Anisotropic Stress as a Signature of Nonstandard Propagation of Gravitational Waves

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We make precise the heretofore ambiguous statement that anisotropic stress is a sign of a modification of gravity. We show that in cosmological solutions of very general classes of models extending gravity—all scalar-tensor theories (Horndeski), Einstein-aether models, and bimetric massive gravity—a direct correspondence exists between perfect fluids apparently carrying anisotropic stress and a modification in the propagation of gravitational waves. Since the anisotropic stress can be measured in a model-independent manner, a comparison of the behavior of gravitational waves from cosmological sources with large-scale-structure formation could, in principle, lead to new constraints on the theory of gravity.

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Over the last decade, we have established beyond reasonable doubt that, in its recent past, the expansion of the universe has been accelerating. This has challenged our beliefs about the theory of gravity: the only possibility available in general relativity with nonexotic matter is a cosmological constant, which would suffer from severe fine-tuning issues. Alternatively, the mechanism could be dynamical, i.e., feature at least one new degree of freedom. These dynamics would modify the predictions of concordance cosmology and give us a means to carry out precision tests of gravity at extremely large scales.

Frequently, in extended models of gravity, perfect fluids apparently carry anisotropic stress: there is gravitational slip, i.e., the values of the two scalar gravitational potentials sourced by matter are not equal. This affects structure formation and weak lensing. Recently, it was shown that the ratio of the two potentials is actually a model-independent observable [1,2], which Euclid should be able to measure to a precision of a few percent, depending on the precise assumptions [3]. This begs the question as to what detecting or not detecting anisotropic stress actually means.

In this Letter, we show that the propagation of *tensor* modes (gravitational waves, GWs) is also modified whenever the anisotropic stress is present at first order in perturbations sourced by perfect-fluid matter. We demonstrate this relationship in the context of three very large classes of extensions of the gravitational sector: general scalar-tensor theories (Horndeski [4,5]), Einstein-aether models [6–8], and bimetric massive gravity [9,10]. GWs are the only propagating degrees of freedom in general relativity, and it is natural to define modified gravity models as those where the gravitational waves are modified in such a nontrivial manner. Since imperfect fluids with anisotropic stress also split the two gravitational potentials but do not modify the propagation of tensor modes, this definition allows us to separate modifications of gravity from imperfect fluids.

The emphasis of this Letter is not on new calculations (see, e.g., the review [11]), but rather on new relations which are very general, were not noted before in the literature and could have a significant impact on tests of gravity on cosmological scales.

Assumptions.—We assume that the universe is well-described by small linear perturbations living on top of a spatially flat Friedmann metric. We take the line element for the metric on which matter and light propagate as

$$ds^2 = a^2(\tau)\{-(1+2\Psi)d\tau^2 + (1-2\Phi)[\delta_{ij}+h_{ij}]dx^i dx^j\},$$

where τ is the conformal time, a the scale factor, Φ and Ψ are the scalar gravitational potentials, and h_{ij} is the traceless spatial metric (tensor) perturbation, i.e., the gravitational wave. We assume that the matter sector can be described as a fluid arising from the averaging of the motion of particles. We comment on the effect of this fluid's being imperfect. We use the prime to denote a derivative with respect to conformal time.

The presence of anisotropic stress results in a difference in values between the two scalar potentials and can be described through the gravitational slip,

$$\eta \equiv \frac{\Phi}{\Psi}. \quad (1)$$

In concordance cosmology, $\eta = 1$, with small corrections appearing from neutrino free streaming. At second order in perturbations, anisotropic stress also always appears even when the matter consists completely of dust [12], but in the late Universe should be smaller than $|\eta - 1| \lesssim 10^{-3}$ [12,13].

On the other hand, various modifications of gravity [such as $f(R)$ [14], $f(G)$ [15] or DGP [16]] do feature an $O(1)$ correction to the slip parameter at linear order in perturbations, at least at some scales and even in the presence of just a perfect-fluid matter. It is, however, well-known that the value of η can be modified by a change of frame, e.g., a conformal rescaling of the metric, making its value seemingly ambiguous.

