# Conversion of an Electromagnetic Wave into a Periodic Train of Solitons under Cyclotron Resonance Interaction with a Backward Beam of Unexcited Electron-Oscillators

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The possibility of the conversion of intense continuous microwave radiation into a periodic train of short pulses by means of resonant interaction with a beam of unexcited cyclotron electron oscillators moving backward is shown. In such a system there is a certain range of parameters where the incident stationary signal splits into a train of short pulses and each of them can be interpreted as a soliton. It is proposed to use this effect for amplitude modulation of radiation of short wavelength gyrotrons.

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The problem of transforming microwave radiation into a sequence of coherent short nanosecond pulses is important for a number of applications including plasma diagnostics, radars, particle accelerators, spectroscopy, etc. In Refs. [1,2] optically controlled switches are used for these purposes; those switches are based on the effect of induced photoconductivity in semiconductor elements implemented in a resonant system. In the present Letter we propose an alternative method based on cyclotron resonance absorption of microwave radiation by an initially rectilinear electron beam interacting with a backward propagating wave.

The specifics of the interaction of short electromagnetic pulses with initially rectilinear electron beams under the cyclotron resonance condition has been investigated in Refs. [3,4]. It was shown that, starting from a certain threshold power of an incident pulse, linear cyclotron absorption is replaced by the effect of self-induced transparency, when the electromagnetic pulse propagates without damping. In fact, similar to optics [5–9], the initial pulse transforms itself into a soliton whose amplitude and duration depend on its velocity. The present Letter deals with the nontrivial dynamics arising when a quasistationary incident signal interacts with a counterpropagating rectilinear electron beam under the cyclotron resonance condition. As shown below, in such a system the continuous signal decomposes itself into a train of short pulses, and each of them can be interpreted as a soliton. It is important to note that the described effect occurs only when the relativistic dependence of the gyrofrequency on the particle energy is taken into account [10,11]. Moreover, the phase velocity of the wave should be significantly different from the speed of light in order to avoid mutual compensation of electron phase shifts caused by the changes in gyrofrequency and the recoil effect, which is typical for autoresonance regimes [12]. Under such an assumption, an initially rectilinear electron beam could be considered as a nonlinear resonance medium.

Let us consider the interaction of an initially rectilinear annular electron beam guided by a homogeneous magnetic field $\vec{H} = \vec{z}_0 H_0$ with a backward electromagnetic wave (Fig. 1) in a cylindrical waveguide with a radius $R$ under the cyclotron-resonance condition

$$\omega + hv_0 \approx \omega_H, \tag{1}$$

where $v_0 = \beta_0 c$ is the axial velocity of particles, $\omega_H = eH_0/mc\gamma$ is the electron gyrofrequency, and $\gamma$ is the relativistic mass factor. The electromagnetic field in the situation under study can be presented in the form

$$\vec{E} = \mathrm{Re}(\vec{E}^s(\vec{r}_\perp)A(z,t)\exp(i\omega t + ihz)), \tag{2}$$

where $A(z,t)$ is the slowly varying wave amplitude and the function $\vec{E}^s(\vec{r}_\perp)$ describes the transverse structure of radiation corresponding to a $\mathrm{TE}_{mn}$ waveguide mode. The electron-wave interaction can be described by the equations [3,4]

$$\frac{\partial a}{\partial Z} - \frac{\partial a}{\partial \tau} = p, \qquad \frac{\partial p}{\partial Z} + ip(\delta + |p|^2) = a. \tag{3}$$

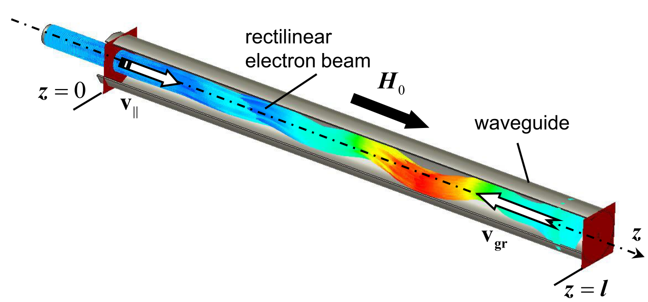


FIG. 1 (color online). Schematic of the interaction space with electron trajectories found in PIC simulation.

