Revealing Single-Trap Condensate Fragmentation by Measuring Density-Density Correlations after Time of Flight

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We consider ultracold bosonic atoms in a single trap in the Thomas-Fermi regime, forming many-body states corresponding to stable macroscopically fragmented two-mode condensates. It is demonstrated that upon free expansion of the gas, the spatial dependence of the density-density correlations at late times provides a unique signature of fragmentation. This hallmark of fragmented condensate many-body states in a single trap is due to the fact that the time of flight modifies the correlation signal such that two opposite points in the expanding cloud become uncorrelated, in distinction to a nonfragmented Bose-Einstein condensate, where they remain correlated.

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Introduction.—The textbook definition of Bose-Einstein condensation consists in the existence of exactly one $\mathcal{O}(N)$ (i.e., macroscopic) eigenvalue of the single-particle density matrix (SPDM) [1–3], where $N \gg 1$ is the total number of particles. When interactions become sufficiently strong, the condensate is depleted by scattering processes [3,4]. A fundamental question then arises: Upon increasing the interaction beyond a certain threshold, do fragmented condensates with two or more $\mathcal{O}(N)$ eigenvalues of the SPDM exist [5], or does the system cross over directly from a single condensate to nonmacroscopic fragments?

The phenomenon of fragmentation is well known when the externally applied potential provides deep double wells [6], or for the periodic extension deep optical lattices [7], where the fragmented phase bears the name Mott insulator. However, there has been the prevalent belief that for experiments performed with ultracold atoms of one given species in *single* (e.g., harmonic) traps, a nonfragmented Bose-Einstein condensate is obtained, despite these experiments usually being conducted in the Thomas-Fermi (TF) limit, for which the kinetic energy is small compared to trapping and interaction energies. That is, macroscopic condensate fragmentation is supposed in these experiments not to occur before three-body recombination [8] rapidly destroys the condensate.

On the other hand, recent work has demonstrated that condensate fragmentation is a genuine many-body phenomenon and is intrinsically not describable within a simple mean-field theory (within an effective Gross-Pitaevskii theory) [9–12]. In a single trap, fragmentation occurs for repulsive interactions in the ground state [9] and for experimentally accessible TF parameters [10,11], against the expectation that for repulsive interactions, no fragmentation is obtained [13]. In the TF limit, interaction thus can lead to the population of several macroscopically occupied orbitals. The (quasi-)continuity of distribution amplitudes in Fock space has been shown to be responsible for the stability of fragmentation, also against thermal fluctuations [14]. This is in strong contrast with the unstable fragmentation occurring, e.g., in spin-orbit coupled gases [15] or spinor gases [16], for which fragmented states are (superpositions of) exact Fock states [17], i.e., have sharply peaked distributions in Fock space.

An outstanding open question concerns the *detection* of fragmentation in a single trap, that is, to verify conclusively that it has indeed taken place. Fragmentation in the superfluid-Mott transition on optical lattices is detected by the decrease of the visibility of the structure factor peaks [7]. This first-order correlation function measure of coherence, directly related to the SPDM in position space $\rho_1(\boldsymbol{r},\boldsymbol{r}')$ [18], is not operative in a single trap. This is primarily because, in general, the macroscopically occupied natural orbitals (for a definition see below) will significantly overlap in a potentially complicated fashion, in distinction to the multiple-well scenario, where they are well separated [6,7]. Unequivocally assigning fragmentation to the measured signal will thus be severely hampered. This difficulty becomes particularly relevant when the degree of fragmentation is relatively small.

Detecting density-density correlations is by now a standard tool to discriminate one many-body phase from the other [19]; the correlations can be measured both *in situ* [20] and *ex situ*, that is, after time of flight (TOF); see, e.g., Ref. [21]. Motivated by this fact, we propose a readily implemented experimental procedure to determine whether a given condensate has fragmented. It is demonstrated that density-density correlations after TOF give a clear and unequivocal signature for the fragmentation. As we will show, counterintuitively, the essentially noninteracting expansion, which necessarily diminishes the density, magnifies the characteristic signature of fragmentation.

