Mode-Coupling Instability in a Fluid Two-Dimensional Complex Plasma

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A theory of the mode-coupling instability (MCI) in a fluid two-dimensional complex plasma is developed. In analogy to the point-wake model of the wake-mediated interactions commonly used to describe MCI in two-dimensional crystals, the layer-wake model is employed for fluids. It is demonstrated that the wake-induced coupling of wave modes occurs in both crystalline and fluid complex plasmas, but the confinement-density threshold, which determines the MCI onset in crystals, virtually disappears in fluids. The theory shows excellent qualitative agreement with available experiments and provides certain predictions to be verified.

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Two-dimensional (2D) complex plasmas serve as an excellent natural model system for studies of generic phenomena occurring in classical liquids and solids [1–3], such as equilibrium and nonequilibrium melting [4,5], diffusion [6,7], onset of plastic deformations [8], dynamics of dislocations [9], etc. Two-dimensional complex plasmas are usually obtained in radio-frequency (rf) plasma discharge chambers [10–13]. The negatively charged monodisperse microparticles levitate above a flat horizontal rf electrode due to the balance between gravity and the electrostatic force exerted by the inhomogeneous vertical electric sheath field. The combination of the two forces provides vertical confinement. The sheath field also drives a strong vertical plasma flow, which in turn produces a perturbed region downstream from each particle—the so-called "plasma wake" [14–20]. The wakes exert attractive forces on the neighboring particles and make the pair interactions nonreciprocal, which has profound consequences for the stability of these systems [21–25].

The stability of a 2D plasma crystal is determined by three parameters: the areal density of particles; the strength of the vertical confinement (which is effectively controlled by the rf discharge power); and the frictional damping rate (which is due to neutral gas collisions and, thus, is proportional to the gas pressure). We have shown in previous work [24–26] that a universal curve divides the confinement-density plane into stable and unstable regions (see, particularly, Fig. 7 and Eqs. (8) and (11) of Ref. [26]): At high confinement strength and/or low density, the collective behavior of the system is always stable and can be described by an effective Hamiltonian. Crossing the curve, which identifies the *confinement-density threshold*, results in the onset of mode-coupling instability (MCI) [24–26]. The MCI is triggered by resonant coupling of the horizontal (longitudinal acoustic) and vertical (transverse optical) phonon modes of the crystal, due to the nonreciprocal particle-wake interactions [24]. The mode coupling sets in only if the dispersion relations of the two modes intersect, which, in turn, occurs only if the confinement-density threshold is crossed. The instability can be suppressed by increasing the damping rate; thus, there is also a *damping threshold* for any given value of the confinement strength and density. The instability converts the free energy of the flowing plasma into particle thermal energy, resulting in melting of the crystal. All of these phenomena are now well understood and presented quantitatively in numerous theoretical [24–29] and experimental [26,30–35] papers.

However, the remarkable continuing evolution of the system after melting has never been understood. Both experiment and simulation show that the particle temperature keeps increasing exponentially after melting [33,34], eventually saturating at an extremely high (~keV) level. This suggests that an MCI-like instability continues to operate after the crystal has melted. This Letter presents the first theoretical treatment of just such an instability in the fluid state (all previous work on the MCI has been for 2D crystals). The theory is based on a *layer-wake model*, which is an adaptation to the continuum fluid of the point-wake model [25,26,36] commonly employed to describe the wake-mediated interactions in 2D plasma crystals. We demonstrate that a wake-induced mode coupling instability does persist in a 2D fluid layer. Moreover, the instability is stronger in the fluid state: the growth rate is larger, and the confinement-density threshold virtually disappears. The theory shows excellent qualitative agreement with the available experimental data, and makes definite predictions to be tested in future experiments.

Waves in a fluid layer.—Let us consider a system of particles of charge $Q(<0)$ introduced in a plasma. The electrons and ions are assumed to provide screening which is characterized by some effective screening length λ. The electrostatic potential φ of such a system is described by the Poisson equation

$$\nabla^2\varphi - \lambda^{-2}\varphi = -4\pi Q n, \tag{1}$$

where $n(\mathbf{r}, t)$ is the particle number density which can vary both in space and time. We assume that, in equilibrium, particles form a horizontal layer, uniform and of infinite extent in the xy plane, with vertical density profile $n_0(z)$. With a linear perturbation $n_1(\mathbf{r}, t)$ also present, the complete density is $n(\mathbf{r}, t) = n_0(z) + n_1(\mathbf{r}, t)$.