 

Here the electron motion equations were reduced to well-known equations for nonisochronous oscillators [7] by using assumptions of weak relativism $\gamma \approx 1 + |p|^2/mc^2$ and a low electron beam density. In Eq. (3), the following dimensionless variables and parameters are used:

$$\tau = \sqrt{G}\omega(t - z/v_0)\frac{\beta_{gr}\beta_0}{\beta_{gr} + \beta_0}, \qquad Z = \frac{\sqrt{G}\omega z}{c},$$
$$a = \frac{\sqrt{(1+\beta_{ph}^{-1}\beta_0)}}{2\sqrt{2}G^{3/4}\beta_0^{3/2}\gamma_0}\frac{eAJ_{m-1}(\kappa R_0)}{mc\omega}.$$

$p = (\sqrt{\mu}(p_x + ip_y)/G^{1/4}mc\gamma_0\beta_0)e^{-i(\omega t + hz)}$ is the normalized transverse momentum of electrons, $\mu = \beta_0(1 - \beta_{ph}^{-2})/2(1 + \beta_{ph}^{-1}\beta_0)$ is the parameter of nonisochronism that takes into account the difference between the wave phase velocity $v_{ph} = \beta_{ph}c$ and the speed of light $c$, $\delta = (1 + \beta_{ph}^{-1}\beta_0 - \omega_{H0}/\omega)\beta_0^{-1}G^{-1/2}$ is the initial cyclotron resonance mismatch, $v_{gr} = \beta_{gr}c$ is the wave group velocity, and

$$G = \frac{eI_b}{mc^3}\frac{2\mu(1+\beta_{ph}^{-1}\beta_0)^2}{\gamma_0\beta_{ph}^{-1}\beta_0^3}\frac{J_{m-1}^2(\nu_n R_0/R)}{J_m^2(\nu_n)(\nu_n^2 - m^2)}$$

is the form factor written under the assumption that the electron beam is hollow with an injection radius $R_0$; $I_b$ is the electron current, $J_m(x)$ is the Bessel function, $m$ is the azimuthal index of the operating mode, $\nu_n$ is the $n$th root of the equation $dJ_m(x)/dx = 0$. Below we assume that at the initial instant of time all electrons have the zero transverse momentum: $p(Z = 0) = 0$. The incident signal enters the system via the cross section $Z = L$, where $L = \sqrt{G}\omega l/c$ is the normalized length of the interaction space.

As shown in Ref. [4], Eqs. (3) possess a solitonlike solution, which in the case of the exact cyclotron resonance $\delta = 0$ can be written for the intensity $I = |a|^2$ of the electromagnetic field as

$$I = \frac{4}{(1+U)^{3/2}}\mathrm{sech}\left[\frac{2}{\sqrt{1+U}}(Z - U\tau)\right], \qquad (4)$$

where $U$ is the soliton velocity. According to Eq. (4), the distinctive feature of the counterpropagating interaction under the cyclotron resonance condition is the existence of a stationary ($U = 0$) soliton

$$I = 4\mathrm{sech}(2Z), \qquad (5)$$

with the peak intensity $I^* = 4$. Strong solitons with the peak intensity exceeding $I^*$ propagate in the direction opposite to the electron beam ($U < 0$). In turn, weak solitons with $I_{peak} < I^*$ propagate in the same direction ($U > 0$) as the electron beam.

In the case of an incident cw signal $a(Z = L) = a_{in} = \sqrt{I_{in}}$, several regimes of interaction can be observed depending on the normalized intensity of the input signal $I_{in}$ and the normalized length of the interaction space $L$. Zones of different regimes are shown in the plane ($I_{in}$, $L$) for $\delta = 0$ in Fig. 2. The stationary regimes of interaction are realized in zones I and II. In the case of an initial intensity $I_{in} < I^*$ for any length of interaction space (zone I) the ordinary regime of the cyclotron absorption takes place when the amplitudes of the field and transverse momentum decrease monotonically from the collector to the cathode end of the interaction space, as shown in Fig. 3(a).

In zone II the field amplitude and the transverse momentum experience periodic variations along the longitudinal coordinate. In this region, along with regimes of the cyclotron absorption with $I_{in} > I_{out}$, the regimes of nonlinear bleaching can be realized [see Fig. 3(b)]. In such regimes, the penetration of electromagnetic waves through the area of the cyclotron absorption without energy loss takes place; i.e., the intensities of the electromagnetic wave at the entrance ($Z = L$) and at the exit ($Z = 0$) are equal: $I_{in} = I_{out}$. Dashed curves 1, 2, and 3 in zone II (Fig. 2) correspond to regimes of nonlinear bleaching with one, two, and three spatial variations of the field amplitude in the interaction space. The existence of such regimes was first described in Ref. [13], where the cyclotron resonance interaction of an electromagnetic wave with a counterpropagating electron beam was studied in the frame of the stationary approach. It is quite unexpected that in the passive system the field amplitude inside the interaction

