



High-Precision Test of Landauer's Principle in a Feedback Trap

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We confirm Landauer's 1961 hypothesis that reducing the number of possible macroscopic states in a system by a factor of 2 requires work of at least $kT \ln 2$. Our experiment uses a colloidal particle in a time-dependent, virtual potential created by a feedback trap to implement Landauer's erasure operation. In a control experiment, similar manipulations that do not reduce the number of system states can be done reversibly. Erasing information thus requires work. In individual cycles, the work to erase can be below the Landauer limit, consistent with the Jarzynski equality.

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In 1961, Rolf Landauer proposed a fundamental link between information theory and physical systems [1]: erasing information in a macroscopic or mesoscopic system is an irreversible process that should require a minimum amount of work, $kT \ln 2$ per bit erased, where T is the system temperature and k is Boltzmann's constant. This work is dissipated into a surrounding heat bath. At the time, the motivation was to understand the minimum power a computer requires to function. Surprisingly, logical operations—the computations themselves—can be coded using logically reversible gates that, in principle, can be realized in a thermodynamically reversible operation, with no dissipation [2]. But eventually, a computer's memory must be reset to its original state, and such an operation is, according to Landauer, inherently dissipative. As the only inherently dissipative operation, it determines the theoretical minimum power required to run a computer.

Landauer's principle acquired further significance some years later, when Charles Bennett (and, independently, Oliver Penrose) noted that it resolves the long-standing threat to the second law of thermodynamics posed by Maxwell's demon [3–6]. In modern language, a demon acquires information about a system, lowering its entropy and raising its free energy, and then uses this acquired free energy to do work. Unless some aspect of the demon's operation is dissipative, it will use the energy of the surrounding heat bath to do work, violating the second law of thermodynamics. Szilard [7], Brillouin [8], and others [5] proposed that the measurement step is inherently dissipative. However, as Bennett showed, measurements can, in principle, be done without work [4]. If measurements and calculations do not require work, the only other possibility consistent with the second law is that the erasure step, required to return the computer to its original state, is dissipative. Thus, Landauer's principle resolves the paradox created by Maxwell's demon. Although these theoretical arguments have persuaded most physicists, there has been persistent skepticism from a variety of authors, within and without the physics community [9–11]. The continuing

controversy makes clear experimental tests of Landauer's principle particularly important.

Landauer's principle remained untested for over 50 years. Tests have recently become possible because of two key recent developments: The first advance is conceptual—a method for estimating the work done on a particle and the heat dissipated by that particle that is based solely on the trajectory $x(t)$ and a knowledge of the potential $U(x, t)$. In particular, it does not rely on measuring the minute amounts of heat ($\approx 10^{-21}$ J) involved in the erasure of a single bit of information. The method was first proposed by Sekimoto [12,13] and tested, for example, by studying a colloidal particle in an aqueous medium [14]. Extensions have led to a new field, the stochastic thermodynamics of small systems [15,16]. By focusing solely on the trajectory and the potential, one can isolate and measure the quantities of direct interest, removing the contributions of work and dissipation from ancillary devices—computer, camera, illumination, etc.—that are irrelevant to calculating the work done by the potential on the particle and the heat dissipated into the surrounding bath.

The second advance is technical—the development of ways to impose user-defined potentials on a small particle undergoing Brownian motion. One way, for example, uses the localized potential forces created by optical tweezers formed from a highly focused laser beam. Then, either by shaping the beam by diffractive optical elements or by rapidly moving the beam between two or more locations, a more complicated potential, such as a double well, can be created. Such an approach was recently used to make a first test of Landauer's principle, under partial-erasure conditions [17]. A related approach [18] had earlier been used to explore a Szilard engine [5] that converts information to work, a process that may be regarded as an indirect test of Landauer's principle.