Equilibrium 2D plasma crystals observed in experiments are usually monolayers. Typical vertical displacements due to thermal particle motion are ~ 10 μm (i.e., comparable to the grain size and, therefore, hardly detectable [31]), so $n_0(z)$ can be well approximated by the delta function. But when the crystal undergoes nonequilibrium melting, by external forcing [5] or due to MCI [26], and the kinetic temperature T reaches dozens of eV, the vertical spreading of the layer becomes quite noticeable. The average density profile in this case is described by the Boltzmann distribution in the parabolic potential well $\frac{1}{2}m\Omega_v z^2$, where m is the particle mass and Ω_v is the eigenfrequency of the vertical confinement. This yields

$$n_0(z) = \frac{\rho_0}{\sqrt{\pi}L}\exp\left(-\frac{z^2}{L^2}\right), \tag{2}$$

where $L = \sqrt{2T/m\Omega_v^2}$ is the thickness of the layer and ρ_0 is the areal density. Assuming that the horizontal size of the layer does not change with temperature, we obtain ρ_0 equal to the inverse of the area of the primitive lattice cell; for a triangular lattice with the nearest-neighbor distance Δ, we get $\rho_0 = (2/\sqrt{3})\Delta^{-2}$. Obviously, for small temperatures (or strong confinements) $n_0(z) \to \rho_0\delta(z)$.

For the sake of clarity, we shall first assume that the layer thickness L is much smaller than any relevant spatial scale of the problem, so that the vertical density profile of the layer can be represented by the delta function. Deviation from equilibrium is then characterized by small perturbations of the areal density and the levitation height. Since the fluid layer in equilibrium is uniform and isotropic in the xy plane, one can specify that the perturbations with frequency ω and wave number k propagate in the x direction, so that the areal density is of the form $\rho(x, t) = \rho_0 + \rho_1(x, t)$, where $\rho_1(x, t) = \rho_a e^{ikx - i\omega t} + \text{c.c.}$ and $\rho_a \ll \rho_0$. Similarly, the height perturbation (with respect to $z = 0$) is $\xi(x, t) = \xi_a e^{ikx - i\omega t} + \text{c.c.}$ and $\xi_a \sim \rho_a$.

The electrostatic potential is a superposition of equilibrium and varying components, $\varphi(\mathbf{r}, t) = \varphi_0(z) + \varphi_1(x, z, t)$. The equilibrium potential of the layer is the solution of the one-dimensional Poisson equation (prime denotes the z derivative)

$$\varphi_0'' - \lambda^{-2}\varphi_0 = -4\pi Q\rho_0\delta(z), \tag{3}$$

which yields $\varphi_0(z) = 2\pi Q\rho_0\lambda e^{-|z|/\lambda}$. For the varying component, we suppose that $\varphi_1 \propto \xi$, so that $\partial^2\varphi_1/\partial x^2 = -k^2\varphi_1 + O(\xi^2)$. Then, from Eq. (1), we obtain (to the same accuracy) the equation for φ_1,

$$\varphi_1'' - (\lambda^{-2} + k^2)\varphi_1 = -4\pi Q\rho_0[\delta(z - \xi) - \delta(z)] - 4\pi Q\rho_1\delta(z - \xi), \tag{4}$$

where Eq. (3) was utilized. Let us introduce the following normalization:

$$\frac{\varphi}{2\pi Q\rho_0\lambda} \to \varphi, \qquad \mathbf{r}/\lambda \to \mathbf{r}, \qquad k\lambda \to k,$$

so that the equilibrium component of the potential is

$$\varphi_0(z) = e^{-|z|}. \tag{5}$$

We write the formal solution of Eq. (4) as $\varphi_1 = Ae^{-K|z-\xi|} + Be^{-K|z|}$, where $K = \sqrt{1 + k^2}$. The factors A and B are determined by the discontinuity of the first derivative at $z = 0$ and $z = \xi$, due to the delta functions on the rhs of Eq. (4): $\varphi_1'|_{0_+} - \varphi_1'|_{0_-} = -2$ and $\varphi_1'|_{\xi_+} - \varphi_1'|_{\xi_-} = -2(1 + \rho_1/\rho_0)$. This gives the solution

$$\varphi_1(x, z, t) = K^{-1}\left(e^{-K|z-\xi|} - e^{-K|z|} + \frac{\rho_1}{\rho_0}e^{-K|z-\xi|}\right), \tag{6}$$

which satisfies the assumption $\varphi_1 \propto \xi$ in the entire xz plane except for the segments $\min\{0, \xi\} < z < \max\{0, \xi\}$. For the purposes of our problem, we only need the solution outside of the segments [37].

