

**Kurzyński *et al.* Reply** The preceding Comment [1] by Tichy and Andersen is based on the apparent violation of the exclusivity principle by our three-boson system [2]. It is stated that following our reasoning the three events considered by us are not exclusive, which seems to disprove the Letter's conclusion. Moreover, our assumption (ii) of noncontextuality is claimed to be too strong and unreasonable. Finally, a hidden-variable model describing our system is proposed.

Let us recall that the exclusive principle asserts that the sum of probabilities of three pairwise exclusive events cannot exceed 1 [3,4]. Quantum theory is shown to satisfy this principle if the exclusive events can be expressed as pairwise orthogonal projectors. Although this is certainly a reasonable formulation of exclusive events, the exclusivity introduced in the Letter is more counterfactual. Note that our approach is also shared by other researchers, such as Yu and Oh [5], who show that a certain four events are exclusive following their assumptions despite the corresponding projectors being nonorthogonal.

The question of (noncontextual) hidden variables is interesting only for measurements that are argued to be independent under a reasonable hidden-variable model. In the case of Bell's inequality, a local hidden variable is chosen to comply with the classical understanding of special relativity, which directly implies that spacelike separated measurements do not influence each other. If one adopts a nonlocal hidden-variable model, then even the spacelike separated measurements are no longer guaranteed to be independent.

The noncontextuality assumption in the Letter was based on the following two observations. First, the quantum theoretical description of bunching phenomena does not require any interaction between the two bosons and is fully described by a single-particle Hamiltonian. This motivated us to describe the bunching phenomenon via a model that, just like the quantum description, does not involve interactions. In addition, what is important for the inequality is that the particle exchange symmetries do not allow for signaling or disturbance; i.e., introducing another photon in the other beam splitter port should not change the marginal scattering probability of the first photon. This is indeed upheld, as each photon still has a 50% chance for transmission or reflection. In a sense, the exchange interaction resembles the "spooky action at a distance"—it is not a real physical interaction, since it does not allow for information transfer. Under such a model we can regard the measurements in the Letter as independent.

We argue that the hidden-variable model presented in the Comment [1] does not prove that their measurements are independent. In fact, the model is clearly contextual. Tichy and Andersen's model assigns a hidden variable  $0 < \lambda_i < 1$  to each boson. The questions  $A_i$  are defined as, will photon " $i$ " be reflected or transmitted through the beam splitter? Since for each  $A_i$  there are only two possible (exclusive) answers, we

can label them as  $+1$  and  $-1$ . When we measure  $A_i$  and  $A_j$ : if  $\lambda_i > \lambda_j$ , Tichy and Andersen's model assigns  $A_i = +1$  and  $A_j = -1$  (equivalent to  $\underline{a} \ a_j$  in the Letter); and if  $\lambda_j > \lambda_i$ , the model assigns  $A_i = -1$  and  $A_j = +1$  ( $\underline{a} \ a_i$ ). If we consider preassigned  $\lambda_i$ 's, such that  $\lambda_1 < \lambda_2 < \lambda_3$ , then measuring  $A_2$  and  $A_3$  would result in  $A_2 = -1$ ; however, measuring  $A_2$  and  $A_1$  would result in  $A_2 = +1$ . Therefore, the outcome of  $A_2$  is context dependent. Note that the above model when applied to the standard contextuality scenario will not only maximally violate the Specker's inequality, but will also maximally violate the Klyachko-Can-Binicioglu-Shumovsky inequality [6] up to its no-disturbance and arithmetic bound of  $-5$ , which is also not allowed by the exclusivity principle.

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- [1] M. C. Tichy and C. K. Andersen, preceding Comment, *Phys. Rev. Lett.* **113**, 138901 (2014).
- [2] P. Kurzyński, A. Soeda, J. Thompson, and D. Kaszlikowski, *Phys. Rev. Lett.* **112**, 020403 (2014)
- [3] A. Cabello, *Phys. Rev. Lett.* **110**, 060402 (2013)
- [4] T. Fritz, A. B. Sainz, R. Augusiak, J. Bohr Brask, R. Chaves, A. Leverrier, and A. Acín, *Nat. Commun.* **4**, 2263 (2013)
- [5] S. Yu and C. H. Oh, *Phys. Rev. Lett.* **108**, 030402 (2012)
- [6] A. A. Klyachko, M. A. Can, S. Binicioglu, and A. S. Shumovsky, *Phys. Rev. Lett.* **101**, 020403 (2008).