Here, we adopt a more flexible approach that uses feedback loops to create a virtual potential. We implement a version of the anti-Brownian electrokinetic (ABEL) feedback trap to test Landauer's principle. As illustrated

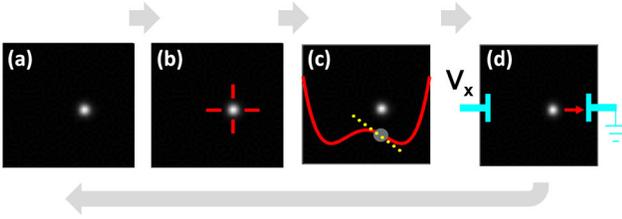


FIG. 1 (color online). Schematic of feedback trap operation. (a) Acquisition of an image of a fluorescent particle. (b) Determination of particle position from that image using a centroid algorithm. (c) Evaluation of feedback force $F_x = -\partial_x U(x, t)$ at the observed position \bar{x} . (d) Application of electric force, with voltage set by electrodes (light blue), held constant during the update time $t_s = 10$ ms. The long gray arrow indicates repetition of the cycle.

In Fig. 1, the trap acquires an image of fluorescent particle diffusing in an aqueous solution and uses image processing to estimate its position. Rather than using a physical potential, such as that produced by optical or magnetic tweezers, in a feedback trap, voltages are applied across two sets of electrodes, creating an electrical force that moves the particle. In the present study, the force is chosen to create a virtual potential $U(\bar{x}_n)$ that is a discrete approximation to a corresponding continuous potential $U(x)$ [19–22]. The potential is virtual because it is imposed by the feedback loop and is based on the measured position at \bar{x}_n , rather than the corresponding (unknown) actual position, x_n . The feedback trap allows the exploration of particle dynamics in a nearly arbitrary potential, where the only constraint is that the local relaxation time of the potential be significantly greater than the update time of the feedback loop. The local relaxation time is the time for a particle to relax in a potential of curvature $\kappa(x)$, where x is the local position of the particle [23]. A virtual potential may also be time dependent $U(\bar{x}_n, t_n)$; the time variations must also be slow compared to the feedback update time t_s . If the feedback update time is short, the dynamics in virtual potentials asymptotically approach those of the corresponding continuous potential [21].

In our experimental setup [22], a camera takes images of 200-nm fluorescent particles in an inverted epifluorescence microscope. Particles diffuse in two dimensions, as the 800-nm thick cell limits vertical motion. For inserting the electrodes, the cell has two pairs of holes that are roughly orthogonal. Fluorescent particles are illuminated by a 5-mW, 532-nm laser. The computer estimates a particle’s position in a camera image using a modified centroid algorithm and then generates a feedback force based on the inferred position. The feedback force is applied as an electric force that is generated by applying voltage differences to two pairs of electrodes. The feedback loop is updated every $t_s = 10$ ms, with force updates at the middle of a 5-ms camera exposure. The delay between observing a position and applying the calculated force is also 10 ms.

Since the feedback forces are generated by applying voltage differences to two pairs of electrodes, one must translate the voltages into forces properly. The two sets of quantities are connected by a 2×2 mobility matrix μ that relates the two nominally orthogonal potentials to x and y displacements [23]. In the feedback trap, displacements are affected by slow drifts, most likely due to voltage offsets created by electrochemical reactions at the electrodes and by the voltage amplifier. Removal of these drifts is essential for quantitative measurement of thermodynamic parameters such as work. Here, we estimate and correct for them in real time, using a recursive maximum likelihood (RML) method [26] for relating displacements to applied voltages. The RML method gives an unbiased estimate of the particle’s properties, which are further used for imposing the “virtual” potential and measuring work with high precision. See [27] for a full discussion.

The ability to measure work with the high precision and the flexibility to choose the potential in a feedback trap gives it a key advantage in testing Landauer’s principle. Previous tests, based on the rapid manipulation of optical tweezers, did not have full control of the potential shape [17]. As a result, they were unable to achieve complete erasure, and corrections were necessary to connect to the $kT \ln 2$ result predicted by Landauer for the full erasure of 1 bit of information. For their reported erasure success rates of 0.75–0.95, the asymptotic Landauer limit is $\approx (0.13\text{--}0.49)kT$, which is significantly lower than the full-erasure limit ($kT \ln 2 \approx 0.69kT$) [28]. Follow-up studies used the Jarzynski relation to infer the Landauer value from finite-time cycles [24] and explored the energetics of symmetry breaking [29]. Here, a higher barrier prevents spontaneous hops across the barrier, ensuring complete erasure and approach to the limiting value of work, $kT \ln 2$. Equally important, we are also able to perform a control experiment where, using similar manipulations in the potential that are chosen so as not to compress the phase space, the required work goes to zero at long cycle times, consistent with a reversible operation. We thus directly show the link between phase-space compression and the loss of reversibility. A very recent complementary study has explored Landauer’s principle in the context of a Szilard engine [7] that extracts $kT \ln 2$ of heat to create one bit of information [30].