In Refs. [1,2], it was shown that comparing the evolution of redshift-space distortions of the galaxy power spectrum with weak-lensing tomography allows us to reconstruct η as a function of time and scale in a model-independent manner. Such an operational definition removes the frame ambiguity, since the measurement picks out the particular metric on the geodesics of which the galaxies and light move. It is the gravitational slip in that metric that is being measured by such cosmological probes. With the ambiguity of frame removed, the gravitational slip is a *bona fide* observable, rather than just a phenomenological parameter. Fixing the metric also determines what is considered a gravitational wave: we call these the propagating spin-2 perturbations of the metric on which matter moves. (In the case of massive gravity, we are referring to the helicity-2 modes of the metric coupled to matter.)

In this Letter, we assume that the gravitational sector is extended by one of three classes of models featuring a single extra degree of freedom: (1) a very general scalar-tensor theory belonging to the Horndeski class [4], (2) Einstein-aether theory featuring an extra vector and spontaneous violation of Lorentz invariance, or (3) bimetric massive gravity. We will discuss each of these in turn and show that similar conclusions hold.

Modified gravity defined.—Dynamical models of late-time acceleration can feature interactions between the new degree of freedom and curvature or metric (scalar-tensor or Einstein-aether) and the two metrics (bimetric). On a cosmological background, these interactions can alter the speed of propagation of gravitational waves (c_T), make the effective Planck mass (M_*) evolve in time [17] or add a mass μ, giving

$$h''_{ij} + (2+\nu)\mathcal{H}h'_{ij} + c_T^2 k^2 h_{ij} + a^2\mu^2 h_{ij} = a^2\Gamma\gamma_{ij}, \quad (2)$$

where h_{ij} is the tensor wave amplitude in either of the two polarizations, $\mathcal{H} \equiv a'/a$ is the Hubble rate in conformal time. The deviations away from standard behavior are contained in $\nu \equiv \mathcal{H}^{-1}(d \ln M_*^2/dt)$, the *Planck mass run rate*, and c_T, the speed of tensor waves, with both of these quantities defined in the Jordan frame of the matter. (Note that no observable quantity depends on M_* itself, since a changed Planck mass can always be reabsorbed into the definition of masses if it is constant.) We will show that scalar-tensor and Einstein-aether models can change ν and c_T. On the other hand, in massive bigravity, the equation is modified by the mass of the graviton μ. The transverse-traceless tensor γ_{ij} is a source term for the gravitational waves. In the case of bimetric massive gravity, γ_{ij} is the gravitational wave in the second metric and the two tensor modes mix as they propagate. When the matter fluid has anisotropic stress, this appears as the source term γ_{ij}, but it never modifies the homogeneous part of Eq. (2). However, this anisotropic stress is itself coupled to the gravitational waves and can lead to dissipation for horizon-scale GW modes [18,19].

As we stressed above, Eq. (2) describes the evolution of the gravitational waves of the Jordan-frame metric. This choice is unique if our observations (e.g., redshifts, time delays) are taken to result from the geometry of the Universe. We should also note that, for bimetric massive gravity, the Einstein frame with standard gravitons does not exist even on a perturbative level. On the other hand, the issue of which of the two metrics matter couples to is an important one, which has to be fixed to define the model properly.

As is frequently said, anisotropic stress is a feature of modified gravity. For any gravity theory at the linear level, the anisotropy constraint in the Newtonian gauge takes the form

$$\Phi - \Psi = \sigma(t)\Pi + \pi_m, \quad (3)$$

with Π a function of a particular combination of background and linear perturbation variables, depending on the theory. The quantity $\sigma(t)$ is a background function only, depending on the parameters of the Lagrangian. The π_m is the scalar anisotropic stress sourced by the matter fluid. This appears whenever the perfect-fluid approximation breaks down and the particle distribution contains higher moments than those described by a perfect fluid. For example, free streaming in neutrinos gives such a term even in concordance cosmology, but such contributions are very small in the late Universe.

