Catalytic Coherence

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Because of conservation of energy we cannot directly turn a quantum system with a definite energy into a superposition of different energies. However, if we have access to an additional resource in terms of a system with a high degree of coherence, as for standard models of laser light, we can overcome this limitation. The question is to what extent coherence gets degraded when utilized. Here it is shown that coherence can be turned into a catalyst, meaning that we can use it repeatedly without ever diminishing its power to enable coherent operations. This finding stands in contrast to the degradation of other quantum resources and has direct consequences for quantum thermodynamics, as it shows that latent energy that may be locked into superpositions of energy eigenstates can be released catalytically.

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Introduction.—Coherence is a resource that enables us to implement coherent operations on quantum systems, a canonical example being the use of lasers to put atoms in superposition between two energy levels [1], or analogously for radio fields and nuclear spins [2]. When we excite an atom we need another system, an "energy reservoir," where the energy is taken. What kind of energy reservoir would we need in order to put an atom in superposition between two energies? With a bit of thought one can realize that this is impossible if the atom and the reservoir initially have definite energies. (See [3], Sec. I, for details. This observation can be understood in the wider context of "reference frames" and symmetry preserving operations [52–54].) One way to resolve the apparent contradiction with the above claim, that such superpositions indeed can be generated, is to realize that this usually is achieved via, e.g., lasers or radio fields. These are often modeled as coherent states, typically described as superpositions of the energy eigenstates of the electromagnetic field [1,55,56] (although this can be debated; see, e.g., [57–62]). In other words, the coherence of the laser is a resource that enables us to put the atom in superposition or, more generally, to perform operations that coherently mix energies.

The main result of this investigation is that coherence can be turned into a catalyst, in the sense that it enables otherwise impossible tasks, without itself being consumed. Although maybe reminiscent of entanglement catalysis [63], this stands in contrast to other quantum resources, e.g., reference frames for measurements, which appear to degrade upon use [52,64–66]. We establish catalytic coherence within two models. The first model (the doubly infinite energy ladder) is convenient to analyze but is somewhat unphysical in that its Hamiltonian has no ground state. The second model (the half-infinite ladder) amends this problem. We furthermore numerically investigate remnants of catalytic coherence in the Jaynes-Cummings (JC) model [67,68]. Finally, we apply catalytic coherence to "work extraction" in the context of quantum thermodynamics.

The doubly-infinite energy ladder.—Catalytic coherence is achieved via a specific design of the interaction between the system and the energy reservoir. While this construction can be used to generate general coherent operations on N-level systems (see [3], Sec. II) here we focus on two-level systems. Let S be a system for which the Hamiltonian H_S has eigenvalues $h_0 = 0$ and $h_1 = s > 0$, corresponding to the eigenstates $|\psi_0\rangle$ and $|\psi_1\rangle$. The Hamiltonian of the reservoir E is $H_E = s\sum_{j\in\mathbb{Z}} j|j\rangle\langle j|$, where s is the energy spacing in the ladder, $\{|j\rangle\}_{j\in\mathbb{Z}}$ is an orthonormal basis, and $\mathbb{Z}$ denotes the set of integers. Regarded as a Hamiltonian, H_E is slightly odd in that it does not have any ground state. We shall remedy this problem shortly, but due to several convenient properties we use this model for the initial analysis. As a first step we define the "shift operator" $\Delta = \sum_{j\in\mathbb{Z}}|j+1\rangle\langle j|$. As one can see, this unitary operator translates every state "rigidly" along the energy ladder (see Fig. 1). With the aid of Δ we define the following family of unitary operators on $\mathcal{H}_S \otimes \mathcal{H}_E$:

$$\begin{aligned} V(U) &= \sum_{n,n'=0,1} |\psi_n\rangle\langle\psi_n|U|\psi_{n'}\rangle\langle\psi_{n'}| \otimes \Delta^{n'-n} \\ &= \sum_{j\in\mathbb{Z}} V_j(U), \\ V_j(U) &= \sum_{n,n'=0,1} |\psi_n\rangle\langle\psi_n|Q|\psi_{n'}\rangle\langle\psi_{n'}| \otimes |j-n\rangle\langle j-n'|, \quad (1) \end{aligned}$$

where U is an arbitrary unitary operator on $\mathcal{H}_S$. (This type of interaction has previously been considered in quantum thermodynamics [69].) By construction, all $V(U)$ commute with $H_S + H_E$; i.e., they are energy conserving. Furthermore, they commute with all Δ^a and thus act uniformly over the energy ladder (see [69] for discussions on this). The family of all $V(U)$ serves as the set of "allowed operations" in our model. In the following we investigate what kind of transformations we can implement on S and how this depends on the coherence in the reservoir E.

