3/4-Efficient Bell Measurement with Passive Linear Optics and Unentangled Ancillae

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It is well known that an unambiguous discrimination of the four optically encoded Bell states is possible with a probability of 50% at best, when using static, passive linear optics and arbitrarily many vacuum-mode ancillae. By adding unentangled single-photon ancillae, we are able to surpass this limit and reach a success probability of at least 75%. We discuss the error robustness of the proposed scheme and a generalization to reach a success probability arbitrarily close to 100%.

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Introduction.—Bell measurements (BMs), projections of two-qubit states on the Bell basis, form important components of many protocols in quantum computation [1] and communication [2]. Some of the most prominent examples are quantum teleportation [3–5], entanglement swapping [6], and dense coding [7]. In this Letter, we consider Bell states in dual-rail encoding, which is the most convenient and common encoding in optical quantum computation, typically realized through two orthogonal polarization modes. It is well known that an unambiguous BM in this encoding, utilizing fixed arrays of passive, linear optical elements, arbitrarily many vacuum ancilla modes, and photon number resolving detectors (PNRDs), cannot reach a success probability higher than 50% [8,9]. While nonlinear optical interactions of cubic [10] or quartic [11] order do, in principle, allow for a complete BM, such schemes are very inefficient in practice. Techniques from linear-optics quantum computation [1] also enable one to achieve near-unit BM efficiencies, but at the expense of complicated entangled ancilla states and feed-forward operations. Much more recently, two new schemes towards more practical and efficient BMs were presented. On the one hand, Grice [12] demonstrated that a 100%-efficient BM can be approached without feed-forward, but with sufficiently many entangled (Bell- and GHZ-type) ancilla states, which are still fairly expensive and can be generated only probabilistically. On the other hand, active optical elements such as squeezers, without feed-forward and without any ancillae, allow for BMs with above-1/2 efficiency [13]. In fact, squeezing still transforms the mode operators linearly, and it has also become a viable experimental resource, but such squeezing-enhanced BMs have not yet been shown to reach success probabilities greater than 64.3% [13].

We present in this Letter a scheme that reaches a success probability of 75% without using any one of the experimentally challenging methods mentioned above. The only resources required are 50∶50 beam splitters, PNRDs, and unentangled single photons as ancillae [14–19].

We further discuss a generalization to reach success probabilities close to 100%. This extension is an adaption of the scheme by Grice [12] from ancilla states with at most one photon per mode to those with up to two photons, and unfortunately, it also needs entanglement in the added ancillae. Numerical investigations strongly suggest that these states cannot be obtained from unentangled states with passive linear optics, but techniques with an ancillary atom exist [20]. Finally, we investigate the robustness of our scheme to typical experimental errors such as imperfect photon sources and lossy detectors.

3/4-*Efficient Bell measurement.*—The Bell states in dual-rail encoding are given by

$$|\psi^\pm\rangle = \frac{1}{\sqrt{2}}(|1001\rangle \pm |0110\rangle), \tag{1}$$

$$|\phi^\pm\rangle = \frac{1}{\sqrt{2}}(|1010\rangle \pm |0101\rangle). \tag{2}$$

We label the four optical modes from A to D. The simplest way to do a BM that reaches the $1/2$ limit for linear optics is to use two beam splitters, whose action on the mode creation operators is defined by

$$\begin{pmatrix} a_1^\dagger \\ a_2^\dagger \end{pmatrix} \to \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}\begin{pmatrix} a_1^\dagger \\ a_2^\dagger \end{pmatrix}. \tag{3}$$

Throughout this Letter, a beam splitter always refers to this phase-free 50∶50 beam splitter. Applying two of these to combine modes A with C and B with D, respectively, converts the Bell states to the following forms:

$$|\psi^+\rangle \to \frac{i}{\sqrt{2}}(|1100\rangle + |0011\rangle), \tag{4}$$

$$|\psi^-\rangle \to \frac{1}{\sqrt{2}}(|1001\rangle - |0110\rangle), \tag{5}$$

$$|\phi^\pm\rangle \to \frac{i}{2}(|2000\rangle + |0020\rangle \pm |0200\rangle \pm |0002\rangle). \tag{6}$$