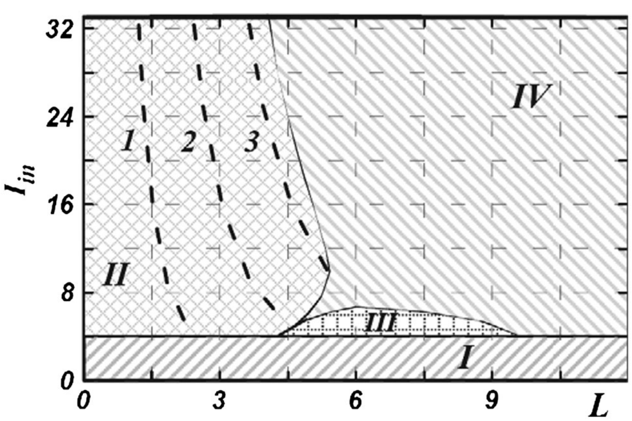


FIG. 2. Zones of different interaction regimes of cw radiation with backward rectilinear electron beam under the exact cyclotron resonance condition $\delta = 0$. I: zone of stationary regimes with monotonic dependence of the field intensity on the normalized longitudinal coordinate (zone of normal cyclotron absorption). II: zone of stationary regimes with periodic dependence of the field intensity on the normalized longitudinal coordinate; dashed line 1,2,3 corresponds to the regime of nonlinear bleaching with the indicated number of longitudinal variations. III: zone of periodic self-modulation regimes. IV: zone of chaotic self-modulation regimes.

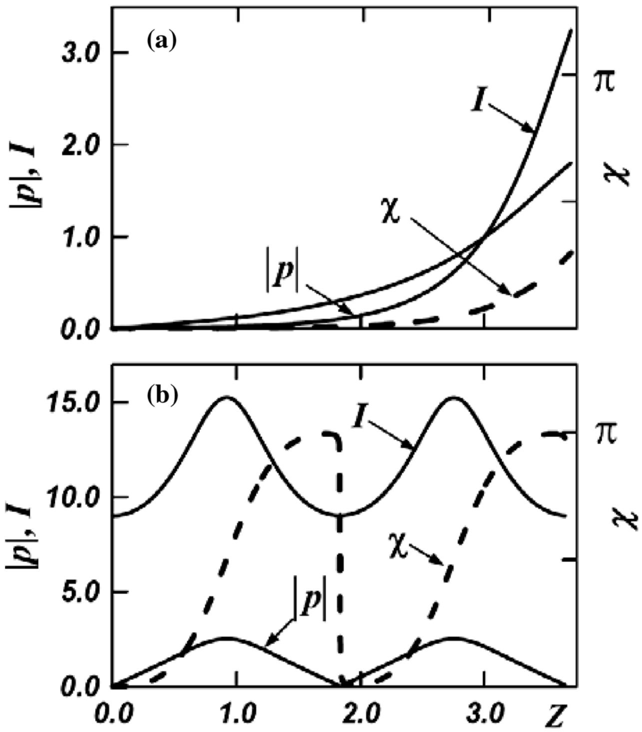


FIG. 3. Longitudinal distributions of normalized field intensity $I$, absolute value of transverse momentum $|p|$, and mutual phase difference $\chi$ between signal and electron rotation momentum. (a) Regime of stationary cyclotron absorption (zone I, for $L = 3.7$, $I_{\mathrm{in}} = 3.24$). (b) Regime of stationary nonlinear bleaching with two longitudinal variations (dashed curve 2 in zone II, for $L = 3.7$, $I_{\mathrm{in}} = 9$).