We first introduce some terminology. Expanding the field operator as $\hat{\psi}(\boldsymbol{r}) = \sum_i \psi_i(\mathbf{r})\hat{a}_i$ and writing the SPDM in its eigenbasis $\rho_1(\boldsymbol{r},\boldsymbol{r}') = \sum_i \lambda_i \psi_i^*(\boldsymbol{r})\psi_i(\boldsymbol{r}')$, the corresponding orbitals $\psi_i(\boldsymbol{r})$ are called *natural*. We then have $\langle \hat{a}_i^\dagger \hat{a}_j \rangle = 0$, $\forall i \neq j$, and the eigenvalue $\lambda_i = \langle \hat{a}_i^\dagger \hat{a}_i \rangle$ is the occupation number of the natural orbital $\psi_i(\mathbf{r})$. A many-body state with more than one $\lambda_i = \mathcal{O}(N)$ is a fragmented condensate. We perform the calculation below for two macroscopically occupied orbitals, assuming that the thermal portion of atoms is negligible. The SPDM is then a (truncated) 2×2 matrix, and the degree of fragmentation is defined by $\mathcal{F} = 1 - |\lambda_0 - \lambda_1|/N$. When both eigenvalues are $\mathcal{O}(N)$, $\mathcal{F}$ is finite and becomes maximal (unity) when they are both equal to $N/2$. Considering two macroscopic fragments is partly motivated by the recent study [11], finding a stepwise increase of the number of fragments from the single condensate upon increasing the interaction coupling.

For two orbitals (modes), the Fock space many-body state reads

$$|\Psi\rangle = \sum_{l=0}^{N} C_l \frac{(\hat{a}_0^\dagger)^{N-l}(\hat{a}_1^\dagger)^l}{\sqrt{(N-l)!l!}}|0\rangle \equiv \sum_{l=0}^{N} C_l |N-l, l\rangle. \quad (1)$$

We assume the rather generic condition on the many-body amplitudes C_l, see Ref. [9], that they have a sharply peaked continuum limit distribution for the moduli, e.g., the Gaussian $|C(l)| = (\pi a^2)^{-1/4} \exp[-(l - N/2 - \mathcal{S})^2/(2a^2)]$. Here, the width of the distribution $a \propto \sqrt{N}$ and the shift $\mathcal{S}$ are given in terms of the parameters of a two-mode Hamiltonian in the trap, e.g., of the form $\hat{H} = \epsilon_0 \hat{a}_0^\dagger \hat{a}_0 + \epsilon_1 \hat{a}_1^\dagger \hat{a}_1 + (A_1/2)\hat{a}_0^\dagger \hat{a}_0^\dagger \hat{a}_0 \hat{a}_0 + (A_2/2)\hat{a}_1^\dagger \hat{a}_1^\dagger \hat{a}_1 \hat{a}_1 + [(A_3/2)\hat{a}_0^\dagger \hat{a}_0^\dagger \hat{a}_1 \hat{a}_1 + \text{H.c.}] + (A_4/2)\hat{a}_1^\dagger \hat{a}_1 \hat{a}_0^\dagger \hat{a}_0$, where ϵ_i are single-particle energies and A_i are interaction couplings depending on the orbitals and the two-body interaction. We then have a maximum at l_0 ($= N/2 + \mathcal{S}$ for the Gaussian distribution), whose relative width becomes very small when $N \gg 1$. Note that there are no single-particle tunneling terms $-\frac{1}{2}\Omega \hat{a}_0^\dagger \hat{a}_1 + \text{H.c.}$ and number-weighed tunneling terms $\propto \hat{n}_0 \hat{a}_0^\dagger \hat{a}_1 + \text{H.c.}$ or $\propto \hat{n}_1 \hat{a}_0^\dagger \hat{a}_1 + \text{H.c.}$ when the two modes have even (0) and odd (1) parity, respectively (see also below). We set the pair-exchange coupling $A_3 > 0$ (which is naturally of the same order as the other A_i in a single trap [9,10]). From energy minimization and the discrete time-independent Schrödinger equation $EC_l = \frac{1}{2}A_3(d_l C_{l+2} + d_{l-2}C_{l-2}) + [\epsilon_0(N-l) + \epsilon_1 l + \frac{1}{2}A_1(N-l) \times (N-l-1) + \frac{1}{2}A_2 l(l-1) + \frac{1}{2}A_4(N-l)l]C_l$, connecting l "sites" in Fock space differing by 2, we obtain $\text{sgn}(C_l C_{l+2}) = -1$ ($C_l \in \mathbb{R}$ [22]). This entails a fragmented condensate many-body state due to the consequent condition $\text{sgn}(C_l C_{l+1}) = \pm(-1)^l$ [9,10].

