Mechanism for Thermal Relic Dark Matter of Strongly Interacting Massive Particles

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We present a new paradigm for achieving thermal relic dark matter. The mechanism arises when a nearly secluded dark sector is thermalized with the standard model after reheating. The freeze-out process is a number-changing $3 \to 2$ annihilation of strongly interacting massive particles (SIMPs) in the dark sector, and points to sub-GeV dark matter. The couplings to the visible sector, necessary for maintaining thermal equilibrium with the standard model, imply measurable signals that will allow coverage of a significant part of the parameter space with future indirect- and direct-detection experiments and via direct production of dark matter at colliders. Moreover, $3 \to 2$ annihilations typically predict sizable $2 \to 2$ self-interactions which naturally address the "core versus cusp" and "too-big-to-fail" small-scale structure formation problems.

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Introduction.—Dark matter (DM) makes up the majority of the mass in the Universe; however, its identity is unknown. The few properties known about DM are that it is cold, massive, it is not electrically charged, it is not colored, and it is not very strongly self-interacting. One possibility for the identity of DM is that it is a thermal relic from the early Universe. Cold thermal relics are predicted to have a mass

$$m_{\rm DM} \sim \alpha_{\rm ann}(T_{\rm eq}M_{\rm Pl})^{1/2} \sim {\rm TeV}, \tag{1}$$

where $\alpha_{\rm ann}$ is the effective coupling constant of the $2 \to 2$ DM annihilation cross section, taken to be of order weak processes $\alpha_{\rm ann} \simeq 1/30$ above, $T_{\rm eq}$ is the matter-radiation equality temperature, and $M_{\rm Pl}$ is the reduced Planck mass. The emergence of the weak scale from a geometric mean of two unrelated scales, frequently called the WIMP miracle, provides an alternate motivation beyond the hierarchy problem for TeV-scale new physics.

In this Letter, we show that there is another mechanism that can produce thermal relic DM even if $\alpha_{\rm ann} \simeq 0$. In this limit, while thermal DM cannot freeze-out through the standard $2 \to 2$ annihilation, it may do so via a $3 \to 2$ process, where three DM particles collide and produce two DM particles. The mass scale that is indicated by this mechanism is given by a generalized geometric mean,

$$m_{\rm DM} \sim \alpha_{\rm eff}(T_{\rm eq}^2 M_{\rm Pl})^{1/3} \sim 100\ {\rm MeV}, \tag{2}$$

where $\alpha_{\rm eff}$ is the effective strength of the self-interaction of the DM which we take as $\alpha_{\rm eff} \simeq 1$ in the above. As we will see, the $3 \to 2$ mechanism points to strongly self-interacting DM at or below the GeV scale.

If the dark sector does not have sufficient couplings to the visible sector for it to remain in thermal equilibrium, the $3 \to 2$ annihilations heat up the DM, significantly altering structure formation [1,2]. In contrast, a crucial aspect of the mechanism described here is that the dark sector is in thermal equilibrium with the standard model (SM); i.e., the DM has a phase-space distribution given by the temperature of the photon bath. Thus, the scattering with the SM bath enables the DM to cool off as heat is being pumped in from the $3 \to 2$ process. Consequently, the $3 \to 2$ thermal freeze-out mechanism generically requires measurable couplings between the DM and visible sectors. A schematic description of the SIMP paradigm is presented in Fig. 1.

The phenomenological consequences of this paradigm are twofold. First, the significant DM self-interactions have implications for structure formation, successfully addressing the so called "core versus cusp" and "too-big-to-fail" problems (see, e.g., [3–5]). Second, the interactions between the DM and visible sectors predict significant direct and indirect signatures which may be probed in the near future.

In this Letter, we aim to present a new paradigm for DM, rather than a specific DM candidate. For this reason, we do not explore particular models for the dark sector, but instead use a simplified effective description in order to understand the properties of the DM sector and its interaction with the SM such that the mechanism is viable. A detailed study exploring models for the SIMP mechanism is underway [6].