By the use of photon detectors for each of the modes, it is now possible to perfectly discriminate $|\psi^+\rangle$ and $|\psi^-\rangle$ from each other and from $|\phi^{\pm}\rangle$, whereas $|\phi^{\pm}\rangle$ are indistinguishable from each other. Thus, an overall success probability of 50% can be attained (given an even distribution for the four Bell states).

Our method to obtain higher success probabilities involves the usual linear-optics elements and ancillary photons. To analyze their use, it is convenient to split the modes into two pairs $\{A,B\}$ and $\{C,D\}$. From now on, these mode pairs are treated separately but in exactly the same way. Hence, the state $|\psi^{-}\rangle$ can always unambiguously be discriminated from the other Bell states, since it always sends one photon in each mode pair while the other Bell states send 0 or 2. To discriminate the other three Bell states, only the mode pair, in which two photons are sent, can be useful. Hence, the remaining problem is to discriminate the three states

$$|\alpha\rangle \coloneqq |11\rangle, \qquad |\beta^{\pm}\rangle \coloneqq \frac{1}{\sqrt{2}}(|20\rangle \pm |02\rangle). \tag{7}$$

To achieve this, we use ancillary photons in the state

$$|\Upsilon_1\rangle = \frac{1}{\sqrt{2}}(|20\rangle + |02\rangle) \quad (= |\beta^{+}\rangle). \tag{8}$$

At first glance, this is a highly entangled state, whose creation might be hard, but this state can easily be obtained by sending two identical single photons through a beam splitter (Hong-Ou-Mandel effect [21]). This is the great advantage of our scheme compared to other methods to surpass the 1/2 limit. We only need single photons as ancillae, no nonlinear effects [22], feed-forward techniques [1], squeezing [13], or entangled ancillae [12].

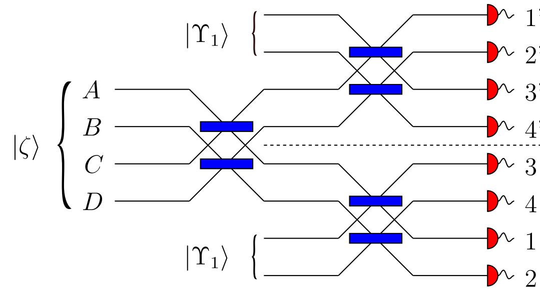

FIG. 1 (color online). Optical setup that identifies an input Bell state $|\zeta\rangle \in \{|\psi^{\pm}\rangle, |\phi^{\pm}\rangle\}$ with a success probability of 75%. The ancillary state $|\Upsilon_1\rangle$ can easily be obtained from $|11\rangle$ with a beam splitter. It is worth emphasizing that this is a static setup. There is no conditional dynamics between the two halves of the setup, so the photon detectors can all be read out simultaneously.

In the following, the concrete use of the ancillary photons is described. For convenience, the modes are now labeled by increasing integers starting with 1 and 2 for the two modes of the relevant pair (see Fig. 1). Mixing these modes with the ancillary modes 3 and 4 at two beam splitters (1 with 3 and 2 with 4) leads to

$$\begin{aligned}|\alpha\rangle|\Upsilon_1\rangle \to \frac{1}{4\sqrt{2}}(&-\sqrt{3}|3100\rangle + i|2110\rangle - |1120\rangle + i\sqrt{3}|0130\rangle - i\sqrt{3}|3001\rangle - |2011\rangle - i|1021\rangle - \sqrt{3}|0031\rangle \\ &-\sqrt{3}|1300\rangle + i|1201\rangle - |1102\rangle + i\sqrt{3}|1003\rangle - i\sqrt{3}|0310\rangle - |0211\rangle - i|0112\rangle - \sqrt{3}|0013\rangle),\end{aligned} \tag{9}$$