The aim of this Letter is to provide an unambiguous definition of modified gravity as one where the propagation of *gravitational waves* [Eq. (2)] is affected. The gravitational slip and gravitational waves are connected since both the anisotropy constraint [Eq. (3)] and the GW evolution equation [Eq. (2)] arise from the spatial–traceless part of the linearized Einstein equations. In the remainder of this Letter, we will demonstrate that the coupling $\sigma(t)$ appearing in the anisotropy equation [Eq. (3)] consists of the quantities that also control the modification of the tensor propagator. This means that modified gravity models popular in the literature are included in our definition.

However, imperfect-fluid matter while acting as a source to both the anisotropy constraint [Eq. (3)] and the GW equation [Eq. (2)], cannot directly modify the homogeneous part of the GW equation. Our definition of modified gravity therefore breaks the ambiguity that arises in the presence of such a source and points to an approach for differentiating modified gravity from imperfect fluids.

Scalar-tensor theories.—In this section, we consider the most general class of theories featuring one extra scalar degree of freedom which has Einstein equations with no more than second derivatives on any background and are universally coupled to matter: the Horndeski class of models. (We have not considered in detail the extension discussed in [20–22], where higher derivatives appear in the Einstein equations, but can be eliminated by solving the constraints.) This class includes the majority of the popular models of late-time acceleration such as quintessence, perfect fluids, $f(R)$ gravity, $f(G)$ gravity, kinetic gravity braiding, and galileons (see, e.g., the reviews [23,24]). The Horndeski Lagrangian is defined as the sum of four terms that are fully specified by a noncanonical kinetic term $K(\phi, X)$ and three arbitrary coupling functions $G_{3,4,5}(\phi, X)$, where $X = -g_{\mu\nu}\phi^{;\mu}\phi^{;\nu}/2$ is the canonical kinetic energy term and where the comma denotes a partial derivative.

We make extensive use of the formulation for linear structure formation in scalar-tensor theories introduced in Ref. [25]. It was shown there that the form of linear perturbation equations for all Horndeski models can be completely described in terms of the background expansion history, density fraction of matter today Ω_{m0}, and four independent and arbitrary functions of time only, $\alpha_K, \alpha_B, \alpha_M$, and α_T, which mix the four functional degrees of freedom of the action, K and G_i. The *Planck mass run rate* α_M and the *tensor speed excess* α_T control the existence of anisotropic stress. Unrelated to the anisotropic stress, if the *braiding* $\alpha_B \neq 0$, then the dark energy will cluster at small scales, with the *kineticity* α_K controlling at what scales this happens.

The anisotropy constraint in the notation of Eq. (3) is [26]

$$\sigma = \alpha_M - \alpha_T, \Pi = H\delta\phi/\dot{\phi} + \alpha_T/(\alpha_M - \alpha_T)\Phi, \qquad (4)$$

where $\delta\phi$ is a perturbation of the scalar field. Note that the split between σ and Π above is arbitrary. The gravitational wave equation [Eq. (2)] is modified through

$$\nu = \alpha_M, \qquad c_T^2 = 1 + \alpha_T, \mu^2 = 0, \qquad \Gamma = 0. \qquad (5)$$

It is clear from Eq. (4) that when both $\alpha_M = \alpha_T = 0$ there is no new contribution to either to anisotropic stress or tensor propagation. In the context of scalar-tensor models and the late Universe with $\pi_m \approx 0$, a detection of anisotropic stress therefore is direct evidence that one or both of the parameters α_T and α_M are different from their concordance values of zero and that gravity is modified in the sense of this work.

