Comment on "New Limit on Lorentz-Invariance- and *CPT*-Violating Neutron Spin Interactions Using a Free-Spin-Precession ^{3}He-^{129}Xe Comagnetometer"

In Ref. [1], the authors use a classical result for the magnetic field created by a uniform magnetization to analyze the effects of magnetic interactions between ^{3}He and ^{129}Xe nuclear spins. We point out that the classical result is not applicable for interaction between nuclear spins: the actual interaction is much weaker. This calls into question the results of their analysis setting limits on Lorentz invariance. We also point out that the Letter does not contain any discussion of systematic errors in the determination of Lorentz-violation coefficients.

In Ref. [1], ^{3}He and ^{129}Xe nuclear-polarized gases are contained in a spherical cell. Magnetic fields created by the polarized spins lead to their mutual interactions and modify their precession frequencies. To include these effects in the analysis, the authors assume that the field $\mathbf{B}$ inside a spherical cell due to a uniformly polarized gas with a magnetization $\mathbf{M}$ is given by

$$\mathbf{B} = \frac{2\mu_0}{3}\mathbf{M}. \tag{1}$$

This classical result is given, for example, in Ref. [2]. It can be obtained from a classical expression for the magnetic field created by a magnetic dipole moment $\mathbf{m}$,

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{3\hat{\mathbf{r}}(\hat{\mathbf{r}}\cdot\mathbf{m})-\mathbf{m}}{r^3} + \frac{2\mu_0\mathbf{m}}{3}\delta(\mathbf{r}), \tag{2}$$

by integrating the field over a sphere with a uniform density of dipoles $n = M/m$. The integral of the first term in Eq. (2) over a sphere vanishes. Fluctuations of the field due to a random distribution of atoms in the sphere have a zero mean for real atoms with a finite size [3,4].

The second term in Eq. (2) gives the classical expression (1), but it is only valid if the spins are completely noninteracting and their density $n(\mathbf{r})$ near each other remains uniform. In real systems, like gases, the spin interaction Hamiltonian becomes $H_2 = 2\mu_0\mathbf{m}_1\cdot\mathbf{m}_2 n_1(\mathbf{r}_1-\mathbf{r}_2)\delta(\mathbf{r}_1-\mathbf{r}_2)/3$. For interactions between electron and nuclear spins, it is parametrized by a factor κ [5],

$$\mathbf{B} = \frac{2\mu_0}{3}\kappa\mathbf{M}, \tag{3}$$

with the value of $\kappa \equiv n(0)/n(\infty)$ ranging from a few to a few hundred for various pairs of alkali-metal and noble-gas atoms due to the attractive interaction between the valence electrons and the noble gas nuclei.

For interactions between nuclear spins $\kappa = 0$ to first order [6], since during atomic collisions the overlap of the nuclei remains strictly zero, $n(0) = 0$. A finite effect can be generated by a second-order-electron Fermi-contact interaction in van der Waals molecules [7]. It is quite small and was only recently observed experimentally [8], giving $\kappa = -0.0014$ for a solution of liquid ^{129}Xe and ^{1}H in pentane. For a ^{3}He and ^{129}Xe gas mixture we expect an even smaller contribution from van der Waals molecules.

Deviations of the cell from a spherical shape can also generate a finite κ [9]. In glass-blown spherical cells, we typically find κ_{eff} ranging from -0.01 to -0.02. In these measurements we use 1.9 cm diameter cells containing 0.8 to 5.5 atm of ^{3}He gas, several torr of ^{129}Xe and Rb vapor heated to 120 °C. We measure the ^{129}Xe precession frequency while the ^{3}He polarization remains parallel or antiparallel to the bias field, so the ^{129}Xe frequency has a first-order sensitivity to κ_{eff}. The size as well as the sign of κ_{eff} depends on the orientation of the cell relative to the magnetic field. The fact that experimental data in Ref. [1] appear to be consistent with $\kappa = 1$ indicates that another source of frequency shift is likely to be present.

In conclusion, Eq. (1) greatly overestimates, likely by nearly 2 orders of magnitude, the magnetic interaction between nuclear spins. This calls into question the interpretation of the experimental results in Ref. [1], which also lacks substantive studies of systematic errors in Lorentz-violating coefficients. The systematic checks typically performed in similar experiments include splitting the Lorentz-violation data into several independent subsets, with different directions of magnetic field, different spin polarizations, different cells, etc., to check for consistency of the central values obtained from independent data subsets and for reliability of error estimates.

In their Reply [10] the authors show the Allan standard deviation for run $j = 6$, the same as in Fig. 2 of Ref. [1]. One can notice that for some other runs in Fig. 1(b) of Ref. [1], particularly runs 2 and 7, the phase deviations do not follow pure Gaussian noise.

M. V. Romalis,[1] D. Sheng,[1] B. Saam[2] and T. G. Walker[3]
[1]Department of Physics, Princeton University
Princeton, New Jersey 08544, USA
[2]Department of Physics, University of Utah
Salt Lake City, Utah 84112, USA
[3]Department of Physics, University of Wisconsin
Madison, Wisconsin 53706, USA

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[1] F. Allmendinger, W. Heil, S. Karpuk, W. Kilian, A. Scharth, U. Schmidt, A. Schnabel, Yu. Sobolev, and K. Tullney, Phys. Rev. Lett. **112**, 110801 (2014).
[2] J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1999).
[3] J. H. Wesenberg and K. Mølmer, Phys. Rev. Lett. **93**, 143903 (2004).
[4] S. G. Rautian, Opt. Spectrosc. **100**, 739 (2006).

[5] S. R. Schaefer, G. Cates, T.-R. Chien, D. Gonatas, W. Happer, and T. Walker, Phys. Rev. A **39**, 5613 (1989).
[6] A. Vlassenbroek, J. Jeener, and P. Broekaert, J. Magn. Reson., Ser. A **118**, 234 (1996).
[7] J. J. Heckman, M. P. Ledbetter, and M. V. Romalis, Phys. Rev. Lett. **91**, 067601 (2003).
[8] M. P. Ledbetter, G. Saielli, A. Bagno, N. Tran, and M. V. Romalis, Proc. Natl. Acad. Sci. U.S.A. **109**, 12393 (2012).
[9] M. V. Romalis and G. D. Cates, Phys. Rev. A **58**, 3004 (1998).
[10] F. Allmendinger, U. Schmidt, W. Heil, S. Karpuk, A. Scharth, Yu. Sobolev, and K. Tullney, following Letter, Phys. Rev. Lett. **113**, 188902 (2014).