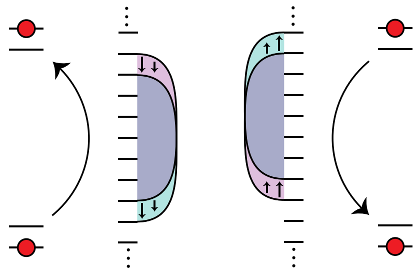

FIG. 1 (color online). *Rigid translation property.* A two-level system interacts with a reservoir whose energy levels can be described as a ladder. By design, the interaction shifts the whole state of the reservoir "rigidly" down or up along this ladder, depending on whether the two-level system absorbs (left part) or donates (right part) energy. For example, by removal of one quantum, the uniform superposition $|\eta_{L,l_0}\rangle = \sum_{l=0}^{L-1}|l_0+l\rangle/\sqrt{L}$ is shifted to $|\eta_{L,l_0-1}\rangle$. If L is large, the difference between $|\eta_{L,l_0-1}\rangle$ and $|\eta_{L,l_0}\rangle$ is small. Hence, the state of the reservoir does not change much by the loss or gain of a quantum, which enables coherent operations on the two-level system. In the limit of large L, these implementations can be made perfect. This is analogous to how coherent states on a bosonic mode can implement coherent operations on an atom via the Jaynes-Cummings model.

Coherence and coherent operations.—Given a state σ on the reservoir we can implement channels on S via

$$\Phi_{\sigma,U}(\rho) = \mathrm{Tr}_E[V(U)\rho\otimes\sigma V(U)^\dagger]. \tag{2}$$

(This generally requires time control; see [3], Sec. II C.) We let $\mathcal{C}(\sigma)$ denote the set of channels $\Phi_{\sigma,U}$ that can be obtained for arbitrary unitary U, given σ. Let us now see what aspects of σ it is that determine $\Phi_{\sigma,U}$. If one inserts Eq. (1) into (2), it turns out that $\Phi_{\sigma,U}$, and thus $\mathcal{C}(\sigma)$, depends on σ only via expectation values of the form $\mathrm{Tr}(\Delta^a\sigma)$ for $a\in\mathbb{Z}$ (which do not determine σ uniquely).

Next, suppose we wish to perform a unitary operation that mixes different energy levels. From Eqs. (1) and (2) one can see that if $\mathrm{Tr}(\Delta^a\sigma)\approx 1$ for $a=-2,\ldots,2$, then $\Phi_{\sigma,U}(\rho)\approx U\rho U^\dagger$. (For general N-level systems, the necessary range of a is determined by the amount of energy that the reservoir would need to donate or absorb.) Hence, when we speak of a "high degree of coherence," this means that the state σ of the reservoir is such that $\mathrm{Tr}(\Delta^a\sigma)\approx 1$ for a broad range of a, with the rationale that this allows us to perform coherent operations to a good approximation.

A concrete example is the family of states $\sigma = |\eta_{L,l_0}\rangle\langle\eta_{L,l_0}|$ of the form $|\eta_{L,l_0}\rangle = \sum_{l=0}^{L-1}|l_0+l\rangle/\sqrt{L}$, i.e., uniform superpositions over a collection of consecutive energy eigenstates. In the limit of large L, the channel $\Phi_{|\eta_{L,l_0}\rangle\langle\eta_{L,l_0}|,U}$ converges to the unitary operation $\rho\mapsto U\rho U^\dagger$ (as one could expect from the Wigner-Arkai-Yanase theorem [70–87]). Hence, this model is powerful enough to implement all unitary operations on S, given a sufficient degree of coherence in the reservoir. Next we show that the degree of coherence does not change over repeated applications.