$$\begin{aligned}|\beta^{\pm}\rangle|\Upsilon_1\rangle \to &\frac{1}{8}[-\sqrt{6}(|4000\rangle + |0040\rangle \pm |0400\rangle \pm |0004\rangle) - 2(|2020\rangle \pm |0202\rangle)] \\ &+ (1 \pm 1)\frac{1}{8}(-|2200\rangle + |2002\rangle + |0220\rangle - |0022\rangle - 2|1111\rangle) \\ &+ (1 \mp 1)\frac{i\sqrt{2}}{8}(|2101\rangle - |1210\rangle + |1012\rangle - |0121\rangle).\end{aligned} \tag{10}$$

Obviously, $|\alpha\rangle$ can be discriminated from $|\beta^{\pm}\rangle$ unambiguously, since every term that originates from $|\alpha\rangle|\Upsilon_1\rangle$ is unique to $|\alpha\rangle$. It is useful for later to characterize these terms: the total number of photons in modes with an odd label n_{odd} is itself odd. States originating from $|\beta^{\pm}\rangle|\Upsilon_1\rangle$ on the other hand have even n_{odd} values. This is also true in the simple BM without ancillae. The improvement lies in the fact that there are some unique terms for $|\beta^{+}\rangle$ and others for $|\beta^{-}\rangle$. The characterization of these terms is a little more involved. The terms originating from $|\beta^{\pm}\rangle$ are of two types: either the photons are equally distributed on even and odd modes or they are all in modes with the same parity. In the latter case, no information about the original state can be obtained. But if there are two photons in odd and even modes each, $|\beta^{\pm}\rangle$ can be discriminated by $n_{\{1,2\}}$ (the total number of photons in modes 1 and 2), which is then even for $|\beta^{+}\rangle|\Upsilon_1\rangle$ and odd for $|\beta^{-}\rangle|\Upsilon_1\rangle$.

Adding up the squares of the amplitudes of the unique terms shows that for each $|\beta^{+}\rangle$ and $|\beta^{-}\rangle$ the probability of measuring a unique constellation of photons in the four modes is 50%.

If the described optical setup was applied to only one of the original mode pairs (e.g., $\{A,B\}$), the success probabilities for $|\phi^{\pm}\rangle$ were only 25% each, since the setup would only be needed half of the time. It is, therefore, crucial to use a second pair of ancillary photons on the other side (pair $\{C, D\}$). The complete optical setup is shown in Fig. 1.

The overall success probability of this BM can easily be calculated to be

$$P_{\text{succ}} = \frac{1}{4}(1 + 1 + 0.5 + 0.5) = \frac{3}{4}. \tag{11}$$

Near-unit efficient Bell measurement.—In this section, the presented approach is generalized by adding more ancillary photons to reach higher success probabilities. This section is similar [23] to Grice's general scheme [12], and we shall use a similar notation here. For better readability, the detailed proofs of the two lemmata in this section are included in the Supplemental Material [24].

As before, only one mode pair, e.g., $\{A, B\}$, is considered. The optical setup is defined recursively. Given the setup on the modes $1, \ldots, 2^N$, the new setup on the modes $1, \ldots, 2^{N+1}$ is constructed as follows: the old setup (without the detectors) is applied to the modes $1, \ldots, 2^N$, and an identical copy is also applied to the modes $2^N + 1, \ldots, 2^{N+1}$ in which the new ancillary state $|\Upsilon_N\rangle$ is stored. Finally, the modes are pairwise mixed: 1 with $2^N + 1$, 2 with $2^N + 2$, and so on (see the Supplemental Material [25]).

