Plasma-Resistivity-Induced Strong Damping of the Kinetic Resistive Wall Mode

Yuling He,[1] Yueqiang Liu,[2] Yue Liu,[1,*] Guangzhou Hao,[3] and Aike Wang[3]
[1]Key Laboratory of Materials Modification by Laser, Ion and Electron Beams (Dalian University of Technology), Ministry of Education, Dalian 116024, China
[2]Culham Centre for Fusion Energy, Culham Science Centre, Abingdon OX14 3DB, United Kingdom
[3]Southwestern Institute of Physics, P.O. Box 432, Chengdu 610041, China
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An energy-principle-based dispersion relation is derived for the resistive wall mode, which incorporates both the drift kinetic resonance between the mode and energetic particles and the resistive layer physics. The equivalence between the energy-principle approach and the resistive layer matching approach is first demonstrated for the resistive plasma resistive wall mode. As a key new result, it is found that the resistive wall mode, coupled to the favorable average curvature stabilization inside the resistive layer (as well as the toroidal plasma flow), can be substantially more stable than that predicted by drift kinetic theory with fast ion stabilization, but with the ideal fluid assumption. Since the layer stabilization becomes stronger with decreasing plasma resistivity, this regime is favorable for reactor scale, high-temperature fusion devices.

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In order to maximize the benefit of the concept of the advanced tokamak—which aims at high pressure, steady state plasma operation—the stability, or stabilization, of the resistive wall mode (RWM) is a critical issue. The absence of unstable, low n (n is the toroidal mode number) RWMs provides a viable way of achieving high normalized β, which in turn ensures both high fusion gain and high noninductive, bootstrap current fraction in a tokamak plasma. These are the key motivations for understanding, both in experiments and in theory, the physics and control of the RWM.

Because the mode originates from the ideal external kink instability, so far most of the theory and modeling efforts have adopted the ideal fluid assumption (i.e., with vanishing plasma resistivity), with few exceptions [1–3]. The resistive plasma resistive wall mode theory was developed, in the aforementioned work, in the pure fluid approximation. On the other hand, recent theory [4–9] and experiments [10–12] seem to suggest a strong damping of the RWM by drift kinetic resonant effects, with both thermal [4,8,13] and energetic particles [7,9,14]. The kinetic theory, often combined with the fluid description to form a hybrid formulation, so far has been to assume an ideal plasma.

In this Letter, we examine the combined effects of the (local) resistive layer physics and the (global) drift kinetic resonance damping, of fast ions, on the stability of the RWM. To accomplish this, we first derive an extended RWM dispersion relation, based on the energy conservation. We then numerically solve the dispersion relation, which is highly nonlinear with respect to the mode's eigenvalue, on an example of a cylindrical plasma. Certain approximations have to be adopted following this approach. The most severe one is probably the inconsistency of the evaluation of the perturbed drift kinetic energy perturbation from the trapped fast ions, with the assumption of the cylindrical geometry. Nevertheless, we believe that this does not affect the qualitative physics of the drift kinetic damping, as confirmed by full toroidal computations [7,14,15]. Moreover, the resistive layer contribution, as we shall show, is rigorously evaluated in our example.

Our approach follows that for the resistive internal kink mode devised by Biglari and Chen [16]. We extend the well-known dispersion for the kinetic RWM by adding the resistive layer energy dissipation term δW_{RL},

$$\tilde{\gamma}^2\delta K + \delta W_p + \frac{\delta W_v^{\infty} + \gamma\tau_w^*\delta W_v^b}{1 + \gamma\tau_w^*} + \delta W_k + \delta W_{\mathrm{RL}} = 0, \quad (1)$$

where $\tilde{\gamma} \equiv (\gamma + in\omega_0)\tau_A$ is the Doppler-shifted (complex) eigenvalue of the mode, with ω_0 being the toroidal rotation frequency of the plasma (which is assumed to be uniform here). τ_A is the Alfvén time to be defined later. δK is the plasma inertia. δW_p is the potential energy perturbation inside the plasma, while δW_v^{∞} and δW_v^b represent the vacuum energy perturbation without and with an ideal conducting wall, respectively. The whole third term in the above equation represents the vacuum energy including the eddy-current-induced dissipation in a resistive wall, with the effective wall time of τ_w^* (to be defined later). The fourth and fifth terms are the energy dissipation terms associated with the drift kinetic resonances and the resistive layer, respectively.