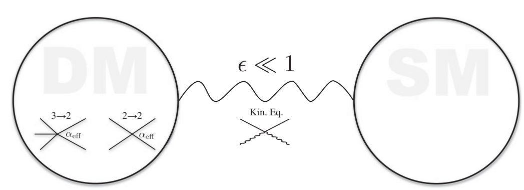

FIG. 1. A schematic description of the SIMP paradigm.

The $3 \to 2$ *mechanism.*—As mentioned above, the $3 \to 2$ annihilation mechanism predicts a mass range for the DM, just as the standard $2 \to 2$ annihilation mechanism predicts the TeV scale. The estimate of the indicated mass scale is presented here, and is later verified by solving the Boltzmann equation explicitly.

It is useful to express quantities in the freeze-out estimate in terms of measured quantities. In particular, the DM number density is given by

$$n_{\rm DM} = \frac{\xi m_p \eta s}{m_{\rm DM}} = \frac{c T_{\rm eq} s}{m_{\rm DM}}, \tag{3}$$

where $\xi = \rho_{\rm DM}/\rho_b \simeq 5.4$ [7], m_p is the proton mass, s is the entropy density of the Universe, and η is the baryon to entropy ratio (see, e.g., [8] for further definitions). In the second equality, the number density is expressed in terms of the matter-radiation equality temperature, $T_{\rm eq} = \xi m_p \eta / c \simeq 0.8$ eV, where $c \equiv (\xi/1+\xi)\frac{3}{4}(g_{*,\rm eq}/g_{*s,\rm eq}) \simeq 0.54$, and $g_{*,\rm eq}(g_{*s,\rm eq})$ is the energy (entropy) effective number of relativistic degrees of freedom at equality time.

Freeze-out roughly occurs when the rate of the $3 \to 2$ process, $\Gamma_{3\to2}$, is equal to the Hubble rate H. The freeze-out condition is given by

$$n_{\rm DM}^2 \langle \sigma_{3\to2} v^2 \rangle|_{T=T_F} = 0.33\sqrt{g_{*,F}}\frac{T_F^2}{M_{\rm Pl}}. \tag{4}$$

We parameterize the $3 \to 2$ cross section by

$$\langle \sigma v^2 \rangle_{3\to2} \equiv \frac{\alpha_{\rm eff}^3}{m_{\rm DM}^5}, \tag{5}$$

where $\alpha_{\rm eff}$ is the effective coupling strength entering the *thermally averaged* cross section. We stress that the effective coupling above can be significantly larger than unity if, for example, the number of DM degrees of freedom is large, if the cross section is nonperturbatively enhanced, or if the $3 \to 2$ process is mediated by a light particle.

The rest of the freeze-out estimate proceeds in a straightforward manner. Using

$$s = \frac{\kappa}{c}T^3, \qquad \kappa = \frac{2\pi^2 c g_{*s}(T)}{45}, \tag{6}$$

and parameterizing the freeze-out temperature as $T_F = (m_{\rm DM}/x_F)$, the DM mass indicated by the $3 \to 2$ process is

$$m_{\rm DM} \simeq 1.4 \alpha_{\rm eff} x_F^{-1} [g_{*,F}^{-(1/2)} x_F^{-1} (\kappa T_{\rm eq})^2 M_{\rm Pl}]^{1/3}. \tag{7}$$

Taking $x_F = 20$ and $\alpha_{\rm eff} = 1$ for a (rather) strongly interacting theory that freezes out while the DM is nonrelativistic, we arrive at

$$m_{\rm DM} \simeq 40\ {\rm MeV} \quad (3 \to 2). \tag{8}$$

Small corrections are found when the more precise Boltzmann equations are solved (see Fig. 2). Thus, in analogy to the standard thermal WIMP, where weak coupling gives rise to the weak scale, the $3 \to 2$ freeze-out mechanism gives rise to strong-scale DM for strong coupling. Lower (higher) DM mass is of course consistent with lower (higher) $\alpha_{\rm eff}$. As we will see, self-interactions of DM along with CMB and BBN constraints point to the strongly interacting limit of large $\alpha_{\rm eff}$. We thus dub this scenario the *strongly interacting massive particle* (SIMP) paradigm.