Stated in terms of the 2^{N+1}-dimensional vector of mode creation operators, we have $\vec{a}^\dagger \to \mathbf{S}_N \vec{a}^\dagger$ with the matrix $\mathbf{S}_N$ given by the recursive relation

$$\mathbf{S}_N = \frac{1}{\sqrt{2}} \begin{pmatrix} \mathbf{S}_{N-1} & i\mathbf{S}_{N-1} \\ i\mathbf{S}_{N-1} & \mathbf{S}_{N-1} \end{pmatrix} \tag{12}$$

and $\mathbf{S}_0 = \mathbb{1}_{2\times 2}$. The used input state is $|\xi\rangle|\Upsilon_1\rangle \ldots |\Upsilon_N\rangle$, a product state of the unknown state $|\xi\rangle \in \{|\alpha\rangle, |\beta^\pm\rangle\}$ and N ancillary states given by

$$|\Upsilon_j\rangle := \frac{1}{\sqrt{2}2^{2^{j-2}}} \left[\prod_{\substack{k=2^j+1 \\ k\,\text{odd}}}^{2^{j+1}} (a_k^\dagger)^2 + \prod_{\substack{k=2^j+1 \\ k\,\text{even}}}^{2^{j+1}} (a_k^\dagger)^2 \right] |\underline{0}\rangle$$
$$= \frac{1}{\sqrt{2}} [|2, 0, 2, 0, \ldots, 2, 0\rangle + |0, 2, 0, 2, \ldots, 0, 2\rangle], \tag{13}$$

where $|\underline{0}\rangle = |0\rangle|0\rangle \ldots |0\rangle$ denotes the multimode vacuum. This recursive procedure is illustrated in Fig. 2.

Just as in the previous section, $|\alpha\rangle$ can be distinguished from $|\beta^\pm\rangle$ using n_{odd}: Since each $|\Upsilon_j\rangle$ adds an even number of photons to every mode (0 or 2) and since even and odd modes are not mixed (see definition of $\mathbf{S}_N$), the parity of n_{odd} does not change from one setup to the next. Thus, n_{odd} is odd for $|\alpha\rangle$ and even for $|\beta^\pm\rangle$ for every N.

The advantage of the additional ancillary photons is that they allow us to reduce the degeneracy of $|\beta^+\rangle$ and $|\beta^-\rangle$ by half in each step. To see this, express the input states for $|\beta^\pm\rangle$ as

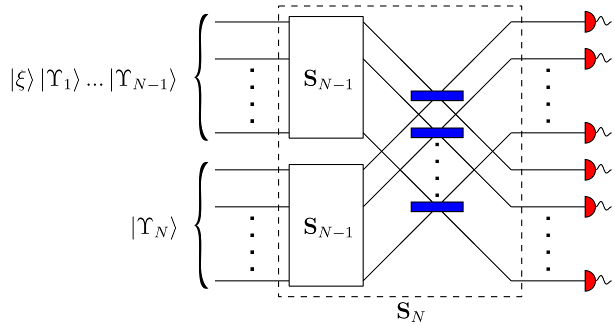

FIG. 2 (color online). Recursive definition of the optical setups for $N \geq 2$. Shown is only the relevant part of the total setup, with ancillae and $|\xi\rangle \in \{|\alpha\rangle, |\beta^\pm\rangle\}$ as the input.

$$|\beta^\pm\rangle|\Upsilon_1\rangle \ldots |\Upsilon_N\rangle$$
$$= |\Xi_N^\pm\rangle + |\Xi_{N-1}^\pm\rangle|\Upsilon_N\rangle + |\Xi_{N-2}^\pm\rangle|\Upsilon_{N-1}\rangle|\Upsilon_N\rangle + \ldots$$
$$+ |\Xi_1^\pm\rangle|\Upsilon_2\rangle|\Upsilon_3\rangle \ldots |\Upsilon_N\rangle + |\Gamma_N^\pm\rangle, \tag{14}$$