Now we can calculate the total potential $\Phi = \varphi_g + \varphi_w$ produced by the layer of particles (grains, "g") and by the wakes ("w"). For the particles, we have $\varphi_g(x, z, t) = \varphi_0(z) + \varphi_1(x, z, t)$. In the framework of the simplest point-wake model [25,26,36], each individual wake has the charge $q(> 0)$ located below the respective particle at the distance δ. Therefore, by shifting a replica of the particle layer downwards by the distance δ and replacing Q with q, we obtain a continuous representation of wakes—the *layer-wake model*. The resulting potential is $\varphi_w(x, z, t) = -\tilde{q}\varphi_g(x, z + \delta, t)$, where $\tilde{q} = |q/Q|$; in the explicit form, from Eqs. (5) and (6), we derive the following expression accurate to $O(\xi)$:

$$\varphi_w(x, z, t) = -\tilde{q}e^{-(\delta+z)} - \tilde{q}e^{-K(\delta+z)}K^{-1}\left(K\xi + \frac{\rho_1}{\rho_0}\right). \tag{7}$$

For the layer of particles, we expand Eqs. (5) and (6) up to the terms providing linear ($\propto \xi$) contributions to the electric field at $z = \xi$, which gives

$$\varphi_g(x,z,t) = 1 + \frac{1}{2}z^2 - K\xi z + K^{-1}\frac{\rho_1}{\rho_0} - \left(1+\frac{\rho_1}{\rho_0}\right)|z-\xi| + O(z^3, z^2\xi). \quad (8)$$

The dynamics of perturbations is governed by the equations of horizontal and vertical motion. For simplicity, we omit the damping term due to neutral gas friction, which is characterized by the damping rate ν (the friction can be taken into account by simply replacing ω with $\omega + \frac{1}{2}i\nu$ in the resulting dispersion relations). The equations for the horizontal and vertical perturbations read

$$\frac{\partial v_x}{\partial t} = -\frac{Q}{m}\frac{\partial \Phi}{\partial x}\bigg|_{z=\xi}, \quad (9)$$

$$\frac{\partial^2 \xi}{\partial t^2} = -\Omega_v^2 \xi - \frac{Q}{m}\left(\frac{\partial \Phi}{\partial z}\bigg|_{z=\xi} - \frac{\partial \varphi_w}{\partial z}\bigg|_{z=0}\right), \quad (10)$$

where the horizontal velocity v_x is related to the density perturbation via the continuity equation

$$v_x = \frac{\omega}{k}\frac{\rho_1}{\rho_0}.$$

The last term in Eq. (10) is the constant "self-action" field produced by wakes, $\partial\varphi_w/\partial z|_{z=0} = \tilde{q}e^{-\delta}$. This term only affects the equilibrium levitation height and, hence, should be subtracted from the perturbed equations. We keep in mind that the absolute value $|z-\xi|$ appears in Eq. (8) as the result of integration across the layer, with the density profile [Eq. (2)] in the limiting form of the delta function. Therefore, the derivatives of $|z-\xi|$ with respect to z or x, taken at $z=\xi$, represent the self-action of an infinitely thin layer and should be set equal to zero. Thus, by calculating components of the electric field from Eqs. (8) and (7) and substituting the results in Eqs. (9) and (10), we finally derive the dispersion relations for the longitudinal (horizontal, "h") and transverse (vertical, "v") wave modes

$$[\omega^2 - \omega_h^2(k)][\omega^2 - \omega_v^2(k)] + \sigma^2(k) = 0. \quad (11)$$

We normalize all frequencies by

$$\omega_{p2D} = \sqrt{\frac{2\pi Q^2 \rho_0}{m\lambda}}, \quad (12)$$

which is the effective plasma frequency in the (2D) layer, so the modes are

$$\omega_h^2(k) = (1 - \varepsilon e^{-(K-1)\delta})\frac{k^2}{K}, \quad (13)$$

$$\omega_v^2(k) = \Omega_v^2 + 1 - K + \varepsilon(Ke^{-(K-1)\delta} - 1), \quad (14)$$