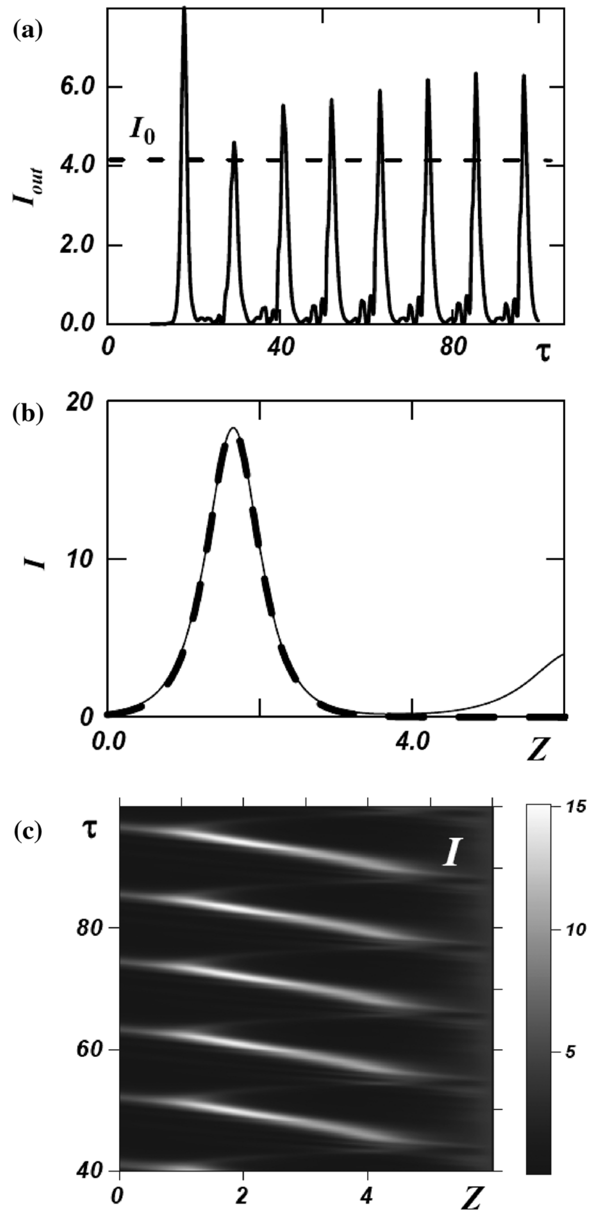


FIG. 4. Regime of periodic self-modulation (zone II, for $L = 6$, $I_{\mathrm{in}} = 4.1$). (a) Output signal at the cross section $Z = 0$. (b) Single soliton profile inside the interaction space in comparison with analytical solution (4) (dashed line). (c) Soliton propagation on the plane $Z$, $\tau$.

space exceeds the level of the incident signal [Fig. 3(b)]. Obviously this fact can be explained by a temporal storage of the electromagnetic energy in the considered resonance system. If one treats the dependence of the field amplitude on the longitudinal coordinate $z$ it is easy to see from Fig. 3 that the change from amplitude growth to decline is accompanied by a proper shift of phase of the signal relative to the electron rotation phase $\chi = \arg(a/p)$. This phase shift is caused by the nonisochronism of the electron motion.

Simulations of nonstationary dynamics described by Eqs. (3) show that the stationary regimes described above are not stable in the whole range of parameters $I_{\mathrm{in}}$ and $L$. In zones III and IV (see Fig. 2) the self-modulation regimes are realized, in which the input cw signal transforms into a periodic (zone III) or chaotic (zone IV) sequence of short pulses. The case of periodic modulation is presented in Fig. 4(a) for $L = 6$, $I_{\mathrm{in}} = 4.1$. In fact, each elementary pulse possesses a solitonlike shape described by expression (4) [see Fig. 4(b), where the dashed line corresponds to the analytical solution (4)]. Formation of a periodic train of solitons is demonstrated in Fig. 4(c). Actually, approaching the cathode end ($Z = 0$) each soliton produces a reflected soliton of low intensity. This weak soliton travels with the electron beam to the collector end ($Z = L$), where it stimulates the formation of a new strong soliton. Then, the process repeats itself periodically. For parameters of zone IV several solitons propagate in the interaction space simultaneously. The mutual influence of solitons in the process of collisions leads to distortion of their shapes and complication of the output signal. As a result, the output signal has the form of a quasichaotic sequence of short pulses.

From a practical point of view, the studied effects can be used for conversion of radiation of high-frequency

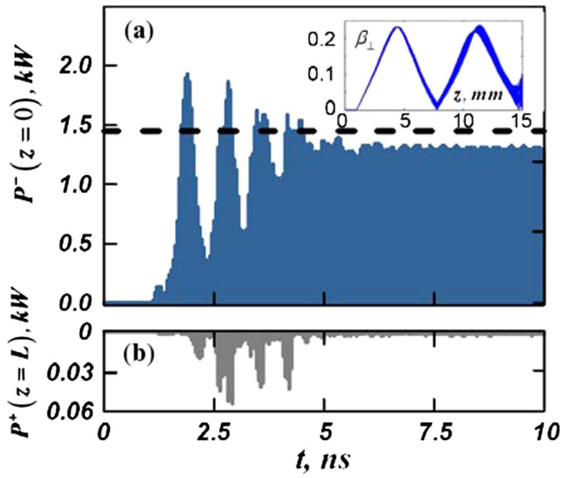


FIG. 5 (color online). PIC simulations of the regime of nonlinear bleaching. Radiation power emitted in (a) negative $P^-(z=0)$ and (b) positive $P^+(z=l)$ directions of the $z$ axis. The inset shows the longitudinal distribution of transverse electron velocities.