For a nominal RWM, the plasma inertia is often of secondary importance (the plasma inertia becomes important when, for instance, the plasma is close to the so-called ideal wall β limit or, equivalently, to the marginal ideal wall position). Neglecting the first term in Eq. (1), we can write

$$\gamma\tau_w^* = -\frac{\delta W^\infty + \delta W_k + \delta W_{\rm RL}}{\delta W^b + \delta W_k + \delta W_{\rm RL}}, \tag{2}$$

where $\delta W^\infty \equiv \delta W_p + \delta W_v^\infty$ and $\delta W^b \equiv \delta W_p + \delta W_v^b$.

We shall show later on that the above expression (2) is not a heuristic extension of the known RWM dispersion relation [4]. Neglecting the drift kinetic term, Eq. (2) can be shown to be equivalent to the matching condition, as used in the previous RWM study within the resistive fluid theory [1–3], between the inner and outer solutions near the mode's rational surface. Before proceeding to the proof, we first note that the energy dissipation associated with the resistive layer is proportional to the inner layer tearing index $\Delta'_{\rm int}$ [17],

$$\delta W_{\rm RL} = C\Delta'_{\rm int}, \tag{3}$$

with the coefficient C being a positive number. As an example, the above relation can be easily derived for a zero β plasma, by solving the coupled layer equations, for the perturbed poloidal magnetic flux ψ_1 and the plasma radial displacement ξ [17]:

$$\psi_1(x) + \hat{s}x\xi(x)n/m = r_s^2\psi_1''(x)/(\gamma\tau_R), \tag{4}$$

$$(\gamma\tau_A)^2\xi''(x) = \hat{s}x\psi_1''(x)nm/r_s^2, \tag{5}$$

where $x \equiv r - r_s$, with r being the minor radius and r_s denoting the location of the rational surface, $\hat{s} \equiv rq'/q$ being the magnetic shear calculated at the rational surface, and m and n being the poloidal and toroidal mode numbers, respectively. $\tau_A \equiv R\sqrt{\mu_0\rho}/B_z$ is the toroidal Alfvév time and $\tau_R \equiv \mu_0 r_s^2/\eta$ the resistive decay time of the plasma. Following Ref. [16], $\delta W_{\rm RL} \propto \int_{-\infty}^{+\infty}[(\gamma\tau_A)^2|\xi'|^2 - \hat{s}nmx\xi^*\psi_1''/r_s]dx$, which can be shown to be proportional to $\Delta'_{\rm int} = 2\pi[\Gamma(3/4)/\Gamma(1/4)](n\hat{s})^{-1/2}S^{3/4}(\gamma\tau_A)^{5/4}$, where $S \equiv \tau_R/\tau_A$ is the Lundquist number. We point out that the linear scaling between $\delta W_{\rm RL}$ and $\Delta'_{\rm int}$ is valid for the $m > 1$ mode (our case). For the $m = 1$ mode (the internal kink mode), $\delta W_{\rm RL}$ is inversely proportional to $\Delta'_{\rm int}$, as shown in Ref. [16].