Density-density correlations.—We focus from now on quasi-one-dimensional (quasi-1D) condensates, for which the largest degrees of fragmentation can be expected [10]. We also assume that the condensate is deep in the TF regime of large particle numbers [23]. The density expectation value in terms of the axial coordinate z, in the natural basis, reads $\rho(z) = \langle \hat{\psi}^\dagger(z)\hat{\psi}(z)\rangle = N_0|\psi_0(z)|^2 + N_1|\psi_1(z)|^2$, where $N_i = \lambda_i = \langle \hat{a}_i^\dagger \hat{a}_i \rangle$. The density-density correlation function (the two-particle density matrix (TPDM) in position space [24]) then takes the form

$$\begin{aligned}\rho_2(z, z') &= \langle \hat{\psi}^\dagger(z)\hat{\psi}^\dagger(z')\hat{\psi}(z')\hat{\psi}(z)\rangle \\ &= |\psi_0(z)|^2|\psi_0(z')|^2\langle \hat{a}_0^\dagger \hat{a}_0^\dagger \hat{a}_0 \hat{a}_0\rangle + 0 \to 1 \\ &\quad + (|\psi_0(z)|^2|\psi_1(z')|^2 + 0 \leftrightarrow 1)\langle \hat{a}_0^\dagger \hat{a}_1^\dagger \hat{a}_1 \hat{a}_0\rangle \\ &\quad + 2\Re[\psi_0^*(z)\psi_1^*(z')\psi_0(z')\psi_1(z)\langle \hat{a}_0^\dagger \hat{a}_1^\dagger \hat{a}_1 \hat{a}_0\rangle \\ &\quad + \psi_0^*(z)\psi_0^*(z')\psi_1(z')\psi_1(z)\langle \hat{a}_0^\dagger \hat{a}_0^\dagger \hat{a}_1 \hat{a}_1\rangle]. \end{aligned} \quad (2)$$

It is for given orbitals $\psi_i(z)$ prescribed by the TPDM elements $\langle \hat{a}_i^\dagger \hat{a}_j^\dagger \hat{a}_k \hat{a}_l\rangle$, which are, in turn, determined by the many-body amplitudes C_l. The last line contains the pair-exchange term, which decides whether the many-body ground state in a single trap is fragmented [9].

For simplicity, the initial orbitals are assumed to fulfill that $\psi_0(z, 0)$ is an even real function of z with $\psi_0(z) = \psi_0(-z, 0) \in \mathbb{R}$ and that $\psi_1(z, 0)$ is an odd real function of z with $\psi_1(z, 0) = -\psi_1(-z, 0) \in \mathbb{R}$; i.e., they have definite parity in the trap [26]. We define w as a (finite) common width measure of the orbitals, which is, e.g., a variational parameter determined by the competition of interaction and trapping [10]. In what follows, $w = 1$ is used as the unit of length, as well as $\hbar = m = 1$, with m the boson mass.

Calculating the TPDM elements from the continuum limit for C_l, we have to $\mathcal{O}(1/N)$ [27]

$$\begin{aligned} &\langle \hat{a}_0^\dagger \hat{a}_0^\dagger \hat{a}_0 \hat{a}_0\rangle = N_0^2, \qquad \langle \hat{a}_1^\dagger \hat{a}_1^\dagger \hat{a}_1 \hat{a}_1\rangle = N_1^2, \\ &\langle \hat{a}_0^\dagger \hat{a}_1^\dagger \hat{a}_1 \hat{a}_0\rangle = N_0 N_1, \qquad \langle \hat{a}_0^\dagger \hat{a}_0^\dagger \hat{a}_1 \hat{a}_1\rangle = -N_0 N_1. \end{aligned} \quad (3)$$

This result remains valid as long as the C_l distribution is centered at $l_0 \sim \mathcal{O}(N)$ with a width $\ll N$.