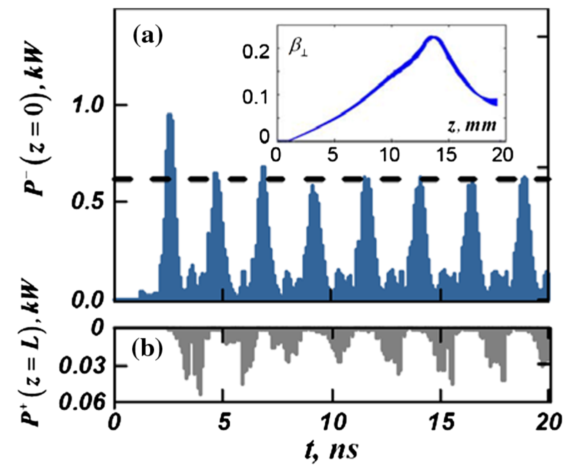


FIG. 6 (color online). Same as in Fig. 5 for the regime of periodic self-modulation.

gyrotrons into a coherent train of nanosecond pulses. It should be noted that the cw electromagnetic wave is coupled in and the pulse train is coupled out based on input and output microwave systems used typically in gyroamplifiers. The results of simulations presented in Fig. 4(a) correspond to modulation of the signal from a 240 GHz gyrotron with power 450 W [14] in the process of interaction with an initially linear annular electron beam having an electron energy of 1 keV, a current of 65 mA, and an injection radius of 0.1 mm. It is assumed that electrons guided by a homogeneous magnetic field with an intensity of 8.9 T move in the waveguide with a radius of 0.4 mm and an ~26 mm length and interact with the $TE_{11}$ wave. For the above parameters, the solitonlike pulses have a duration $\Delta t$ of 0.8 ns and a peak power of 680 W. It should be noted that by varying the electron current it is possible to change the output pulse duration as $\Delta t \sim I_b^{-1/2}$. An additional method for variation of the pulse duration is associated with the variation of the cyclotron resonance mismatch $\delta$ by means of changing the guiding magnetic field. Similar to self-modulation regimes in backward-wave oscillators (BWOs) [15,16], the pulse repetition frequency is determined by the time of passage of radiation through the interaction space in the backward and forward direction. However, in contrast to the conventional BWO, the electron beam in the discussed model forms a passive medium. As a result, a significant part of the energy of the incident radiation is absorbed by the electron beam. For the considered parameters, the time averaged output power amounts to only 30% of the power of the incident cw signal.

The results of the studies of the set of equations (3) averaged over the period of RF oscillations have been confirmed by direct 3D-simulations performed by using the particle-in-cell (PIC) code CST STUDIO SUITE [17]. Simulations were carried out for parameters of the electron beam and the incident cw signal close to those discussed above. The regime of nonlinear bleaching (Fig. 5) was realized for an interaction length of 15 mm and an input power of 720 W. The output power in the steady-state regime is approximately equal to the power of the input signal (dashed line in Fig. 5). As is seen from the inset, the energy exchange between an electromagnetic wave and an electron beam in the stationary regime results in the appearance of two periods in the course of which the electrons acquire and then lose the wave energy along the interaction space. For a length of 20 mm and an input power of 320 W, instead of a stable energy exchange between the wave and electrons, the regime of periodic pulsation occurs [Fig. 6(a)], when the input stationary signal breaks up into solitons. The instantaneous profile of the transverse velocity in a single soliton is shown in the inset. It is important that in the direct PIC simulation we can observe [Fig. 6(b)] emission of both strong (in the $-z$ direction) and weak (in the $+z$ direction) solitons. The weak solitons can also be interpreted as some reflection of the incident signal arising in the case of nonstationary electron-wave interaction. In the case of the stationary interaction such reflections are negligibly small [Fig. 5(b)].

In summary, it has been demonstrated that a very simple system consisting of an electromagnetic wave interacting with a counterpropagating beam of initially unexcited oscillators can exhibit a quite complex behavior. The nonlinearity caused by the nonisochronous motion of electrons in the magnetic field in combination with time delay effects leads to a nontrivial picture of stationary and nonstationary dynamical regimes. The region of parameters is found in which the incident cw signal splits into a train of short pulses, and each of them can be interpreted as a soliton. This effect may be used for deep modulation of radiation of millimeter and submillimeter gyrotrons. We believe that such a modulated signal with a rather broad spectrum can be useful for some applications including spectroscopy [18,19].

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