Figure 2 illustrates the two protocols that we used. At left is the full-erasure protocol, denoted $p = 1$, to indicate that the probability that a particle ends up in the right well, regardless of its initial state (left well or right) is unity. Our erasure protocol is a modified version of that presented by Dillenschneider and Lutz [31]. The cyclic operation has four stages: lower the barrier, tilt, raise the barrier, until. To create the protocol, we impose

$$U(x, t) = 4E_b \left[-\frac{1}{2}g(t)\tilde{x}^2 + \frac{1}{4}\tilde{x}^4 - Af(t)\tilde{x} \right], \quad (1)$$

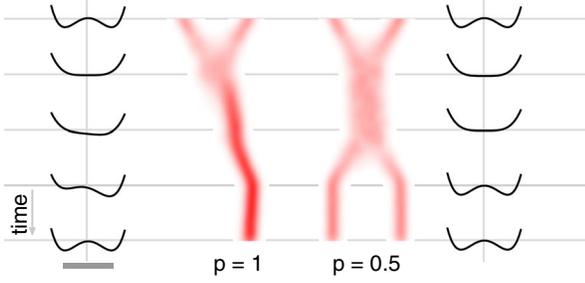


FIG. 2 (color online). Erasure protocol and trajectories for full erasure ($p = 1$) and no erasure ($p = 0.5$). Full erasure requires a strong tilt of the potential towards the desired well ($A = 0.5$). In the no-erasure protocol, the potential is symmetric at every time step ($A = 0$), implying that a particle ends up in a random final state. The image intensity $I(x, t) \propto P(x, t)$ the occupation probability for a particle in a time-dependent, double-well potential and was generated from 30 trajectories for each case using kernel density estimation. We used a Gaussian kernel with standard deviation equal to 0.1 in time and 0.15 μm in space, evaluated on a 500×160 grid. Scale bar at lower left measures 5 μm .

where $\tilde{x} = x/x_m$ and where the energy barrier E_b separates two wells located at $\pm x_m$. The functions $g(t)$ and $f(t)$ control barrier height and tilt, respectively [23]. The full potential is harmonic in the transverse direction: $U_{\text{tot}}(x, y, t) = U(x, t) + \frac{1}{2}k_y y^2$. The virtual potential is evaluated at the observed position $(\tilde{x}_n, \tilde{y}_n)$ at time nt_s .

The associated density plot, where red intensity is proportional to the occupation probability $P(x, t)$ of the particle, shows that all trajectories end up in the right well. Figure 2 shows at right the no-erasure protocol, which differs from the full-erasure protocol only in that there is no tilt. From the symmetry, we expect (and experimentally confirm) that there is an equal probability for particles to end up in the left or right well. In this case, no net erasure of information occurs.

The cycle time τ is measured in units of $\tau_0 = (2x_m)^2/D$, the time scale for particles to diffuse between wells at $\pm x_m$. In the experiment, $x_m = 2.5 \mu\text{m}$, $D \approx 1.7 \mu\text{m}^2/\text{s}$, implying $\tau_0 \approx 15$ s. The energy barrier $E_b = 13kT$, which insures that the time between spontaneous hops (dwell time) is two orders of magnitude longer than the longest erasure cycle [23]. The update time $t_s = 10$ ms is fast enough that the discrete dynamics and work measurements are accurate estimates of the continuous equivalents for our set of parameters [32]. In the no-erasure protocol, $A = 0$. For full erasure, $A = 0.5$.

To find the work in one erasure cycle, we evaluate the imposed potential $U[x(t), t]$ at the position of the particle and discretize Sekimoto's formula [12], $W = \int_0^\tau dt (\partial_t U)$, where τ is the erasure cycle time. We then have

$$W(\tau) = -4E_b \sum_{n=0}^{N_s} \left[\frac{1}{2} (\Delta g)_n \tilde{x}_n^2 + A (\Delta f)_n \tilde{x}_n \right], \quad (2)$$