Turning off the trap potential in the weakly confining axial direction only [28] (see Fig. 1), after a short initial period of rapid expansion, for $t \gg 1$, the gas will expand ballistically [29]. One can then apply the noninteracting propagator to the initial orbitals

$$\psi_j(z, t) = \sqrt{\frac{1}{2\pi i w_t^2}} \exp\left[\frac{iz^2}{2w_t^2}\right] \tilde{\psi}_j(z, t), \qquad w_t = \sqrt{t}, \quad (4)$$

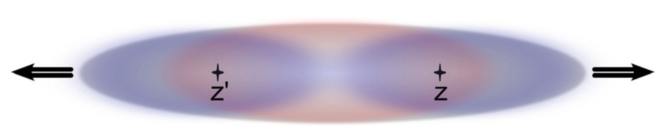

FIG. 1 (color online). Schematic of an axially freely expanding quasi-1D gas in a fragmented condensate many-body state. The two macroscopically occupied orbitals are indicated by red and blue shaded areas. Density correlations are measured at two (opposite) points z and z' in the cloud at some given instant t.

where $\tilde{\psi}_j(z,t) = \exp[-iz^2/2w_t^2]\int dz'\psi_j(z',0)\exp[i(z-z')^2/2w_t^2]$. At late times, $\tilde{\psi}_j(z,t)$ has the meaning of a Fourier transform with respect to the variable pair $(z', z/w_t^2)$ to first order in z'/w_t, with $\psi_j(z',0)$ remaining spatially confined.

Selecting, e.g., two opposite points $z = -z'$, for $t \gg 1$, we obtain the correlation ratio

$$\frac{\rho_2(z,-z,t)}{\rho_2(z,z,t)} = \frac{(|\tilde{\psi}_0(z,t)|^2 N_0 - |\tilde{\psi}_1(z,t)|^2 N_1)^2}{\rho^2(z,t) + 4N_0 N_1 |\tilde{\psi}_0(z,t)|^2 |\tilde{\psi}_1(z,t)|^2}. \quad (5)$$

According to the above formula, the approximately vanishing value of $\rho_2(z,-z,t)/\rho_2(z,z,t)$ for large degree of fragmentation $\mathcal{F}$, visible in Fig. 2, is related to comparable initial curvature radii of modes with given parity, i.e., to comparable dominant Fourier components. Note that $\rho_2(z,-z,t)/\rho_2(z,z,t) = 1\ \forall\, t$ when $\mathcal{F} = 0$, i.e., $N_0 = N$.

We stress that when the pair coherence $\langle \hat{a}_0^\dagger \hat{a}_0^\dagger \hat{a}_1 \hat{a}_1 \rangle +$ H.c. [cf. last term in Eq. (3)] were set positive, the ratio in Eq. (5) becomes unity. The corresponding large difference in the ratio of off-diagonal to diagonal density-density correlations thus allows for the confirmation of the negative sign of the macroscopic pair coherence $\propto \mathcal{O}(N^2)$.

We make our discussion explicit by assuming the following initial orbitals set. The harmonic oscillator ground state is used for the lower single-particle state $\psi_0(z) = \pi^{-1/4}\exp[-z^2/2]$ [30]. For the excited (odd) state, we construct a superposition of two Gaussians of opposite sign and the same width, with symmetrically placed centers a distance d apart. This leads to

$$\psi_1(z) = \frac{1}{\pi^{1/4}} \frac{\sinh(zd/2)\exp[-z^2/2]}{\exp[d^2/16]\sqrt{\sinh(d^2/8)}}. \quad (6)$$

Varying d, this choice serves to illustrate the influence of the overlap of the moduli $|\psi_{0,1}(z)|$ on the correlations. For $d \to 0$, we obtain simply the first excited harmonic oscillator state $\psi_1(z) \to \pi^{-1/4}\sqrt{2}z\exp[-z^2/2]$; for $d \gg 1$, the outer peaks are located where the central Gaussian $\psi_0(z)$ has essentially zero weight; see the top part of Fig. 2.