Thermal equilibrium.—Throughout the above estimate, we have assumed that the dark sector and SM remained in thermal equilibrium. However, the processes that keep the two sectors in thermal equilibrium are the crossing diagrams of the processes that lead to $2 \to 2$ annihilation into the SM. Thus, the assumption of thermal equilibrium might naively imply that the dominant number-changing process for the DM is the $2 \to 2$ annihilation channel. We now find the condition under which the latter is subdominant while thermal equilibrium is maintained.

The ratio of the scattering rate off of SM particles $\Gamma_{\rm kin}$ and the annihilation rate to SM particles $\Gamma_{\rm ann}$ is

$$\frac{\Gamma_{\rm kin}}{\Gamma_{\rm ann}} = \frac{n_{\rm SM}\langle \sigma v \rangle_{\rm kin}}{n_{\rm DM}\langle \sigma v \rangle_{\rm ann}} \simeq \frac{m_{\rm DM}}{\pi^2 \kappa T_{\rm eq}} \simeq 5 \times 10^6, \tag{9}$$

where the second equality uses $\langle \sigma v \rangle_{\rm kin} \sim \langle \sigma v \rangle_{\rm ann}$, and the last equality is derived for $m_{\rm DM} = 40$ MeV. This large ratio is simply understood by the subdominance of the DM number density at $T_F \gg T_{\rm eq}$. Thus, if the SM couples to the DM, the process keeping these two sectors in kinetic equilibrium does not have to be changing the annihilation rate. A similar statement holds in the standard thermal WIMP scenario [9].

In order for the $3 \to 2$ process to control freeze-out, while not heating up the DM, the following inequality must hold up until freeze-out occurs:

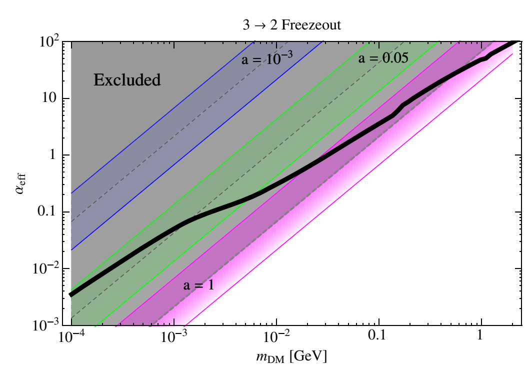

FIG. 2 (color online). $\alpha_{\rm eff}$ vs DM mass (black solid line), derived from the numerical solution to the Boltzmann equation in the $3 \to 2$ freeze-out scenario. The colored regions show the preferred region for the "core vs cusp" and "too-big-to-fail" anomalies for $a = 1$ (magenta), $a = 0.05$ (green), and $a = 10^{-3}$ (blue). The region above the gray-dashed lines is excluded by the bullet-cluster and halo shape constraints, for each value of a.

$$\Gamma_{\rm ann} \lesssim \Gamma_{3\to2} \lesssim \Gamma_{\rm kin}, \tag{10}$$

at $T = T_F$. We parameterize the DM-SM scattering by a small coupling, ϵ, with the relevant energy scale $m_{\rm DM}$, such that

$$\langle \sigma v \rangle_{\rm kin} \sim \langle \sigma v \rangle_{\rm ann} \equiv \frac{\epsilon^2}{m_{\rm DM}^2}. \tag{11}$$