Catalytic coherence in the doubly infinite ladder.—To investigate the catalytic properties of the coherence, we need to determine how the state of the energy reservoir changes when we use it. For that purpose we define the corresponding channel on E,

$$\Lambda_{\rho,U}(\sigma) = \mathrm{Tr}_S[V(U)\rho\otimes\sigma V(U)^\dagger]. \tag{3}$$

By using Eq. (1) one can confirm that

$$\mathrm{Tr}[\Delta^a\Lambda_{\rho,U}(\sigma)] = \mathrm{Tr}(\Delta^a\sigma), \tag{4}$$

for all σ, ρ, U, and a. In other words, the expectation values $\langle\Delta^a\rangle = \mathrm{Tr}(\Delta^a\sigma)$ are invariants under the action of these operations.

Earlier we noted that it is precisely the expectation values $\langle\Delta^a\rangle$ that determine which channels can be implemented. Hence, if we use the reservoir a second time, we can implement the very same channels that we implemented the first time, i.e., $\Phi_{\Lambda(\sigma),U} = \Phi_{\sigma,U}$, and hence $\mathcal{C}(\Lambda_{\rho,U}(\sigma)) = \mathcal{C}(\sigma)$. In other words, we do not degrade the coherence resource in the reservoir by using it. In this sense, coherence is catalytic in this model. One can also prove a stronger type of catalytic property, which does not assume that the reservoir and the systems initially are uncorrelated; see [3], Sec. II. (See also [3], Sec. III, for a reformulation of catalytic coherence in terms of correlations with a reference system.)

Note that the catalytic property holds for all states σ on the reservoir and is *not* limited to states with a high degree of coherence. It should also be emphasized that although $\langle\Delta^a\rangle$ are invariants, the underlying *state* does change (see [3], Sec. II, for an example), which is in contrast to entanglement catalysis [63]. In other words, analogously to how measurements induce back-action on reference frames [88–90], there is indeed a back-action on the energy reservoir. However, as opposed to how certain reference frames appear to degrade due to back-action [52,64–66], this change of state does not affect the usefulness of the coherence in the reservoir.

The half-infinite ladder.—One might worry that the catalytic property is an anomaly related to the lack of ground state, as a broadening distribution otherwise would hit the bottom. In the following we show that the capacity to induce channels can be maintained indefinitely also in a model that has a proper ground state. To this end, we cut away the lower half of the doubly infinite ladder and thus obtain (the spectrum of) the harmonic oscillator $H_E^+ = s\sum_{j=0}^{+\infty} j|j\rangle\langle j|$. We define a new class of unitary operations on S and E as

$$V_+(U) = |\psi_0\rangle\langle\psi_0|\otimes|0\rangle\langle 0| + \sum_{l=1}^{+\infty} V_l(U), \tag{5}$$

with V_l as in Eq. (1). By comparison one can see that $V(U)$ and $V_+(U)$ act identically on all states with at least one quantum, i.e., $\langle l|\sigma|l\rangle = 0$ for $l=0$. (For a general S this "border zone" would be larger; see [3], Sec. IV.)

Protocol for a catalytic half-infinite ladder.—A simple protocol can maintain the coherence properties of the reservoir indefinitely (see Fig. 2). We assume an initial state σ_{in} such that $\langle l|\sigma_{\mathrm{in}}|l\rangle = 0$ for $l=0, 1$; i.e., it contains at least two quanta. We let E and S interact via some arbitrary choice of unitary $V_+(U)$. As we know from the above reasoning, the effect is identical to $V(U)$. The new state $\bar{\sigma}$ on the energy reservoir is such that $\langle l|\bar{\sigma}|l\rangle = 0$ for $l = 0$. Now, consider an ancillary two-level system A, with the two eigenstates $|a_0\rangle$ and $|a_1\rangle$ corresponding to the energies 0 and s, respectively. We assume that A initially is in the excited state $|a_1\rangle$. By applying the operation $V_+(U_A)$ for $U_A = |a_0\rangle\langle a_1| + |a_1\rangle\langle a_0|$, the state $\bar{\sigma}$ is translated one rung up along the ladder to a new state σ_{out}. Hence, the reservoir is again in a state with at least two quanta. Since these operations have all been performed on states safely away from the ground state, it follows that $\mathcal{C}(\sigma_{\text{out}}) = \mathcal{C}(\sigma_{\text{in}})$. By iterating this procedure we can conclude that the set of channels that this reservoir can induce is kept intact indefinitely.