Now we consider a cylindrical plasma described in Ref. [18] (with a step function for the plasma current density and a constant pressure). For this equilibrium, all of the perturbed fluid energies from Eq. (2) can be analytically calculated and can be related to the logarithmic derivative jumps of the perturbed flux function ψ_1:

$$\delta W^\infty = -C\Delta'_{\rm nw}, \tag{6}$$

$$\delta W^b = -C\Delta'_{\rm iw}, \tag{7}$$

$$\delta W_{RL} = C\Delta'_{\rm int}, \tag{8}$$

where $C = [(m - nq_0)^2/(q_0^2\tau_A^2)](r_s/m)$ and q_0 is the on-axis safety factor. $\Delta'_{\rm iw}$ and $\Delta'_{\rm nw}$ are again defined at the rational surface, for cases with and without an ideal wall, respectively.

On the other hand, the matching condition leads to [18]

$$\Delta'_{\rm int} = \Delta'_{\rm ext} \equiv \frac{\delta_0 + \gamma\tau_w\delta_\infty}{\psi_0 + \gamma\tau_w\psi_\infty}, \tag{9}$$

where $\delta_0 = \psi_0\Delta'_{\rm nw}, \delta_\infty = \psi_\infty\Delta'_{\rm iw}, \psi_\infty = \psi_0 C_w, C_w \equiv (1 - b^{-2m})/(2m), \tau_w^* = \tau_w C_w$, and b is the minor radius of the wall location normalized by the plasma minor radius. The physical significance of $\delta_0, \delta_\infty, \psi_0, \psi_\infty$ is explained in Ref. [18]. It is straightforward to show that the above matching condition (9) is equivalent to the energy-based dispersion relation (2), in the absence of the drift kinetic term δW_k.

The matching condition (9) was also used in an earlier work [3] to study the resistive plasma RWM stability with the inclusion of the favorable curvature effect [19]. The limitation of this approach, compared to the energy-based approach (2), is the difficulty to include the drift kinetic effects. Therefore, in further study, we shall follow the dispersion relation (2) in order to study the combined effects of the drift kinetic damping (from fast ions) and the plasma resistive damping, for cases without and with the favorable curvature term. To do this, we use the drift kinetic energy term δW_k derived in Refs. [9,20]. Note that this term is derived for an equilibrium with constant safety factor q. Therefore, our further quantitative results are valid only for cases where the resistive layer is located in a narrow region near the plasma boundary.

Subject to certain normalization, as in Refs. [9,20], for the above specified cylindrical equilibrium, all of the perturbed potential energies from Eq. (2) can be analytically calculated:

$$\delta W^\infty = \frac{4\pi}{m}\frac{(m - nq_0)^2}{q_0^2}\left(1 - \frac{1}{m - nq_0}\right), \tag{10}$$

$$\delta W^b = \frac{4\pi}{m}\frac{(m - nq_0)^2}{q_0^2}\left(\frac{1}{1 - b^{-2m}} - \frac{1}{m - nq_0}\right), \tag{11}$$

$$\delta W_{\rm RL}(\gamma) = \frac{4\pi}{m}\frac{(m - nq_0)^2}{q_0^2}\frac{r_s}{2m}\Delta'_{\rm int}(\gamma), \tag{12}$$

$$\begin{aligned}\delta W_k(\gamma) = {}& 12\pi\left(1 - \frac{\alpha_0 B_0}{2}\right)^2\frac{\beta_h R}{Ka}\\ &\times\left\{\frac{2}{7}(A_K - B_K)\Omega\ln\left(1 - \frac{1}{\Omega}\right)\right.\\ &-\frac{2}{7}\left(A_K + \frac{2}{5}B_K\right)\Omega\left[2\left(\frac{1}{5\Omega} + \frac{1}{3\Omega^2} + \frac{1}{\Omega^3}\right)\right.\\ &\left.\left.-\frac{1}{\Omega^{7/2}}\ln\left(\frac{1 + \sqrt{\Omega}}{1 - \sqrt{\Omega}}\right)\right]\right\} + M,\end{aligned} \tag{13}$$

where $\Omega \equiv i\tilde{\gamma}/(\omega_{ds}\tau_A)$, with ω_{ds} being the toroidal precession frequency of trapped fast ions at the birth energy. The coefficients A_K, B_K, and M are taken from Ref. [20]. The inner layer tearing index, including the favorable curvature effect, can be written as