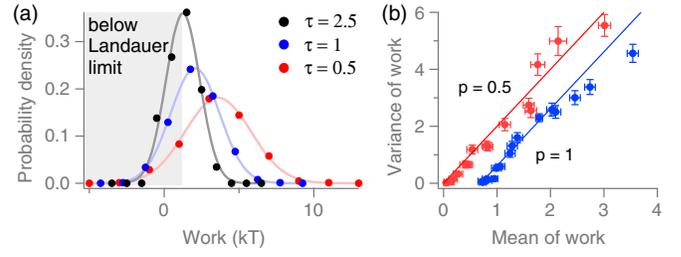


FIG. 3 (color online). (a) Histograms of work series for individual cycles of duration $\tau = 0.5, 1$, and 2.5 , with Gaussian fits shown as solid curves. The gray shaded area shows the part of the probability distribution that is below the Landauer limit. (b) Variance versus mean for the work distribution, in units of kT . Solid lines are plotted from the Jarzynski relation, Eq. (4), and have slope 2.

where $\tilde{x}_n = \bar{x}_n/x_m$, $\Delta g_n \equiv \dot{g}(t_n)t_s$, and $\Delta f_n \equiv \dot{f}(t_n)t_s$ and N_s is the number of steps in the erasure cycle.

Figure 3(a) shows that, for fixed τ , the work in each cycle is stochastic, with $W(\tau)$ empirically distributed as a Gaussian random variable. We estimate the mean work $\langle W \rangle_\tau$ for cycles of time τ by averaging over N measurements. From Fig. 3(a), the standard error of the mean depends only on the total time t_{tot} taken by the N cycles. To keep the standard error of the mean constant for different cycle times, we thus choose $N(\tau) = t_{\text{tot}}/\tau$. With $t_{\text{tot}} = 60$ min, the statistical errors were about $\pm 0.10kT$. To find the asymptotic work, we extrapolate results from finite- τ cycles to infinite cycle times via the expected τ^{-1} finite-time correction [13,33–35]:

$$\frac{\langle W \rangle_\tau}{kT} = \frac{\langle W \rangle_\infty}{kT} + a\tau^{-1}, \quad (3)$$

where $\langle W \rangle_\infty/kT$ is $\ln 2$ for the full-erasure and 0 for the no-erasure protocols.

Figure 4 presents the main results of this study: Part (a) shows the average work as a function of τ for both the full-erasure and no-erasure protocols. The solid lines are fits to Eq. (3). To show the asymptotic form and its limit

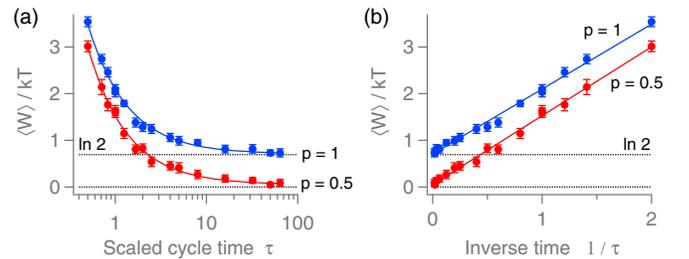


FIG. 4 (color online). Mean work measured in the full erasure ($p = 1$) and no-erasure ($p = 0.5$) protocols. (a) Mean work approaches the Landauer limits for each protocol. Solid line shows fit to asymptotic τ^{-1} correction. (b) Mean work as a function of inverse time. The dimensionless cycle times τ are in units of $\tau_0 = (2x_m)^2/D$.

TABLE I. Summary of results for full-erasure and no-erasure protocols. Work $\langle W \rangle$ is divided by a factor of kT . The full-erasure value is compatible with $\ln 2 \approx 0.693$. The dimensionless parameter a , defined in Eq. (3), gives the erasure scale time, in units of $\tau_0 = (2x_m)^2/D$.

	Asymptotic work (± 0.03)	Scale time a	χ^2 ($\nu = 14$)
Full erasure ($p = 1$)	0.71	1.39	8.2
No erasure ($p = 0.5$)	0.05	1.48	7.5

more clearly, we replot the data as a function of τ^{-1} in Fig. 4(b). The fit parameters are collected in Table I.

We performed 2444 full-erasure and 2169 no-erasure cycles in total and measured the overall success rates of 0.9996 and 0.4965, respectively. The latter is consistent with $0.5 (\pm 0.5/\sqrt{2169} \approx \pm 0.01)$.