Decay of coherence in the Jaynes-Cummings model.— For many theoretical purposes it is enough to know that coherence, in principle, can be made catalytic. It is nevertheless relevant to ask to what extent these phenomena exist in more general types of systems, especially if one considers experimental investigations. Interactions between a two-level atom and a single mode of the electromagnetic field are often modeled via the JC Hamiltonian $H_{\text{JC}} = g\sigma_+ \otimes a + g\sigma_- \otimes a^\dagger$ [67,68]. Here $a, a^\dagger$ are the standard bosonic annihilation and creation operators $[a, a^\dagger] = \hat{1}_E$, and $\sigma_+ = |\psi_1\rangle\langle\psi_0|$, $\sigma_- = |\psi_0\rangle\langle\psi_1|$. Similar to our designed interactions, the JC model also moves a quantum of energy between the atom and the reservoir (as in Fig. 1), but it does not act uniformly over the energy ladder. One can nevertheless find a "shadow" of catalytic coherence in the JC model. The graphs in Fig. 3 suggest that the capacity to repeatedly induce coherent operations decays slower for higher initial average energies, even if the "width" of the initial superposition is fixed (see [3], Sec. V, for details).

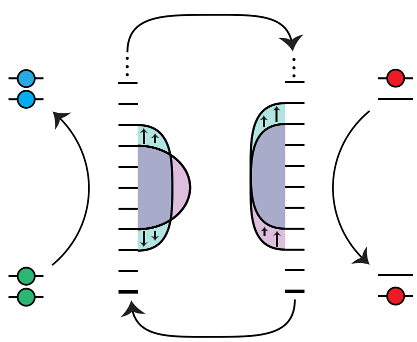

FIG. 2 (color online). *Regenerative catalytic cycles*. When the energy reservoir interacts with the two-level system (left part) the projection of its state onto the number states can increase with at most one level up and down in energy. As long as the projection onto the ground state of the reservoir remains zero, the set of channels that the reservoir can induce on the system stays intact from one interaction to the next. By using another two-level system in a pure excited state (right part) to inject energy into the reservoir, the state is translated "rigidly" up along the energy ladder by one step. By alternating every use of the reservoir with such a pumping, the state of the reservoir can be kept away from the ground state, thus maintaining the coherence properties indefinitely. Note that the state of the energy reservoir does change from one cycle to the next; e.g., the range of number states onto which it projects may become broader for each step. However, the *relevant aspects* of the state, which determine its capacity to induce channels, remain constant.

Application: Coherence in expected work extraction.— Work extraction and the closely related concepts of information erasure and Maxwell's demon have a long history (see, e.g., [91–100]) with recently renewed interests, e.g., in the contexts of resource theories [101–104] and single-shot statistical mechanics [105–112]. The task is to extract as much useful energy as possible by equilibrating a system with Hamiltonian H_S and state ρ with respect to a heat bath of temperature T. Here we consider the question of how much work can be extracted in an average sense. Standard results [93,97,98,113] suggest that the expected work content of a system is characterized by

$$\mathcal{A}_{\text{`standard'}}(\rho, H_S) = kTD(\rho\|G(H_S)), \tag{6}$$

where $D(\rho\|\eta) = \text{Tr}(\rho\ln\rho) - \text{Tr}(\rho\ln\eta)$ is the relative von Neumann entropy, k is Boltzmann's constant, $G(H_S) = \exp(-\beta H_S)/Z(H_S)$, $Z(H_S) = \text{Tr}\exp(-\beta H_S)$, and $\beta = 1/(kT)$. [Equation (6) can be confirmed in a model without an explicit energy reservoir; see [3], Sec. VI.]