and the coupling term is

$$\sigma(k) = \varepsilon k e^{-(K-1)\delta}.$$

The remarkable conclusion from Eqs. (11)–(14) is that the behavior of the longitudinal and transverse waves in a fluid layer is similar to the familiar behavior in a crystal [25,26,29]—the modes are modified, coupled, and destabilized by the wake-mediated interactions. The magnitude of the effect is characterized by the parameter $\varepsilon = \tilde{q}e^{-\delta}$. Normally, this parameter is substantially smaller than unity [26], so the modes themselves are only weakly affected by additional terms $\sim\varepsilon$. (Note that the longitudinal mode is sustained for $\varepsilon < 1$.) There is, however, one striking difference from the mode coupling in crystals: From Eqs. (13) and (14) it is evident that $\omega_h(k)$ monotonically increases with k, while $\omega_v(k)$ monotonically decreases; i.e., the modes always cross [38]. Therefore, in contrast to crystals, the unstable hybrid mode (formed in the vicinity of the crossing) is always present in a fluid layer. The crossing (cr) occurs at the wave number $k_{\rm cr}$, which is determined from the condition $\omega_h(k_{\rm cr}) = \omega_h(k_{\rm cr}) \equiv \omega_{\rm cr}$ yielding [39]

$$k_{\rm cr} \simeq \frac{1}{2}(1+\Omega_v^2) + O(\varepsilon). \quad (15)$$

The k width of the hybrid mode and the maximum growth rate of MCI, γ, are given by $|\omega_h(k) - \omega_v(k)| \le \gamma = \sigma(k_{\rm cr})/\omega_{\rm cr} + O(\varepsilon^2)$.

Properties of MCI in a fluid layer.—In order to compare the fluid and crystalline regimes, it is convenient to relate the respective frequency scales—the effective plasma frequency ω_{p2D} and the dust-lattice (DL) frequency $\Omega_{\rm DL} = \sqrt{Q^2/m\lambda^3}$. By expressing ρ_0 via the lattice constant Δ of a crystalline monolayer, we readily obtain $\omega_{p2D} = \sqrt{4\pi/\sqrt{3}}\Omega_{\rm DL}/\kappa \simeq 2.7\Omega_{\rm DL}/\kappa$, where $\kappa = \Delta/\lambda$ is the screening parameter. In particular, this relation allows us to easily recalculate the value of Ω_v (in units of ω_{p2D}) corresponding to the MCI threshold in a crystal [26]: e.g., for a lattice with $\kappa = 1$ the threshold value is $\Omega_v \simeq 1.5$ (the mode crossing in this case occurs at $k_{\rm cr} \simeq 1.6$). Note that $\sigma(k)$ attains a maximum at $k \simeq \sqrt{1+\delta^{-2}}$ ($\simeq\delta^{-1}$ for typical experiments); for conditions near the threshold, this k is usually close to $k_{\rm cr}$, i.e., $(1+\Omega_v^2)\delta \sim 1$. Therefore, when Ω_v is increased above the threshold, the instability growth rate $\gamma \propto \exp(-\frac{1}{2}\Omega_v^2\delta)$ starts rapidly decreasing, and when it becomes equal to $\frac{1}{2}\nu$, the instability disappears.

Up to this point, we have assumed that the layer thickness L in Eq. (2) is negligibly small compared to the wavelength, i.e., that the temperature is sufficiently small. But if T increases to the point where $kL \gtrsim 1$, the longitudinal dispersion relation should change to the dispersion relation for acoustic waves in a three-dimensional plasma [40]; i.e., $\omega_h(k)$ should tend to a constant value equal

to the plasma frequency ω_p (calculated for the volume density $n_0 \simeq \rho_0/L$); indeed, from Eq. (13), we obtain $\omega_h(k) \simeq \omega_p$ for $k \simeq L^{-1}$. On the other hand, the shear mode cannot be sustained in a three-dimensional gas, so the dispersion of the transverse mode disappears at $kL \gtrsim 1$. Thus, thermal spreading of the fluid layer can inhibit mode coupling and, thereby, set a maximum for the temperature that can be reached in the fluid state via the MCI: The mode crossing and, hence, the hybrid mode are expected to disappear gradually when L becomes as large as $\sim k_{\mathrm{cr}}^{-1}$. By substituting k_{cr} from Eq. (15) and using the expression for L, we estimate the maximum temperature (measured in units of $m\omega_{p2D}^2\lambda^2$) that can be reached when damping is negligible ($\nu \ll \gamma$)

$$T_{\max} \sim \frac{\Omega_v^2}{(1+\Omega_v^2)^2}.$$

For typical conditions of the MCI onset in a crystal, $T_{\max} \sim 1$–3 keV.