The asymptotic work values are compatible with the expected values, $\ln 2 \approx 0.69$ and 0, respectively; the dimensionless scale times a are of order unity, also as expected; and the χ^2 statistics indicate good fits. Thus, we have shown experimentally that the full-erasure protocol, which involves the compression of phase space from two macroscopic states to one, asymptotically requires $kT \ln 2$ of work, while the very similar no-erasure protocol, which has no such phase-space compression, is reversible.

The Gaussian work distributions in Fig. 3(a) have a mean that exceeds Landauer's limit in the erasure experiment. However, individual cycles may have values of the stochastic work that are below the Landauer limit. Indeed, they can even be negative, drawing energy from the bath, in an apparent violation of the second law [36]. As a further check on our results, we note that when work distributions are Gaussian, the Jarzynski equality—in this case, equivalent to linear response theory—implies a relation between the mean $\langle W \rangle$ and variance σ_W^2 of the work distribution [see, for example, Ref. [37], Eq. (21)]:

$$\sigma_W^2 = 2(\langle W \rangle - \Delta F), \quad (4)$$

where $\langle W \rangle$, ΔF , and σ_W are all measured in units of kT and where ΔF equals $\ln 2$ for the full-erasure protocol and 0 for the no-erasure protocol [23].

Figure 3(b) shows these quantities for both protocols. The solid lines are plots (not fits) from Eq. (4). There is good agreement for longer cycle times that becomes poorer for shorter cycles, which have larger mean work values. The shorter cycle times are problematic, both because the asymptotic result, Eq. (4), and the approximation of a virtual to a real potential can break down. The Jarzynski equality has been explored in more detail in the context of Landauer's principle in Ref. [24]. In combination with the expected decrease in mean work as cycle times are lengthened, it explains immediately why the work distribution sharpens for long τ . Because the variance of the

mean estimate and the mean itself [see Eq. (4)] both decrease as τ^{-1} , measuring for a time T leads to the same error-bar estimates, independent of the chosen cycle time τ , assuming that τ is long enough that the distributions are indeed Gaussian. For shorter cycle times, the distributions are expected to be non-Gaussian [31].

In conclusion, the results presented here give the first test of Landauer's principle to confirm directly the predicted erasure energy of $kT \ln 2$ per bit, connecting the seemingly disparate ideas of information and heat flow. The tests also answer the threats to the second law of thermodynamics posed by Maxwell's demon.

The high precision and great flexibility of feedback traps opens doors to many further tests of stochastic thermodynamics and nonequilibrium statistical physics. For example, the optimal protocols for minimizing the work in finite-time operations involve complex, discontinuous manipulations of potentials that would be otherwise hard to impose [34]. The potential from a feedback trap need not even come from a potential, making possible the exploration of nonpotential dynamics [20]. Feedback traps are, of course, also natural settings for exploring nonequilibrium extensions of the Landauer theory [38,39], as well as connections between feedback and thermodynamics [15,16].

Landauer's link between information theory and physical systems is critical for understanding performance limits in nanoscale machines and biological systems. At present, the lowest energies involved in elementary operations in computation such as switching are $\approx 1000kT$ [40], which are approaching the energy scales $[(10 - 100)kT]$ used by biological systems to sense the outside world and make decisions [41]. At these energy scales, the fundamental explorations of equilibrium and nonequilibrium systems made possible by the new methods used here will become increasingly important.

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- [1] R. Landauer, *IBM J. Res. Dev.* **5**, 183 (1961).
- [2] C. H. Bennett, *IBM J. Res. Dev.* **17**, 525 (1973).
- [3] O. Penrose, *Foundations of Statistical Mechanics: A Deductive Treatment* (Pergamon, New York, 1970).
- [4] C. H. Bennett, *Int. J. Theor. Phys.* **21**, 905 (1982).
- [5] H. S. Leff and A. F. Rex, *Maxwell's Demon 2: Entropy, Classical and Quantum Information, Computing* (IOP, London, 2003).
- [6] K. Maruyama, F. Nori, and V. Vedral, *Rev. Mod. Phys.* **81**, 1 (2009).