with

$$|\Xi_j^\pm\rangle := \left(\frac{1}{\sqrt{2}}\right)^{j+1} \frac{1}{2^{2^{j-1}}} \left[\prod_{\substack{k=1 \\ k\,\text{odd}}}^{2^j} (a_k^\dagger)^2 \prod_{\substack{k=2^j+1 \\ k\,\text{even}}}^{2^{j+1}} (a_k^\dagger)^2 \right.$$
$$\left. \pm \prod_{\substack{k=1 \\ k\,\text{even}}}^{2^j} (a_k^\dagger)^2 \prod_{\substack{k=2^j+1 \\ k\,\text{odd}}}^{2^{j+1}} (a_k^\dagger)^2 \right] |\underline{0}\rangle, \tag{15}$$

and

$$|\Gamma_N^\pm\rangle := \left(\frac{1}{\sqrt{2}}\right)^{N+1} \frac{1}{2^{2^{N-1}}} \left[\prod_{\substack{k=1 \\ k\,\text{odd}}}^{2^{N+1}} (a_k^\dagger)^2 \pm \prod_{\substack{k=1 \\ k\,\text{even}}}^{2^{N+1}} (a_k^\dagger)^2 \right] |\underline{0}\rangle. \tag{16}$$

In Eq. (14), the terms are sorted by $n_{\text{odd}} - n_{\text{even}}$: Every $|\Xi_j^\pm\rangle$ leads to an equal number of photons in odd and even modes, but every $|\Upsilon_j\rangle$ adds either 2^j photons to the odd modes or to the even modes. So the term starting with $|\Xi_j^\pm\rangle$ leads to $n_{\text{odd}} - n_{\text{even}} = \pm 2^N \pm 2^{N-1} \ldots \pm 2^{j+1}$ (and 0 for $j = N$). Furthermore, the term $|\Gamma_N^\pm\rangle$ leads to $n_{\text{odd}} - n_{\text{even}} = \pm 2^{N+1}$.

Hence, all terms in Eq. (14) can be distinguished by $n_{\text{odd}} - n_{\text{even}}$. This can be used to discriminate between $|\beta^+\rangle$ and $|\beta^-\rangle$ since for every term except $|\Gamma_N^\pm\rangle$ the $+$case can be distinguished from the $-$case. To see this, consider $|\Xi_N^\pm\rangle$.

Lemma 1: The parity of $n_{\{1,\ldots,2^N\}}$ discriminates $|\Xi_N^\pm\rangle$: $n_{\{1,\ldots,2^N\}}$ is always even for $|\Xi_N^+\rangle$ and always odd for $|\Xi_N^-\rangle$.

To discriminate $+$ from $-$ for the other terms in Eq. (14) starting with a $|\Xi_j^\pm\rangle$, it is necessary that a discrimination from a smaller setup ($j < N$) carries over to the current setup. To this end, let $A_m^{(p,M)} = \{m2^p + 1, \ldots, (m+1)2^p\}$ be the subset of the 2^M output ports, with $0 \le p \le M-1$ and $0 \le m \le 2^{M-p} - 1$. Furthermore, let $A^{(p,M)} = A_0^{(p,M)} \cup A_2^{(p,M)} \cup \ldots \cup A_{2^{M-p}-2}^{(p,M)}$ and $n^{(p,M)}$ be the number of photons in the detector set $A^{(p,M)}$.

Lemma 2: If the 2^M-photon input state $|\Theta_M\rangle$ always leads to an odd (even) value of $n^{(p,M)}$ in the 2^M-photon setup, then the input state $|\Theta_M\rangle|\Upsilon_{M+1}\rangle$ always leads to an odd (even) value of $n^{(p,M+1)}$ in the 2^{M+1}-photon setup.