The hallmark of single-trap condensate fragmentation then becomes apparent upon increasing the degree of fragmentation. As seen from Fig. 2, $\rho_2(z,-\alpha z,t)/\rho_2(z,z,t)$ significantly decreases in the long-time limit for any oppositely located points in the cloud, i.e., $z' = -\alpha z$ with $\alpha > 0$. The robust nature of the proposed indicator is shown by decreasing the orbital overlap significantly; for $d = 4$ in Eq. (6) [see Fig. 2(b)], the result remains similar. Note that the density itself satisfies scaling invariance upon expansion of the cloud. The density-density correlation signal thus obtained is strikingly different from that for a double well, where it exhibits Hanbury Brown–Twiss oscillations for $z' = -z$ and a central peak instead of the central depression seen in Fig. 2 [31,32].

Description with Fock-conjugate phase states.—The above results can be rephrased in terms of a phase-state representation of fragmented condensates [31]. Phase states furnish the most natural tool to transparently describe coherence properties (see, e.g., Refs. [33–37]) and will serve to elucidate that the robustness of the presently discussed fragmented many-body states stems from their being conjugate to fragmented states which are (superpositions of) sharp peaks in Fock space.

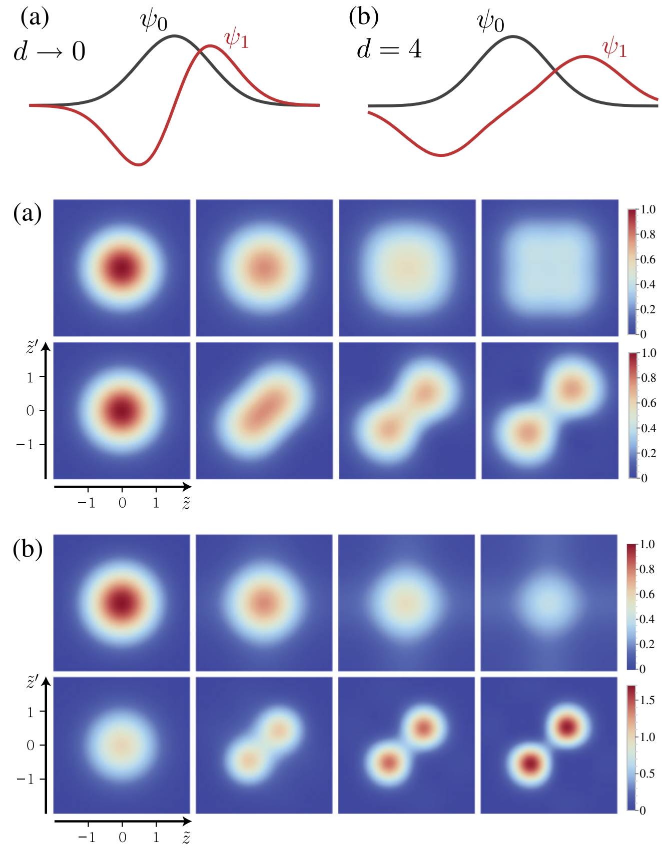

FIG. 2 (color online). Temporal evolution of the density-density correlations $\rho_2(z,z',t)$ for (a) $d \to 0$ and (b) $d = 4$ in Eq. (6). The degree of fragmentation increases from left to right with values $\mathcal{F} = 0, 0.25, 0.5, 0.75$. The top row of the panels is at $t = 0$ and in the original z and z' variables; the bottom row is for $t \gg 1$ and in terms of scaling coordinates $\tilde{z} = z/\sqrt{1+w_t^4}$ and $\tilde{z}'$ correspondingly. The unit of correlations is $N^2/[\pi(1+w_t^4)]$. Note the different color gradings at the top and bottom in (b); for $\mathcal{F} = 0$, the amplitude remains invariant between $t = 0$ and $t \gg 1$.