The exact relation between the above cross sections can be calculated for particular couplings to SM particles. (We note that assuming s-wave annihilations, the above further encodes the DM annihilation rate expected today.) The right-side inequality in Eq. (10), which ensures that the coupling to the SM is strong enough to keep the dark sector and visible sector at a unified temperature, requires

$$\epsilon \gtrsim \epsilon_{\rm min} \equiv 2\alpha_{\rm eff}^{1/2}\left(\frac{T_{\rm eq}}{M_{\rm Pl}}\right)^{1/3} \simeq 1\times 10^{-9}, \tag{12}$$

where the numerical estimates use $\alpha_{\rm eff} = 1$ and $x_F \simeq 20$, as is justified by solving the Boltzmann equation explicitly. The left-side inequality, ensuring that the annihilation of the dark sector to SM states is not efficient at freeze-out, implies

$$\epsilon \lesssim \epsilon_{\rm max} \equiv 0.1\alpha_{\rm eff}\left(\frac{T_{\rm eq}}{M_{\rm Pl}}\right)^{1/6} \simeq 3\times 10^{-6}, \tag{13}$$

with the same choice of parameters as above. We learn that there is a large range of couplings of the DM to the SM in which Eq. (10) is satisfied.

For completeness, we give the Boltzmann equation for $n_{\rm DM}$ when Eq. (10) holds [10]:

$$\partial_t n_{\rm DM} + 3Hn_{\rm DM} = -(n_{\rm DM}^3 - n_{\rm DM}^2 n_{\rm eq})\langle \sigma v^2\rangle_{3\to2} - (n_{\rm DM}^2 - n_{\rm eq}^2)\langle \sigma v\rangle_{\rm ann}. \tag{14}$$

Numerically integrating the above leads to the results of Fig. 2, which agree very well with the estimate of Eq. (7).

A toy model.—To better understand the SIMP paradigm, we now present a weakly coupled toy model for the dark sector which incorporates the $3\to2$ mechanism and leads to stable DM. Consider a Z_3-symmetric theory with a single scalar, χ, defined by

$$\mathcal{L}_{\rm DM} = |\partial\chi|^2 - m_{\rm DM}^2|\chi|^2 - \frac{\kappa}{6}\chi^3 - \frac{\kappa^\dagger}{6}\chi^{\dagger 3} - \frac{\lambda}{4}|\chi|^4. \tag{15}$$

With the above couplings, tree-level $2\to2$ self-interactions and $3\to2$ scattering are induced. For a single scale model, defining g via $\kappa = g m_{\rm DM}$ and taking $\lambda \sim g^2$, the $2\to2$ scattering cross section scales as $g^4/m_{\rm DM}^2$, and the $3\to2$ one as $g^6/m_{\rm DM}^5$, motivating our parametrization of Eq. (5). The stability of the DM is guaranteed by the global symmetry.

Let us now introduce small interactions between the DM and the visible sector. As an example, consider first an interaction with SM fermions f,

$$\mathcal{L}_{\rm int} = \frac{m_f}{\Lambda^2}\chi^\dagger\chi\bar{f}f, \tag{16}$$

which induces both $2\to2$ annihilations and scatterings. Identifying the ϵ defined in Eq. (11) to be of order $\epsilon \simeq \mathcal{O}(m_f m_{\rm DM}/\Lambda^2)$, the $2\to2$ annihilation rate is negligible while kinetic equilibrium is maintained, for $\epsilon_{\rm min} \lesssim \epsilon \lesssim \epsilon_{\rm max}$. One may further check that annihilations such as $\chi\chi f \to \chi^\dagger f$, which are induced by the interactions in Eqs. (15) and (16), are also negligible despite the large number density in the thermal bath. Alternatively, the dark sector may couple to the visible one through photons,

$$\mathcal{L}_{\rm int} = \frac{\alpha_{\rm EM}}{4\pi\Lambda^2}\chi^\dagger\chi F_{\mu\nu}F^{\mu\nu}, \tag{17}$$

in which case, $\epsilon \simeq \mathcal{O}(\alpha_{\rm EM} m_{\rm DM}^2/4\pi\Lambda^2)$.