With the aid of these lemmata, it is now clear that every term except the last in Eq. (14) can be used to discriminate $|\beta^\pm\rangle$: $|\Xi_j^+\rangle$ can be distinguished from $|\Xi_j^-\rangle$ on the 2^{j+1}-port system by the parity of $n_{\{1,\ldots,2^j\}} = n^{(j,j+1)}$. $|\Xi_j^+\rangle|\Upsilon_{j+1}\rangle$ can then be distinguished from $|\Xi_j^-\rangle|\Upsilon_{j+1}\rangle$ by the parity of $n^{(j,j+2)}$. Repeating this shows that $|\Xi_j^\pm\rangle|\Upsilon_{j+1}\rangle\ldots|\Upsilon_N\rangle$ can be distinguished by the parity of $n^{(j,N+1)}$. Thus, only the terms $|\Gamma_N^\pm\rangle$ in Eq. (14) are ambiguous. Their norm is 2^{-N}, and so the probability to unambiguously identify the states $|\beta^\pm\rangle$ is $1 - 2^{-N}$ for both, which leads to a total success rate for the Bell measurement of

$$P_{\text{succ}}^{(N)} = \frac{1 + 1 + 2(1 - 2^{-N})}{4} = 1 - 2^{-N-1}, \tag{17}$$

approaching unity for $N \to \infty$.

As mentioned before, the ancillary states $|\Upsilon_j\rangle$ are highly entangled and for $j \ge 2$ probably cannot be obtained from single-photon states using passive linear optics [26]. This seems to imply that 75% poses a boundary to BMs with unentangled ancillae, just like 50% did for BMs with vacuum ancillae. But it turns out that this is not true. With a lengthy but straightforward calculation, one can see that a probability of $\frac{25}{32} > \frac{3}{4}$ can be reached. To do this, use the setup for $N = 2$ but replace $|\Upsilon_2\rangle$ by the state $|\Upsilon_1\rangle|\Upsilon_1\rangle$, which is obtained by sending $|1111\rangle$ through two beam splitters. Although the gain in success probability surely is not worth the experimental cost, this shows that no conceptual limit has been found yet (Supplemental Material [27]).

Imperfections.—In this section, we investigate the influence of errors on the proposed scheme. Although there is a multitude of possible errors that can occur in quantum optics, we restrict ourselves to two of the main issues: imperfect photon sources and lossy photon detectors. Furthermore, we analyze only the 3/4-efficient BM and not the generalized version.

Ideally, the photon sources produce the pure state $|1\rangle$. Two of these are sent through a beam splitter to obtain the needed ancilla state $|\Upsilon_1\rangle$. In a more realistic scenario, the source will produce a mixed state of the form $\eta_s|1\rangle\langle 1| + (1-\eta_s)|0\rangle\langle 0|$, where η_s denotes the probability of the source producing a $|1\rangle$. Combining two of these at a beam splitter leads to

$$\eta_s^2|\Upsilon_1\rangle\langle\Upsilon_1| + (1-\eta_s)^2|00\rangle\langle 00| + \eta_s(1-\eta_s) \times (|10\rangle\langle 10| + |01\rangle\langle 01|). \tag{18}$$

Since the scheme is not loss resistant, only the first term of this mixture is of use. Thus, the success probability needs to be multiplied with a factor of η_s^2 whenever the ancilla state $|\Upsilon_1\rangle$ is needed.

A lossy photon detector is modeled by a beam splitter with transmittance η_d and one empty entry in front of a perfect PNRD. Since only terms without photon loss are of use, the probability of a successful event is multiplied with a factor η_d for every photon involved (Supplemental Material [28]).

The successful events for the four Bell states can be characterized in the following way:

[$|\psi^+\rangle$]—4 photons in one half of the setup and n_{odd} is odd in this one: $P_{\text{succ}}(|\psi^+\rangle) = \eta_s^2\eta_d^4$,
[$|\psi^-\rangle$]—3 photons in each half: $P_{\text{succ}}(|\psi^-\rangle) = \eta_s^4\eta_d^6$,
[$|\phi^+\rangle$]—4 photons in one half, n_{odd} is even, $n_{\text{odd}} - n_{\text{even}} = 0$ and $n_{\{1,2\}}$ is even: $P_{\text{succ}}(|\phi^+\rangle) = \frac{1}{2}\eta_s^2\eta_d^4$,
[$|\phi^-\rangle$]—4 photons in one half, n_{odd} is even, $n_{\text{odd}} - n_{\text{even}} = 0$ and $n_{\{1,2\}}$ is odd: $P_{\text{succ}}(|\phi^-\rangle) = \frac{1}{2}\eta_s^2\eta_d^4$.