We prove in what follows that the macroscopically occupied modes of the fragmented state correspond to sharp peaks in the distribution function corresponding to the weights of phase states [38]. We define the phase-state representation of $|\Psi\rangle$ as the integral expression

$$|\Psi\rangle = \int_0^{2\pi} \frac{d\phi}{2\pi} C_{\phi,l_0} |\phi, N, l_0\rangle, \quad (7)$$

where $C_{\phi,l_0} = \sum_l C_l \mathcal{N}_{N,l_0;l} e^{-il\phi}$ with the normalization factor $\mathcal{N}_{N,l_0;l} = \sqrt{[(N-l)!l!/N!][N^N/(N-l_0)^{N-l} l_0^l]}$. The basis vectors $|\phi, N, l\rangle = [(\hat{\psi}^\dagger_{\phi,N,l})^N/\sqrt{N!}]|0\rangle$ are created by the l-dependent superposition operators

$$\hat{\psi}^{\dagger}_{\phi,N,l} \equiv \frac{\sqrt{N-l}\hat{a}^{\dagger}_0 + \sqrt{l}e^{i\phi}\hat{a}^{\dagger}_1}{\sqrt{N}}. \tag{8}$$

The phase-state formulation enables us to rewrite any expectation value of an operator $\hat{O}$ in a given many-body state, to a very good approximation [31], as an integral over diagonal matrix elements

$$\langle\hat{O}\rangle \simeq \int_0^{2\pi} \frac{d\phi}{2\pi} |C_\phi|^2 \langle\phi, N, l_0|\hat{O}|\phi, N, l_0\rangle, \tag{9}$$

where the amplitudes $C_\phi = \sum_l C_l e^{-il\phi}$ are the discrete Fourier transforms of the Fock space amplitudes C_l.

Calculating C_ϕ from the C_l distribution of stably fragmented two-mode many-body states, one can show that the latter are accurately represented by two sharp peaks of the modulus (in the limit $N \to \infty$) [31,39]

$$|C_{\pi/2}| = |C_{3\pi/2}| = \frac{1}{\sqrt{2}}. \tag{10}$$

This simple representation of the many-body fragmented state in terms of two distribution peaks of phase difference π essentially stems from the property $\mathrm{sgn}(C_l C_{l+2}) = -1$. The widths of the peaks in phase and Fock space satisfy the conjugation relation $\Delta C_\phi \sim (\Delta C_l)^{-1}$ ($\propto 1/\sqrt{N}$ for the Gaussian $|C_l|$ distribution), so that $\Delta C_\phi \to 0$ for $N \to \infty$. Fragmented two-mode condensates with quasicontinuous C_l distributions hence correspond to superpositions of macroscopic states with a phase difference of π, and the two macroscopically occupied modes of the quantum gas are *globally* exactly out of phase with each other. This property is in sharp contrast with double-well fragmented condensates, where all values of the phase ϕ are equally likely ($|C_\phi| = \mathrm{const}$) [31]. Macroscopically fragmented condensates are also distinct from so-called *quasicondensates* [40] occurring above a temperature $\propto N\omega^2/\mu$, where ω and μ are the longitudinal trapping frequency and chemical potential, respectively, which possess strongly fluctuating phases.

