

Comment on “Contextuality in Bosonic Bunching”

Kurzyński *et al.* proposed a system that allegedly violates the exclusivity principle [1]. As illustrated in Fig. 1 in Ref. [1], three bosons are pairwise combined at a beam splitter at which they bunch and leave in the same random output mode. The events \underline{ab} , \underline{bc} , and \underline{ac} [in which \underline{k} (k) denotes particle k being reflected (transmitted)] are conjectured to be pairwise exclusive, since the reflection of a excludes its transmission. All event probabilities being $1/2$ due to bunching, the sum $3/2$ violates the exclusivity bound of unity and saturates the bound allowed by no disturbance. However, following the authors’ reasoning, the three events are not exclusive, which disproves the conclusions of Ref. [1].

The attribution of exclusivity to the three events relies on assumption (ii): “The scattering properties of each boson on the BS [beam splitter] do not depend on which other fiber is connected to the other BS’s input port and on the choice of the BS’s input port.” (all quotes taken from [1]). Essentially, assumption (ii) states that particles must propagate independently. Such strong assumption is unreasonable to impose on physical theories, since it excludes any interaction. It is violated already by classical systems, and by particles described by quantum mechanics that interact either via a potential or by an effective bosonic (fermionic) exchange interaction.

Despite assumption (ii) being clearly violated by correlated bosons, assumption (ii) is nevertheless upheld in Ref. [1] to corroborate exclusivity: Although the authors realize that “... at least one of the assumptions (i), (ii), and (iii) does not hold,” they do not give up any of them and conclude that “there are events that [...] can be considered as exclusive if one takes into account assumptions (i), (ii), and (iii).” Thus, the exclusivity of the three events—the Letter’s very leitmotiv—is based on the disproved assumption (ii). The formal violation of the exclusivity principle is then neither surprising nor is it characteristic to identical particles or quantum physics.

As noted by the authors, although assumption (ii) is called “noncontextuality,” it is unrelated to accepted “traditional” noncontextuality [2]. The traditional definition is referred to in the abstract, the introduction, and the conclusion, even though a traditionally noncontextual hidden-variable model reproduces all phenomena in Ref. [1]: Each particle j ($j = 1, \dots, 3$) is assigned a random hidden variable $0 < \lambda_j < 1$, two particles j, k that impinge on a beam splitter are ejected through output mode 1 if $\lambda_j > \lambda_k$ [output is (2,0)] or mode 2 if $\lambda_j < \lambda_k$ [output is (0,2)]. The initially, randomly and independently chosen hidden variables fully predetermine the outcomes of all possible measurements, which contradicts the dictum that “it is not possible to assign properties to individual bosons independently of this choice [of measurement setting]” (which clearly refers to traditional contextuality). The model perfectly mimics two-boson bunching and reproduces the violation of the exclusivity bound and the Klyachko-Can-Binicioglu-Shumovsky inequality put forward in Ref. [1]. Despite the authors’ claim that no mechanism for such behavior exists which

keeps the beam splitter as a “deterministic memoryless device” without “intrinsic randomness” and “whose action only depends on values of the variables assigned to individual particles,” our model achieves precisely that.

The saturation of the no-disturbance bound $3/2$ in Ref. [1] is accidental: In a modified model, both particles exit through the first mode if $\lambda_j + \delta > \lambda_k$, the average sum of probabilities becomes $3/2 + \delta - \delta^2/2 > 3/2$. This violation occurs because assumption (1) in Ref. [1] (complementarity), on which the upper bound of $3/2$ relies, is not fulfilled either: The assumption that two events out of $\{\underline{ab}, \underline{bc}, \underline{ac}\}$ are tested simultaneously by a single measurement is based on the violated assumption (ii). By testing a and b , we cannot infer how a and c would have behaved.

Furthermore, the authors claim that “a system of bosonic particles and a set of measurement events” is “capable” to do something that cannot be done “using standard quantum events described by projectors.” However, the events in the setup are described by three nonorthogonal, noncommuting projectors. Within quantum physics, the measurements fail to fulfil the requirements of exclusivity and compatibility assumed for noncontextual inequalities or the exclusivity principle [2]. We doubt that the indistinguishability of identical particles can add new features to quantum contextuality: Exclusivity and compatibility are independent of the implementation of a quantum system by one or several, distinguishable or identical particles. In contrast to identical-particle entanglement, the lack of a well-defined tensor-product structure is unproblematic for (non)contextuality [3]: The Fock space of N bosons or fermions in m modes is a finite-dimensional Hilbert space, equivalent to the Hilbert space of a single particle. Fundamentally speaking, a violation of a theorem that is proven to apply to quantum mechanics is tautologically excluded for any quantum-mechanical system, be it realized by distinguishable particles, bosons, or fermions.

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- [1] P. Kurzyński, A. Soeda, J. Thompson, and D. Kaszlikowski, *Phys. Rev. Lett.* **112**, 020403 (2014).
- [2] A. Cabello, [arXiv:1212.5502](https://arxiv.org/abs/1212.5502).
- [3] A. Cabello and M. T. Cunha, *Phys. Rev. A* **87** (2013) 022126.