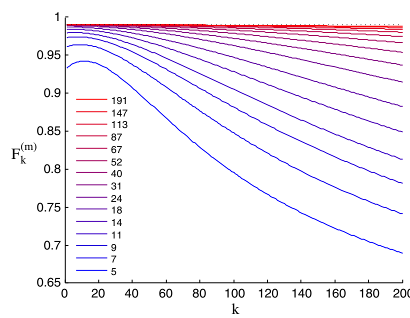

FIG. 3 (color online). *Decay of coherence in the Jaynes-Cummings model*. The JC model exhibits a decay of coherence over repeated use. However, there is a remnant of the catalytic property, in the sense that the "lifetime" of the coherence (counted in number of iterations) appears to increase indefinitely merely by an increase of the average energy of the initial state. These graphs depict the decay of the fidelity by which the superposition $|\phi\rangle = (|\psi_0\rangle - i|\psi_1\rangle)/\sqrt{2}$ can be created from the ground state $|\psi_0\rangle$, in a collection of two-level systems that sequentially interact with one single reservoir, according to the JC model for a fixed time step. Each curve corresponds to a different initial state of the reservoir and shows $F_k^{(m)} = \langle\phi|\Phi(\sigma_m^{(k)})|\phi\rangle$, against the number of times k the reservoir has been used. Each curve corresponds to an initial state $\sigma_m^{(0)} = |\eta_{L,l_0(m)}\rangle\langle\eta_{L,l_0(m)}|$ for $|\eta_{L,l_0(m)}\rangle$ with $l_0(m) = (4m+1)^2 - 24$ and $L = 50$, for $m = 5$, 7, 9, 11, 14, 18, 24, 31, 40, 52, 67, 87, 113, 147, 191. Note that the width of the superposition does not change with m. The dotted line is the value of $(1 + |\langle\eta_{L,l_0(m)}|\Delta|\eta_{L,l_0(m)}\rangle|)/2 = 0.99$, which would be the fidelity reached in the doubly infinite ladder model for these initial states.

However, in light of the above considerations, such approaches appear to implicitly assume access to ideal coherence resources. Recently it has been shown [69] that without any coherence, the optimal expected work content is not characterized by (6), but rather by

$$\mathcal{A}_{\mathrm{diagonal}}(\rho, H_S) = kTD([\rho]_{H_S} \| G(H_S)), \tag{7}$$

where $[\rho]_{H_S} = \sum_l P_l \rho P_l$, and P_l are the projectors onto the eigenspaces of H_S. However, one can show (see [3], Sec. VII) that access to coherence increases the amount of work that can be extracted. Furthermore, in the limit of a large degree of coherence (e.g., large L for $|\eta_{L,l_0}\rangle$), one regains Eq. (6). We can conclude that coherence sails up as an important resource alongside the expected work content. The question is how they relate. How much of one resource can be gained by spending the other? The fact that here we perform the work-extraction analysis entirely within the doubly infinite ladder model implies that we only use the coherence catalytically and do not "spend" it at all.

In relation to this thermodynamic application one may note Ref. [114], where optimal extraction is obtained irrespective of the state of the reservoir, via a larger class of unitary operations that only conserve energy on average, and has the power to create and destroy superpositions of energy eigenstates (see [3], Sec. I B). Note also Ref. [115], which uses a coherent extraction device as a negentropy source to demonstrate a transient efficiency that exceeds the standard Carnot bound for work extraction against a hot and a cold heat bath.

Conclusions and outlook.—We have shown that coherence can be turned into a catalytic resource by a specific design of interactions, and we used this to analyze work extraction. As observed in the single-shot setting [105–112], the expected work content may not always correspond to ordered "worklike" energy (see discussions in [107]). It is an open question if catalytic coherence is associated with a cost of ordered energy (see [3], Sec. VIII). Another question is to quantify expected work extraction with limited access to coherence.

The existence of catalytic coherence raises the question of whether other types of resources [52,64–66], in some sense, can also be turned catalytic. In view of the analogy between embezzling states [116] and coherent states, one can speculate whether there exists some counterpart to catalytic coherence in that setting (see [3], Sec. VIII). One can also ask whether catalytic coherence and entanglement catalysis [63] are special cases of a more general class of catalytic phenomena (see [3], Sec. VIII). On a more general level, the question is under what conditions, and in what sense, a resource can be made catalytic.

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