To conclude, we find that for a low-scale (of order, say, 100 MeV) dark sector with DM described by Eq. (15), the correct relic abundance is obtained if the sector communicates with the visible one (say, through couplings to electrons, muons, or photons) via a new scale in the GeV to 10's of TeV range. Such mediators are thus constrained by LEP [11,12] (see Fig. 3) and are expected to be within reach of ongoing collider experiments.

Signatures.—The paradigm discussed in this Letter not only provides a new mechanism for producing the DM relic abundance, but also predicts interesting and measurable signatures. There are two distinct reasons for this. First, the DM must be in thermal equilibrium with the visible sector. Consequently, it must have non-negligible couplings to SM particles, which in turn predict observable signals. Second, the nonvanishing five-point interaction required for the $3\to2$ annihilations also implies sizeable self-couplings which alter the predictions for structure formation.

We begin with structure formation. The persistent failure of N-body simulation to reproduce the small-scale structure of observed galactic halos has led to the "core versus cusp" and "too-big-to-fail" problems. This motivates self-interacting DM with a strength [13–16]

$$\left(\frac{\sigma_{\rm scatter}}{m_{\rm DM}}\right)_{\rm obs} = (0.1\text{–}10)\ {\rm cm^2/g}. \tag{18}$$

On the other hand, bullet-cluster constraints [17–19] as well as recent simulations which reanalyze the constraints from halo shapes [14,16], suggest the limits on the DM self-interacting cross section (at velocities $\gtrsim$300 km/sec) are

$$\frac{\sigma_{\rm scatter}}{m_{\rm DM}} \lesssim 1\ {\rm cm^2/g}. \tag{19}$$

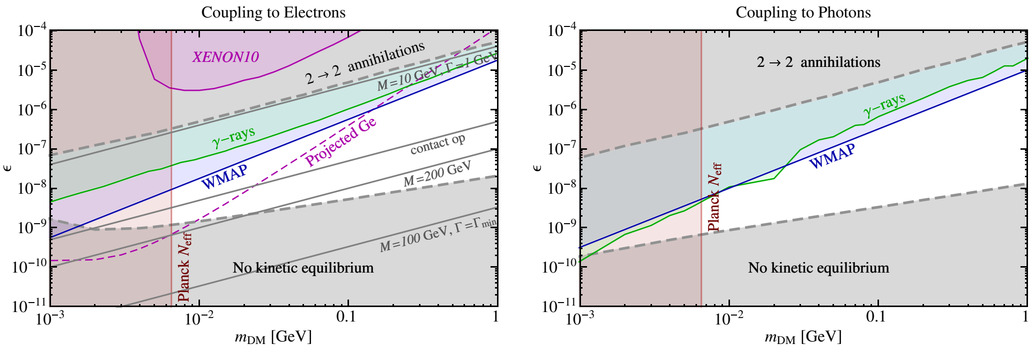

FIG. 3 (color online). The bounds on ϵ vs $m_{\rm DM}$. In both panels, the grey regions (outlined by thick dashed lines) represent the range of parameters in which kinetic equilibrium with the SM is not maintained (lower gray region), and where the standard $2 \to 2$ annihilation to the SM is not subdominant to the $3 \to 2$ process (upper gray region). *Left, coupling to electrons:* Additional exclusion limits from: direct detection in Xenon10 (purple region), and the expected future bound from a germanium-based electron recoil experiment (dashed purple); CMB and low redshift data constraints for electrons (blue region); modification of $N_{\rm eff}$ (red region); indirect detection of γ rays (green region); direct production at LEP for a variety of mediator mass, M, and width, Γ (solid gray). *Right, coupling to photons:* Additional exclusion limits from: indirect detection of γ rays (green region); conservative CMB and low redshift data constraints (blue region); modification of $N_{\rm eff}$ (red region).