The phase-state formalism facilitates an interpretation of the strong suppression of $\rho_2(z, z', t)$ along $z = -z'$ in Fig. 2 as follows. For simplicity of the following argument and notational brevity, we put $N_0 = N_1$ ($\mathcal{F} = 1$, $l_0 = N/2$) and set $\psi_1(z)$ to be the first excited harmonic oscillator state ($d \to 0$). Each of the Hilbert space vectors $|\pi/2, N, N/2\rangle$ and $|3\pi/2, N, N/2\rangle$ is a coherent state, according to the definition in Eq. (8), for the orbitals $\psi_0(z) + i\psi_1(z)$ and $\psi_0(z) - i\psi_1(z)$, respectively, omitting the normalizing $1/\sqrt{2}$. After TOF ($t \gg 1$), the orbitals transform into $\tilde{\psi}_0(\tilde{z}, t) + i\tilde{\psi}_1(\tilde{z}, t)$ and $\tilde{\psi}_0(\tilde{z}, t) - i\tilde{\psi}_1(\tilde{z}, t)$, where the scaling coordinate $\tilde{z} = z/\sqrt{1 + w_t^4}$, and up to an irrelevant common phase factor. Again, $\tilde{\psi}_0(\tilde{z}, t)$ is a Gaussian and now $i\tilde{\psi}_1(\tilde{z}, t)$ is the first excited harmonic oscillator state. Thus, $\tilde{\psi}_0(\tilde{z}, t) \pm i\tilde{\psi}_1(\tilde{z}, t)$ have most weight at positive and negative z for upper and lower signs, respectively. From Eq. (9), $\langle\hat{O}\rangle = \frac{1}{2}\langle\pi/2, N, N/2|\hat{O}|\pi/2, N, N/2\rangle + \frac{1}{2}\langle 3\pi/2, N, N/2|\hat{O}|3\pi/2, N, N/2\rangle$, which decomposes into a sum of correlation functions calculated with respect to the two coherent states. Since, generally, $\rho_2(z, z') \simeq \rho(z)\rho(z')$ for coherent states up to $\mathcal{O}(1/N)$ terms, the resulting correlations will correspondingly be concentrated in the region $z, z' > 0$ due to $|\pi/2, N, 0\rangle$ and in the $z, z' < 0$ region due to $|3\pi/2, N, 0\rangle$, but will almost vanish for $z > 0$, $z' < 0$ and $z < 0$, $z' > 0$. A similar argument can be carried out for $N_0 \neq N_1$ and finite d, so that we obtain complete agreement with Fig. 2. By the same argument, it can be shown that an absorption image of the density alone will not allow for the unique inference that the single-trap condensate has fragmented.

Conclusion and outlook.—We have proposed an experimental tool using standard density-density correlation analysis to verify whether an ultracold, strongly interacting gas of bosons in a single trap is a fragmented condensate. The spatiotemporal behavior of density-density correlations changes dramatically with the sign and magnitude of pair correlations between the modes. Single-trap condensate fragmentation is therefore a *genuine* many-body phenomenon, in that it necessitates the observation of second-order correlations. By contrast, for multiple-well fragmentation, structure factor measurements, and hence first-order correlations, suffice to detect fragmentation: The externally imposed spatial separation of the fragments already entails the direct observability of vanishing off-diagonal long-range order.

The predicted decrease of the ratio of off-diagonal to diagonal density-density correlations with time should be measurable even for relatively small degrees of fragmentation $\mathcal{F}$. We anticipate that values of $\mathcal{F}$ down to the level of about 10%–20% should be measurable with current experimental precision.

For future work, we envisage investigating the full counting statistics of fragmented condensates. By their very nature, there is no inverse mapping of correlation functions to a unique many-body state. While correlation functions can reliably measure *global* features of the many-body state like the degree of fragmentation, they cannot reveal local features in the Fock space distributions because they integrate over such distributions. A single-shot analysis might supply a one-to-one mapping of the many-body state to measured quantities going beyond the predominantly Fock-state-based analyses existing so far [41]. Finally, many-body condensate fragmentation into a finite number of macroscopic pieces potentially increases the matter wave bunching towards the Hanbury Brown–Twiss value for a thermal cloud of bosons [42].

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[27] Note that the magnitude and sign of the pair coherence ensure that the (suitably regularized) variance $\langle[\hat{\rho}(z)-\rho(z)]^2\rangle \simeq \rho_2(z,z)-\rho^2(z)$ is $\mathcal{O}(N)$, implying that the trapped fragmented condensate is an (approximate) eigenstate of the density operator.
[28] This is, for example, possible by rapidly switching currents on a suitably patterned microchip trapping a quasi-1D gas; see, e.g., I. Bouchoule, N. J. Van Druten, and C. I. Westbrook, in *Atom Chips*, edited by J. Reichel and V. Vuletić (Wiley VCH, Weinheim, 2011), p. 331.
[29] Putting the orbital width parameter w to be of order $R_{\rm TF}$, the TF size of the initial cloud (suitably generalized for a fragmented condensate [10]), ballistic expansion begins when $t \gg \mathcal{O}(R^2_{\rm TF})$; see also Ref. [25].
[30] We have verified that the density-density correlations after TOF demonstrate the same qualitative behavior at late times, choosing, e.g., a TF ground-state wave function $\psi_{0,\rm TF} = [3(4-z^2)/32]^{1/2}$, for $-2 < z < 2$, and 0 elsewhere, instead of a Gaussian.
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