This leads to an overall success probability of

$$P_{\text{succ}}(\eta_s, \eta_d) = \frac{1}{2}\eta_s^2\eta_d^4 + \frac{1}{4}\eta_s^4\eta_d^6. \tag{19}$$

It is clear that success rates higher than 1/2 can only be obtained with sufficiently good photon sources and detectors. But instead of looking at the bound of an ideal BM without ancillae, i.e., with perfect PNRDs, it makes more sense to compare this result with the success rates of a simple BM as given in the first section, which suffers the same errors. For this simple BM, no ancillary photons are needed, but the two photons still need to be detected. In order to be better than the simple BM, the experimental parameters of our setup, thus, need to meet the requirement $\eta_s\eta_d \le \sqrt{\sqrt{3} - 1} \approx 0.86$.

So far, only the ancillary state $|\Upsilon_1\rangle$ has been considered useful for the BM. A straightforward analysis, however, shows that also the two-mode vacuum [second term in Eq. (18)] can be used as an ancilla. Although the vacuum does not help in identifying $|\beta^\pm\rangle$, the information about $|\alpha\rangle$ remains intact, unlike the cases with exactly one photon in the ancilla [third term in Eq. (18)]. Therefore, for heralded photon sources [29,30], one could turn the latter cases into vacuum as well through a feed-forward process, thus, beating the simple BM independent of the actual experimental parameters.

Conclusion.—We have shown that with the aid of single-photon ancillae the 1/2 limit for BMs with static, passive linear optics can easily be surpassed and a success probability of more than 3/4 is possible. This increased success rate has practical relevance, for example, in the creation of cluster states or in quantum repeaters. From a conceptual point of view, it is more important that 25/32 poses a new maximal value for linear-optics BMs without conditional dynamics, entangled ancillae, or active components (squeezing).

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[14] Compared to Grice [12], who uses two extra entangled photons (one extra Bell pair) to reach 75% BM efficiency, in our scheme we will need four extra unentangled photons to obtain a value of 75%. The main practical advantage of our scheme over Grice's [12] then becomes manifest when deterministic single-photon sources are employed [15–19], as opposed to a probabilistically generated Bell pair. In order to benefit from our approach, such unconditional single-photon sources do not have to produce ideal pure Fock states. For instance, purities of more than 90% for detector efficiencies greater than 95% would suffice in principle, as we will show in the second-to-last section of this Letter. Of course, four unconditionally prepared ancilla photons can also be turned into one ancilla Bell pair by using the methods of linear-optics quantum computation [1]. However, this transformation is again nondeterministic, depending on the detection of two photons at the output of two nondeterministic, nonlinear sign shift gates. For the case of heralded single photons, our scheme would need a four-photon detection to herald four ancilla photons, whereas the standard linear-optics approach [1] for a Bell-pair creation would require a six-photon detection.
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[23] Important differences of our generalized scheme from Grice's include: our setup is divided into two halves, where each half gives a new unambiguous-state discrimination problem for three 2-mode states, and this problem is addressed with ancillae having 0 or 2 photons in each mode.
[24] See the Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.113.140403, sections A and B for the proofs.
[25] See the Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.113.140403, section C for an illustration of $N = 1 \to N = 2$.
[26] We made various numerical tests with MATHEMATICA to obtain the needed ancillae for $N = 2$ from single-photon states $|1111\rangle$ and also tried feed-forward techniques using an ancillary photon, but none was successful.
[27] See the Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.113.140403, section D for a more detailed explanation on how to reach 25/32 and possibilities to generalize this result.
[28] See the Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.113.140403, section E for a derivation of this statement.
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