$$\Delta'_{\rm int}(\gamma) = A_R\tilde{\gamma}^{5/4}[1 - (\pi/4)D_R B_R \tilde{\gamma}^{-3/2}], \quad (14)$$

with the coefficients $A_R \equiv 2\pi[\Gamma(3/4)/\Gamma(1/4)](n\hat{s})^{-1/2} \times (1+2q^2)^{1/4}S^{3/4}$ and $B_R \equiv (n\hat{s})(1+2q^2)^{-1/2}S^{-1/2}$ evaluated at the rational surface. D_R is defined in Ref. [19] and is normally negative for tokamak plasmas.

The dispersion relation (2) is strongly nonlinear with respect to the mode's eigenvalue γ. We now proceed to numerically solve this dispersion relation for our test equilibrium. We choose a case with $q_0 = 1.42$ and consider the $n = 1, m = 2$ RWM. Before showing the numerical results, we note an interesting feature of Eq. (2) with respect to the resistive layer term. As the plasma resistivity approaches zero (the Lundquist number S approaches infinity), the dispersion relation does not recover the ideal fluid result. In fact, the ideal fluid case is recovered at the vanishing Lundquist number. This is associated with the fact that the ideal plasma theory for the RWM assumes no jump of the ψ_1 through the rational surface (the ideal kink solution). Physically, the ideal assumption does not allow the magnetic reconnection to occur, which therefore leads to a qualitatively different solution compared to the case where the latter is allowed, even at a very small plasma resistivity. On the other hand, the width of the resistive layer increases with plasma resistivity. In the limit of the vanishing S value, the width of the tearing layer is so large that the RWM again behaves like a kink mode. This is the physics reason why the ideal fluid dispersion relation is recovered only at the vanishing Lundquist number.

The aforementioned behavior is clearly seen in the numerical results shown in Fig. 1. We observe a strong reduction of the growth rate of the mode by the resistive layer energy dissipation, compared to the ideal fluid theory prediction. On the other hand, within the resistive model, the mode's growth rate increases with the plasma resistivity. In fact, it can be shown analytically that this increase is monotonic without the favorable curvature term ($D_R = 0$).

The inclusion of the favorable curvature effect ($D_R < 0$) further reduces the mode's growth rate. Interestingly, at a sufficiently large value of $-D_R$, the growth rate of the RWM becomes complex even in a static plasma, as shown in Fig. 2. This is similar to the linear tearing mode, which can become a rotating mode even in a static plasma, when D_R is sufficiently negative.

Inclusion of the drift kinetic effect, in our case from the precessional resonance damping of fast ions, further decreases the growth or damping rate of the RWM. One example is shown in Fig. 3, where we fix the wall radius at $1.2a$ as well as the Glasser term with $D_R = -0.001$, while

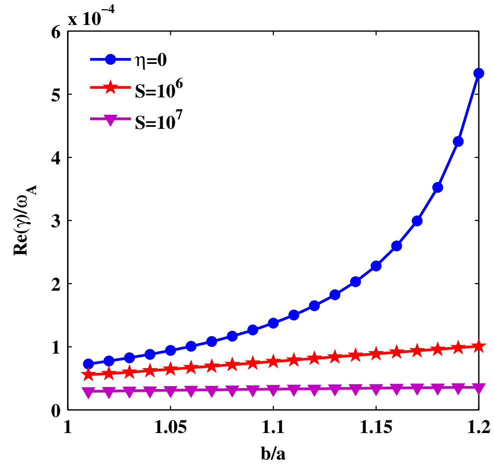

FIG. 1 (color online). Growth rates (normalized by the Alfvén frequency) of the RWM versus the wall minor radius, predicted by the ideal fluid ($\eta = 0$) and the resistive fluid ($S = 10^6$ and $S = 10^7$) theory. A static plasma ($\omega_0 = 0$) is assumed. Neither the drift kinetic effect nor the Glasser effect ($D_R = 0$) is included.