When the particle temperature T is sufficiently large, the acoustic longitudinal mode becomes an ordinary (gas-dynamic) sound wave, with the pressure provided by the thermal motion of particles. To account for this effect, we simply need to add a term $k^2 v_T^2$ to the rhs of Eq. (13) [41] (where the thermal velocity $v_T = \sqrt{T/m}$ should be normalized by $\omega_{p2D}\lambda$). The longitudinal mode then becomes essentially nondispersive at larger k, with $\omega_h(k) \simeq k v_T$. On the one hand, this effect could restore the hybrid mode at $kL \gtrsim 1$ and cause the temperature to grow above $T_{\max}$. On the other hand, dissipation of regular sound due to viscosity and thermal conductivity [42] rapidly increases with temperature (since the mean free path for interparticle Coulomb collisions scales as $\propto T^2$), so the effect might be suppressed by the resulting damping. It should also be noted that the ordinary (nonresonant) two-stream instability between flowing ions and stationary particles occurs in a thick uniform layer, and is known to heat the particles in that regime [43]. Thus, the situation regarding instabilities driven by ion streaming through a layer of a finite thickness is complex, and yet to be fully explored.

Our discussion has been based on a simple layer-wake model, which is the extension (to a continuum fluid) of the point-wake model that has previously been used successfully to treat MCI in 2D crystals. Models of this type provide a qualitatively reasonable "far-field" representation of the wake potential, but may not be adequate in situations where moving particles approach the wake "focus" of their neighbors. If the layer thickness L is comparable to or larger than the wake length δ, the mode coupling may show substantial dependence on details of the wake-field structure. It is also possible that kinetic effects which are not included in our fluid model, such as particle discreteness and Landau damping, could play a significant role in this regime.

To illustrate the range of temperatures where the layer model is valid, let us take $T_{\max}$ as a convenient point at which the layer thickness is $L \sim k_{\mathrm{cr}}^{-1}$. Then, for $T \lesssim T_{\max}$ the sufficient condition of applicability can be written as $k_{\mathrm{cr}}\delta \gg 1$, and, by virtue of Eq. (15), we conclude that the product $(1+\Omega_v^2)\delta$ must be substantially larger than unity. As we already pointed out, its value is usually about unity near the MCI threshold in a crystal. Hence, the model is expected to be valid up to $T \sim T_{\max}$ when Ω_v is somewhat increased above the threshold. Moreover, the theory is always applicable if damping causes the temperature to saturate at $T \ll T_{\max}$.

Conclusions.—We have shown that wake-induced mode coupling operates in a similar way in both crystalline and fluid 2D complex plasmas. However, the MCI threshold virtually disappears in the fluid regime, and the instability growth rate can be even larger in the fluid than in the crystal for, otherwise, the same conditions. This would appear to explain the experiment reported in Fig. 3(a) of Ref. [34], where a period of exponential growth in the kinetic temperature coincided with melting of the crystal, and then a second period of exponential growth, at a larger growth rate, occurred after melting. Furthermore, the theory presented here explains the propagation of melting fronts commonly observed in experiments with crystalline monolayers (see, e.g., Ref. [33], Figs. 3 and 4): MCI is triggered in a crystal when the density threshold is reached, so the melting usually starts in the center of a monolayer, where the density is the highest. Once "ignited," the melting front propagates outwards, where the density is normally well below the (crystalline) MCI threshold [44]. In this way, MCI melting is analogous to exothermic reactions, e.g., combustion [42,45,46].

Another remarkable prediction of the theory is that there are conditions for which the fluid state is viable (because MCI keeps the fluid hot), but the crystalline state is also viable (because the value of the confinement frequency is above the MCI threshold in a crystal). In experiments, the confinement strength is a monotonically increasing function of the rf discharge power P [47]. Therefore, hysteresis is to be expected when melting is controlled by varying the power: The value of P at which the crystal melts must be systematically smaller than P at which the system recrystallizes. Moreover, a slightly "undercritical" crystal (i.e., a stable crystal near the MCI threshold) could undergo sporadic melting due to a sufficiently strong mechanical perturbation, e.g., by a laser beam. All these predictions suggest a series of new experiments that will be of general interest.

It should also be noted that the theory presented in this paper is based on a simple model for the wakes. Further investigation will be required to determine if significant effects result from more accurate representation of the wake structure, or from kinetic effects omitted from the present fluid model. It is expected that this work will stimulate

theoretical and simulation activity to put this on a firmer and more quantitative basis.

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