The above constraint leaves a viable region for the preferred strength of DM self-interactions.

The SIMP scenario naturally predicts a sizable contribution to the above $2 \to 2$ scatterings. One may parametrize it by defining $a \equiv \alpha_{2\to 2}/\alpha_{\rm eff}$, such that

$$\frac{\sigma_{\rm scatter}}{m_{\rm DM}} = \frac{a^2 \alpha_{\rm eff}^2}{m_{\rm DM}^3}, \tag{20}$$

where a can be computed in a given theory, and one expects $a = \mathcal{O}(1)$. This is indeed the case for the toy model discussed above, for a wide range of values of the couplings of Eq. (15). For the $3 \to 2$ SIMP scenario, the constraint, Eq. (19), points to the strongly interacting regime with DM masses at or below the GeV scale. Interestingly, this region in parameter space automatically solves the small-structure anomalies discussed above. Indeed, one may use Eqs. (18) and (19) together with the relation Eq. (7) to derive a preferred range of $\alpha_{\rm eff}$. Taking into account the numerical corrections as found using the Boltzmann equation [Eq. (14)], we arrive at

$$0.3\left(\frac{a}{0.2}\right)^2 \lesssim \alpha_{\rm eff} \lesssim 8\left(\frac{a}{0.2}\right)^2. \tag{21}$$

In Fig. 2, we show the full region preferred by the small-scale structure anomalies, and the region excluded by bullet-cluster and halo-shape constraints, for a variety of values of a.

Models of strongly interacting DM that can accommodate the structure formation anomalies have been proposed in the literature [20–30]; however, most of them rely upon either a new long range force or a nonthermal mechanism to explain the DM relic abundance (see [31–35] for additional constraints that arise with long range forces). In contrast, the SIMP mechanism offers simplicity in the generation of the relic density and naturally points to the correct scale of self-interactions once the relic abundance is fixed to the observed value.

We now move on to the constraints on the coupling between the SIMP and SM particles. In addition to those of Eq. (10), there are constraints from direct detection, indirect detection, and cosmological data. To this end, we consider separately effective couplings of the SIMP to electrons or photons: 1. Coupling to electrons: We take the interaction of Eq. (16) with $f = e^-$. Bounds on ϵ, defined through Eq. (11), as a function of the mass, are then derived from (I) the requirements of Eq. (10), (II) Xenon10 electron ionizations data [36] and the projection for a germanium-based electron recoil experiment [37], (III) CMB data [38], (IV) modification to neutrino $N_{\rm eff}$ [39] from Planck data [7], and (V) indirect detection of FSR radiation off the $\chi\chi \to ee$ process [40], and (VI) direct production constraints from LEP [11]. Our results are depicted in the left panel of Fig. 3. Constraints from supernovae cooling [41–43] are not depicted as they are irrelevant in the allowed parameter space. 2. Coupling to photons: We now take the interaction of Eq. (17). The relevant bounds on ϵ in this case come from (I) Eq. (10), (II) indirect detection of annihilation into photons [40], (III) CMB data [38] (assuming an unsuppressed absorption efficiency [44]), and (IV) modification to neutrino $N_{\rm eff}$ [39] from Planck data [7]. Our results are depicted in the right panel of Fig. 3. Constraints from electron ionization data at Xenon10 are not depicted since they arise either from a photon loop, in which case the bound is weak, or from a tree level process to an $e^+e^-\gamma$ final state, in which case a dedicated study is required due to the dependence of the form factor on the momenta of the outgoing photon. We comment that the Planck $N_{\rm eff}$ bound can be evaded if the DM couples simultaneously to electrons, photons and neutrinos, in which case a lower bound on the DM mass of $\mathcal{O}$(MeV) arises from BBN, unless the DM is a real scalar [39].

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