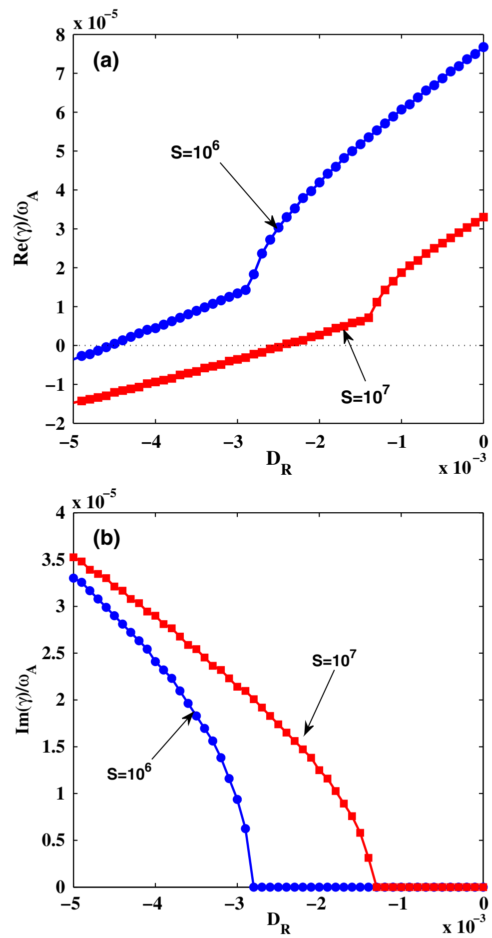

FIG. 2 (color online). The (a) growth rate and (b) real frequency (normalized by the Alfvén frequency) of the RWM versus D_R, predicted by the resistive fluid ($S = 10^6$ and $S = 10^7$) theory. A static plasma ($\omega_0 = 0$) is assumed. No drift kinetic effect is included.

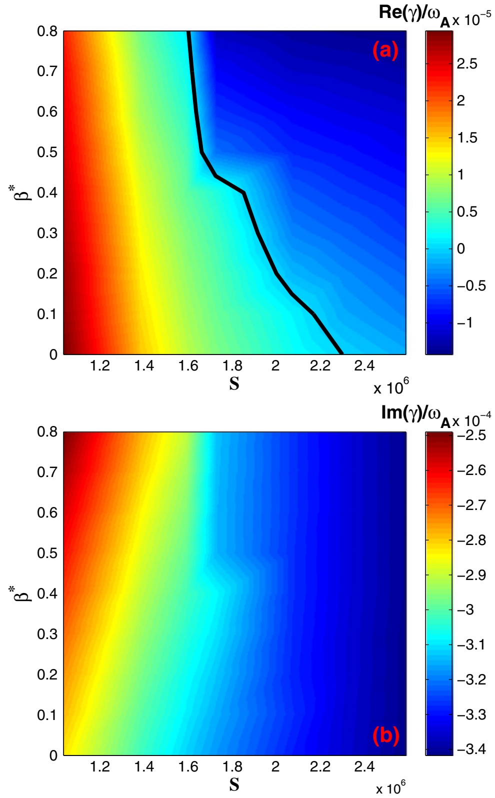

FIG. 3 (color online). The (a) growth rate and (b) real frequency (normalized by the Alfvén frequency) of the RWM with varying energetic particle pressure $\beta^* \equiv \beta_h/\beta_{th}$ and the Lundquist number S. A finite β plasma model is assumed with the presence of the Glasser effect ($D_R = -0.001$). A plasma toroidal flow frequency of $\omega_0/\omega_{ds} = -0.1$ is assumed. The resistive wall is located at a minor radius of $1.2a$. The solid curve in (a) indicates the marginal stability of the RWM.

