{L}\cdot\vec{\theta}} + \text{cc}\right]\phi(\vec{\theta}). \tag{16}$$

Since the orientation varies over all possible values, the observed B modes will be statistically isotropic when averaged over the whole field, and the SI-violation test suggested above will give a null result. Still, the observed B modes will exhibit *local* departures from SI.

We now explain how to detect this position-dependent local SI violation. The polarization pattern in Eq. (16) yields B modes,

$$\tilde{B}(\vec{l}) = \frac{i}{2}[R_{\vec{L}}\tilde{\phi}(\vec{l}-\vec{L})e^{2i\varphi_{\vec{l}}} - R^*_{\vec{L}}\tilde{\phi}(\vec{l}+\vec{L})e^{-2i\varphi_{\vec{l}}}]. \tag{17}$$

Before proceeding, recall that the B modes due to inflationary gravitational waves are expected to be Gaussian and statistically isotropic, which implies that $\langle \tilde{B}(\vec{l})\tilde{B}^*(\vec{l}')\rangle = 0$ for $\vec{l} \neq \vec{l}'$. However, we now find that the polarization pattern in Eq. (16) has expectation values,

$$\begin{aligned}\langle \tilde{B}(\vec{l})\tilde{B}^*(\vec{l}')\rangle = \frac{1}{4}[&|R_{\vec{L}}|^2(C^{\phi}_{|\vec{l}-\vec{L}|} + C^{\phi}_{|\vec{l}+\vec{L}|})\delta_{\vec{l},\vec{l}'} \\ &-(R^*_{\vec{L}})^2 C^{\phi}_{|\vec{l}+\vec{L}|} e^{-2i(\varphi_{\vec{l}}+\varphi_{\vec{l}'})}\delta_{\vec{l}',\vec{l}+2\vec{L}} \\ &-(R_{\vec{L}})^2 C^{\phi}_{|\vec{l}-\vec{L}|} e^{2i(\varphi_{\vec{l}}+\varphi_{\vec{l}'})}\delta_{\vec{l}',\vec{l}-2\vec{L}}],\end{aligned} \tag{18}$$

where C^{ϕ}_l is the power spectrum of the modulation field $\phi(\vec{\theta})$, and $\delta_{\vec{l},\vec{l}'}$ is shorthand for $(2\pi)^3\delta_D(\vec{l}-\vec{l}')$, the Dirac delta function. The first term in Eq. (18), the only one that is nonvanishing for $\vec{l} = \vec{l}'$, provides the (angle-averaged) B-mode power spectrum for the map. Roughly speaking, it is the amplitude power spectrum C^{ϕ}_l smeared in l space by L. As argued above, this first term indicates that there is no departure from statistical isotropy when power is averaged over the entire map.

The second two terms in Eq. (18), though, describe the local SI violation of a polarization field due to the small-scale modulation of a longer-range orientation field. They indicate a cross correlation of a Fourier mode of wave vector $\vec{l}$ with those of wave vectors $\vec{l}' = \vec{l} \pm 2\vec{L}$. The appearance of $2\vec{L}$ (rather than just $\vec{L}$) is related to the hexadecapolar nature of the power asymmetry.

Equation (18) implies that each pair of Fourier amplitudes $\tilde{B}(\vec{l})$ and $\tilde{B}(\vec{l}')$ with $\vec{l}' - \vec{l} = 2\vec{L}$ provides an estimator,

$$\widehat{(R^*_{\vec{L}})^2} = -4\frac{\tilde{B}(\vec{l})\tilde{B}^*(\vec{l}')e^{2i(\varphi_{\vec{l}}+\varphi_{\vec{l}'})}}{C^{\phi}_{|\vec{l}+\vec{L}|}}, \tag{19}$$

for the Fourier amplitude $R^*_{\vec{L}}$ (or, actually, its square) of the orientation amplitude. One then adds the estimators from each such $\vec{l}$, $\vec{l}'$ pair with inverse-variance weighting to obtain the optimal estimator for $(R^*_{\vec{L}})^2$. The procedure is directly analogous to that for weak-lensing, cosmic-birefringence, and patchy-screening reconstruction, and we leave the details to be presented elsewhere.

If any $R_{\vec{L}}$ (for any wave vector $\vec{L}$ that can be accessed with the map) is found to be nonzero with statistical significance, this indicates a likely contamination from foreground. Naturally, when searching for deviation from SI in multiple independent L modes, the "look elsewhere effect" must be properly taken into account. It should be possible, however, in a map that covers a sufficiently large region of sky with a sufficient signal-to-noise ratio, to measure a large number of amplitudes for $E + iB$ and $E - iB$ modes and thus to reconstruct the orientation-angle map $P^{o}_{ab}(\vec{\theta})$ as a function of position on the sky.

Reassuringly, in the limit $L \to 0$, where the orientation angle becomes uniform (and taking $R_{\vec{L}}$ to be real so that the orientation is aligned with θ_x), Eq. (18) simplifies to

$$\langle \tilde{B}(\vec{l}) \tilde{B}^*(\vec{l}') \rangle = \frac{R^2_{\vec{L}}}{2} C^{\phi}_{l} (1 - \cos 4\varphi_{\vec{l}}) \delta_{\vec{l},\vec{l}'}, \qquad (20)$$

recovering the expected hexadecapolar power anisotropy.

To conclude, we have argued that polarization from dust is likely to give rise to non-Gaussianity in the B modes they induce, which appears as a local hexadecapolar departure from statistical isotropy. A simple test that will seek this SI violation in the event that the orientation of the dust-induced polarization is roughly constant was presented. We also showed how an orientation that varies across the survey region can be sought. Here we have only sketched out how these tests can be done. Much more work will be needed before they are implemented in real data. This will include the full development of the optimal estimators, full-sky formalisms, tools to deal with imperfect sky coverage, etc. Still, these developments should parallel the analogous developments for, e.g., weak lensing. The estimators for the effects we deal with here differ in detail from those, e.g., for weak lensing (here we seek a local hexadecapolar SI violation, while lensing induces a quadrupolar effect), but some thought should be given to possible confusion in a low–signal-to-noise scenario.

We do not advocate that the foreground diagnostics we discuss here replace multifrequency component separation. Rather, they can be implemented in the event of limited multifrequency information or, in the event that multifrequency maps uncover a cosmic signal, as a way to check for consistency or identify residual foreground contamination in the maps.

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