- [7] L. Szilard, *Z. Phys.* **53**, 840 (1929).
- [8] L. Brillouin, *J. Appl. Phys.* **22**, 334 (1951).
- [9] L. B. Kish and C. G. Granqvist, *Europhys. Lett.* **98**, 68001 (2012).
- [10] M. Hemmo and O.R. Shenker, *The Road to Maxwell's Demon: Conceptual Foundations of Statistical Mechanics* (Cambridge University Press, Cambridge, England, 2012).
- [11] J.D. Norton, *Entropy* **15**, 4432 (2013).
- [12] K. Sekimoto, *J. Phys. Soc. Jpn.* **66**, 1234 (1997).
- [13] K. Sekimoto, *Stochastic Energetics* (Springer, New York, 2010).
- [14] V. Blickle, T. Speck, L. Helden, U. Seifert, and C. Bechinger, *Phys. Rev. Lett.* **96**, 070603 (2006).
- [15] U. Seifert, *Rep. Prog. Phys.* **75**, 126001 (2012).
- [16] *Nonequilibrium Statistical Physics of Small Systems: Fluctuation Relations and Beyond*, edited by R. Klages, W. Just, and C. Jarzynski (Wiley-VCH, New York, 2013).
- [17] A. Bérut, A. Arakelyan, A. Petrosyan, S. Ciliberto, R. Dillenschneider, and E. Lutz, *Nature (London)* **483**, 187 (2012).
- [18] S. Toyabe, T. Sagawa, M. Ueda, E. Muneyuki, and M. Sano, *Nat. Phys.* **6**, 988 (2010).
- [19] A. E. Cohen and W. E. Moerner, *Appl. Phys. Lett.* **86**, 093109 (2005).
- [20] A. E. Cohen, *Phys. Rev. Lett.* **94**, 118102 (2005).
- [21] Y. Jun and J. Bechhoefer, *Phys. Rev. E* **86**, 061106 (2012).
- [22] M. Gavrilov, Y. Jun, and J. Bechhoefer, *Proc. SPIE Int. Soc. Opt. Eng.* **8810**, 881012 (2013).
- [23] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/PhysRevLett.113.190601>, which includes Refs. [24,25].
- [24] A. Bérut, A. Petrosyan, and S. Ciliberto, *Europhys. Lett.* **103**, 60002 (2013).
- [25] P. Hänggi, P. Talkner, and M. Borkovec, *Rev. Mod. Phys.* **62**, 251 (1990).
- [26] K. J. Åström and B. Wittenmark, *Adaptive Control*, 2nd ed. (Dover, New York, 2008).
- [27] M. Gavrilov, Y. Jun, and J. Bechhoefer, *Rev. Sci. Instrum.* **85**, 095102 (2014).
- [28] The asymptotic work W of a “reset” operation with success rate p is $W = kT[\ln(2) + p\ln(p) + (1-p)\ln(1-p)]$.
- [29] É. Roldán, I. A. Martínez, J. M. R. Parrondo, and D. Petrov, *Nat. Phys.* **10**, 457 (2014).
- [30] J. V. Koski, V. F. Maisi, J. P. Pekola, and D. V. Averin, *Proc. Natl. Acad. Sci. U.S.A.* **111**, 13786 (2014).
- [31] R. Dillenschneider and E. Lutz, *Phys. Rev. Lett.* **102**, 210601 (2009).
- [32] M. Gavrilov, Y. Jun, and J. Bechhoefer (unpublished).
- [33] T. Schmiedl and U. Seifert, *Europhys. Lett.* **81**, 20003 (2008).
- [34] E. Aurell, K. Gawędzki, C. Mejía-Monasterio, R. Mohayadee, and P. Muratore-Ginanneschi, *J. Stat. Phys.* **147**, 487 (2012).
- [35] V. Blickle and C. Bechinger, *Nat. Phys.* **8**, 143 (2012).
- [36] G. M. Wang, E. M. Sevick, E. Mittag, D. J. Searles, and D. J. Evans, *Phys. Rev. Lett.* **89**, 050601 (2002).
- [37] C. Jarzynski, *Annu. Rev. Condens. Matter Phys.* **2**, 329 (2011).
- [38] M. Esposito and C. Van den Broeck, *Europhys. Lett.* **95**, 40004 (2011).
- [39] T. Sagawa, *J. Stat. Mech.* (2014) P03025.
- [40] E. Pop, *Nano Res.* **3**, 147 (2010).
- [41] G. Lan, P. Sartori, S. Neumann, V. Sourjik, and Y. Tu, *Nat. Phys.* **8**, 422 (2012).