In principle, one could imagine that there may exist models defined by a choice of the functions α_i in which the scalar perturbation arranges itself dynamically in such a configuration that no gravitational slip appears, even though one of $\alpha_{M,T}$ is not zero. This would be a very particular situation or one requiring a very tuned choice of model parameters. For example, it happens at the asymptotic future —and static—pure de Sitter limit. It can be shown that it is, in fact, impossible to have such a cancellation in a model where the scalar has real dynamics. We defer the proof to a more technical follow-up study.

Einstein-aether theories.—Einstein-aether models [27,28] are a class of theories which feature an extra vector degree of freedom (the *aether*) u^μ. They are a subclass of general vector theories requiring that u^μ be given a constant and timelike vacuum expectation value $u_\mu u^\mu = -1$ and that it be minimally coupled. This chooses a preferred frame, violating Lorentz symmetry. The infrared limit of Hořava-Lifshitz (HL) models [29–31]—relevant for late-time cosmology—is closely related, with the vector field forced to be hypersurface orthogonal and thus providing a natural slicing for the space-time [32].

The Lagrangian can be written in a basis of four operators, through a kinematic decomposition of $\nabla_\mu u_\nu$ [32]: the squares of acceleration, expansion, twist and shear, and their associated dimensionless coefficients c_a, c_θ, c_ω, and c_σ, respectively. (In the language of Ref. [32], these correspond to $c_a \equiv -c_1 + c_4$, $c_\theta \equiv \frac{1}{3}(c_1 + c_3) + c_2$, $c_\omega = c_1 - c_3$, $c_\sigma \equiv c_1 + c_3$.)

The extra dynamical degree of freedom at the linear level is the perturbation of the spatial components u^i of the vector u^μ, which can be decomposed into longitudinal and transverse parts as $u^i = \partial^i u + \hat{u}^i$. The longitudinal part modifies the anisotropy constraint [33], which in the notation of Eq. (3) is

$$\Pi = \left(\frac{u}{a^2}\right)', \qquad \sigma = -c_\sigma. \qquad (6)$$

At the same time, the parameters of the tensor equation [Eq. (2)] are given by

$$\nu = 0, \qquad c_T^2 = (1 + c_\sigma)^{-1}, \mu^2 = 0, \qquad \Gamma = 0. \qquad (7)$$

In conclusion, the modifications of both the anisotropy constraint and the tensor wave equation are driven by the same coupling c_σ of the shear. If c_σ appears in the action, it will modify both the anisotropic stress and the gravitational wave propagation. Thus a detection of anisotropic stress in the late Universe with $\pi_m \approx 0$ in the context of these models also implies that gravity is modified in the sense of this work.

Bimetric massive gravity.—The bimetric massive gravity model features two dynamical metrics, g_1 and g_2, each with its own Einstein–Hilbert term in the action. In addition, a potential term describes nonderivative interactions between the two metrics, $U(g_1, g_2; a_i)$. The five constants a_i parametrize these interactions and are the theory's free parameters. The interactions inevitably give mass to one of the two metrics [34], and the theory in general propagates a massless and a massive spin-two field [35], and it provides a nonlinear extension of the Fierz-Pauli theory [36], which is free of the so-called Boulware-Deser ghost [9,37–40].

One usually considers the matter fields to be coupled to one of the metrics, which we shall call g_1.

Bimetric gravity provides a natural extension of the so-called dRGT massive gravity, with the latter being a subcase of the former, in the limit where the second metric becomes nondynamical. Cosmological solutions for dRGT and bimetric theories have been studied in, for instance, Refs. [10,41–45] and [46–51,51], respectively, with the aim of explaining the current acceleration of the Universe without the need of an explicit cosmological constant in the action. It has been shown however that in dRGT, homogeneous and isotropic backgrounds are not solutions of the background equations of motion [52], or when these solutions exist, they suffer from strong coupling [42], ghost [53,54], or nonlinear instabilities [54,55], and we will therefore concentrate on the bimetric version only. (In fact, a gradient instability for the new helicity-0 mode in the bimetric setup appears to exist for for some choices of parameters [51,56] but not others [57]. Whenever healthy solutions exist, the conclusions of this Letter hold.)