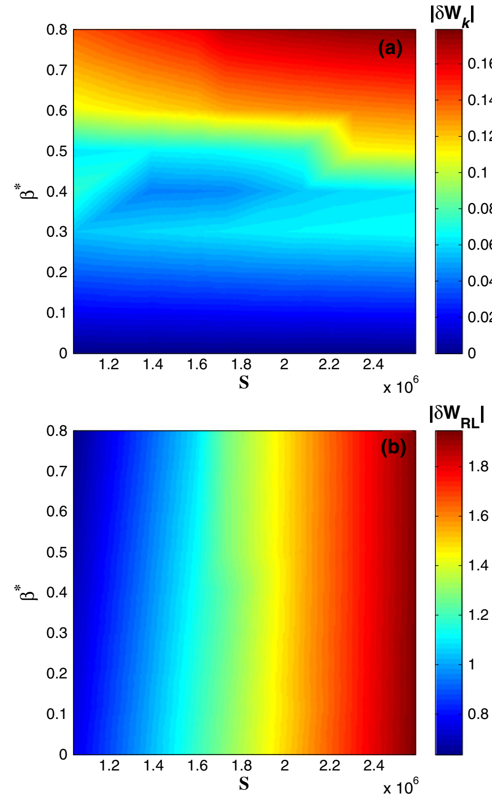

FIG. 4 (color online). The amplitude of (a) the perturbed drift kinetic energy $|\delta W_k|$ and (b) the perturbed energy associated with the resistive layer $|\delta W_{\mathrm{RL}}|$, with varying energetic particle pressure β^* and Lundquist number S. The other plasma conditions are the same as in Fig. 3.

varying the two key parameters representing the drift kinetic effect from the energy particles or EPs (the normalized EP pressure β^*) and the resistive layer effect (the Lundquist number S), respectively. Furthermore, we assume a relatively small plasma toroidal rotation frequency of $\omega_0 = -0.1\omega_{ds} = -4.46 \times 10^{-4}\omega_A$. Increasing the flow speed tends to enhance the drift kinetic damping from the EPs and hence further stabilizes the mode. Interestingly, the drift kinetic contribution from EPs and the resistive layer contribution act synergistically on the mode damping by increasing both the EP pressure fraction and the Lundquist number, as shown in Fig. 3(a). We find a critical curve in the β^*-S plane, above which the RWM becomes stable as a result of this synergy effect. The real frequency of the mode, shown in Fig. 3(b), is predominantly introduced by the resistive layer effect (note that the mode frequency scales mainly with S but is almost independent of β^*). In fact, a comparison of the amplitude of δW_k and δW_{RL}, shown in Fig. 4, reveals that the resistive layer can give an order of magnitude larger contribution to the perturbed energy. Note that these energy components are self-consistently evaluated, using the eigenvalue of the mode as the solution of the RWM dispersion relation. These energy components are also subject to the same normalization (the perturbed plasma inertial energy). The resistivity-induced strong damping of the RWM seems to be a systematic observation from our cylindrical example.

In conclusion, the resistive fluid theory provides a more optimistic prediction of the RWM stability. The additional stabilization from the energy dissipation associated with the resistive layer, which is further enhanced by the favorable curvature effect, can be significant, and in fact can be even stronger than that from the drift kinetic damping by fast ions. It is important to note that the two damping effects work in a synergistic manner. Since the resistive damping is more effective with the reduction of the plasma resistivity, we may expect a strong damping of the RWM in the reactor scale, high-temperature plasmas.

Our present model includes neither all of the kinetic physics nor the full geometrical effects. In particular, we do not consider the drift kinetic damping from thermal particles, which can be strong at slow plasma flow [4,5]. We have assumed a simple cylindrical geometry to enable analytic treatment. Advanced numerical study, using resistive fluid-kinetic hybrid codes such as MARS-K [13], needs to be carried out for realistic geometry.

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*liuyue@dlut.edu.cn

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