We use the setup and notation of Ref. [56] (for a similar analysis see also Ref. [58]), choosing both the background metrics to be homogeneous and isotropic. At the linear level, the theory predicts the existence of anisotropic stress for the scalar Newtonian potentials of the matter metric g_1, giving the anisotropy constraint the form

$$\sigma = a^2 m^2 f_1, \qquad \Pi = E_2, \tag{8}$$

in the notation of Eq. (3). E_2 is the scalar coming from the tensor perturbation of the second metric g_2. The function f_1 is a background-dependent function that depends on the ratio between the scale factors of the two metrics and the constant parameters a_i.

The equation for gravitational waves [Eq. (2)] is modified through

$$\nu = 0, \qquad c_T^2 = 1, \mu^2 = m^2 f_1, \qquad \Gamma = m^2 f_1. \tag{9}$$

Massive bigravity models change neither the Planck mass nor the speed of gravitational waves. They do give gravitons a mass and an interaction term. As we can easily see, the coefficients modifying the anisotropy constraint and the graviton equation of motion are all proportional to $m^2 f_1$. Yet again, if anisotropic stress is observed in the late Universe with $\pi_m \approx 0$ in the context of these models, we must conclude that gravity is modified in the sense of this work.

Conclusions and implications.—In this Letter, we have shown that a very close relationship exists between two properties of general extensions of gravity which until now have not been considered together: when anisotropic stress is apparently sourced by perfect-fluid matter perturbations at linear level, the propagation of gravitational waves is modified. Such a relationship generally exists in all Horndeski theories with an extra scalar, Einstein-aether theories featuring an extra vector field and bimetric massive gravity, featuring a second rank-2 tensor field—this covers a very large fraction of all the extensions of gravity with homogeneous backgrounds. We conjecture that this is a feature of all models *in general configurations*, and we choose to use this physics as the unambiguous definition of modified gravity.

We note here that the anisotropic stress and clustering of the new degree of freedom—frequently described as a change to the effective Newton's constant—are both completely independent quantities, the presence of which is not contingent on each other.

The relationship between tensor propagation and gravitational slip is a result of both being part of the spatial-traceless part of the linearized Einstein equations: the same corrections in the action modify the anisotropy constraint and the action for the graviton.

We stress that this relationship would hold whenever gravity is modified, not only at low redshifts where extensions of gravity are frequently utilized as dynamical models of acceleration. For example, during recombination, if models of gravity with apparent anisotropic stress from perfect fluids are introduced, one would then need to adjust the behavior of gravitational waves. At the same time, this new anisotropic stress would change the lensing and the integrated Sachs-Wolfe effect. All these effects would modify the CMB spectrum, in particular, the B-mode polarization [33,59,60].

This deep relationship between anisotropic stress and tensor modes implies that measurements of large-scale structure and of gravitational waves can give independent information on the properties of each other. For example, a comparison between the time of arrival of neutrinos and gravitational waves from some energetic event is a probe of the speed of tensor modes c_T and their mass μ [61]. A luminosity distance from standard sirens imputed from the decay of the amplitude of the gravitational waves probes ν, μ, and Γ [62]. Such observations are clearly extremely challenging and futuristic, but may one day be possible. (Tests such as the binary pulsar [63] probe the coupling of matter sources to gravitational waves and therefore are not necessarily sensitive to the modification in propagation described here.) On the other hand, the slip parameter η in some models can be an order-one ratio of small numbers [e.g., in $f(R)$ gravity, where the permitted parameter values are $\alpha_M = -\alpha_B \lesssim 10^{-5}$ [64], while $\eta = 1/2$ inside the Compton scale]. Measurements of anisotropic stress can be more informative about tensor modes than direct probes of gravitational waves in such a case. Ultimately, it should be possible to combine them to disambiguate the various properties of the theory of gravity at cosmological scales. We leave the discussion of how feasible this is to future work.

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