

Collapse Arrest in Instantaneous Kerr Media via Parametric Interactions

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We demonstrate, theoretically and experimentally, that a four-wave mixing parametric interaction is able to arrest the collapse of a two-dimensional multicolor beam in an instantaneous Kerr medium. We consider two weak idlers interacting via a third order nonlinearity with two pump beams and we show that a class of collapse-free quasisolitary solutions can be experimentally observed in a normal dispersion Kerr glass. This observation is sustained by rigorous theoretical analysis demonstrating the stability of the observed self-trapped beams.

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The nonlinear Schrödinger equation (NLSE) describes many physical systems in optics and condensed matter. Although the NLSE sustains solitary propagation in $1 + 1$ dimensions $[(1 + 1)D]$ [1,2], the nonlinear wave propagation is unstable and undergoes collapse in higher dimensionality: the possibility of arresting the blowup has remained a hot topic [3,4] since the first studies of wave collapse [5,6]. In optics, stable solitary propagation in $2D + 1$ NLSE-like systems can be observed in materials showing nonlinear saturation, nonlocality [1,3,7,8], in engineered materials, e.g., in lattices [3,9], in stacked layer systems [10,11], or by acting on the beam amplitude and phase profile, e.g., in vortex or spiraling solitons [12,13]. Controlling the blowup of bell-shaped beams in homogeneous Kerr media, however, remains a challenge. Coping with the collapse and postcollapse of the beam is indeed necessary whenever high-power propagation is involved [14–18].

In this Letter we demonstrate the possibility of managing the collapse dynamic of an optical beam propagating in a pure Kerr material by addressing its mutual interaction via parametric four-wave mixing (FWM) with an additional control beam.

Although the nonlinear coupling of two waves by cross-phase modulation (XPM) always leads to a reduction of the power collapse threshold [19], a parametric interaction can counteract the collapse [20,21]. Parametric solitons sustained by three wave mixing have a large impact in nonlinear optics [22], but they are scarcely addressed in the FWM case, mostly because their effect can be small with respect to self-focusing [23,24].

Collapse arrest has been theoretically addressed in the case of highly mismatched coupling between a beam and its weak third harmonic (TH) [24]; this parametric interaction induces an equivalent $\chi^{(5)}$ nonlinearity [25,26] that can counteract the collapse [2] for a proper phase mismatch. TH

generation (THG) has been observed to enhance the self-guiding property of femtosecond filamentation in the framework of a rich dynamics involving dissipative and temporal effects [27] and, more recently, ionization effects [28]. An experiment on the characterization of high-order Kerr nonlinearities in gases [29] has been interpreted as a THG-induced higher order nonlinearity [30]. The challenge in the observation and the control of the TH cascaded $\chi^{(5)}$ nonlinearity is ultimately connected to the large mismatch between the fundamental and the TH that the refractive index dispersion usually imposes to the interaction.

Here we introduce a propagation geometry that supports collapse-free propagation of $(2 + 1)D$ quasisolitons, i.e., solitary waves with weakly oscillating radiative tails, in a regime that can be found in a large class of materials by properly selecting the spectral content of the pump beams.

We consider the nonlinear interaction of two pumps with electric field envelopes $P_{(1,2)}(X, Y, Z)$ slowly varying along Z , with frequencies $\omega_{(p1,p2)} = \omega_0 \mp \Delta\omega$, expressed by their frequency difference $\Delta\omega$ with respect to their central frequency ω_0 . Their wave vectors are $k_{(p1,p2)} = \omega_{(p1,p2)}n/c$, with c speed of light in vacuum and n refractive index.

When the two pumps are close in frequency, the most significant parametric contribution usually involves the generation of two idlers $S_{(1,2)}(X, Y, Z)$ with frequencies regulated by the two FWM interactions $\omega_{(s1,s2)} = 2\omega_{(p2,p1)} - \omega_{(p1,p2)} = \omega_0 \pm 3\Delta\omega$ and wave vectors $k_{(s1,s2)}$. Each interaction is associated with a phase mismatch $\Delta k_1 = 2k_{p2} - k_{p1} - k_{s1}$, $\Delta k_2 = 2k_{p1} - k_{p2} - k_{s2}$. A third FWM interaction involving the mixing $\omega_{s1} + \omega_{s2} = \omega_{p1} + \omega_{p2}$ and the phase matching $\Delta k_3 = k_{p1} + k_{p2} - k_{s1} - k_{s2} = \Delta k_1 + \Delta k_2$ also occurs.

We start by defining a set of equations for the linearly copolarized waves with the wave vector $k_0 = \omega_0 n/c$, the

central frequency ω_0 , and the scaling quantity $\sigma = \Delta\omega / (2\omega_0)$. The nonlinear constant is $\gamma_0 = 3\chi^{(3)}\omega_0^2/c^2$, $\chi^{(3)}$ being the nonlinear third order susceptibility:

$$\begin{aligned} \frac{2ik_0}{1 \mp \sigma} \partial_z P_{(1,2)} + \frac{\nabla_{x,y}^2 P_{(1,2)}}{(1 \mp \sigma)^2} + \gamma_0 K_{(P_1, P_2)} P_{(1,2)} \\ + \gamma_0 F_{(P_1, P_2)} = 0, \\ \frac{2ik_0}{1 \pm 3\sigma} \partial_z S_{(1,2)} + \frac{\nabla_{x,y}^2 S_{(1,2)}}{(1 \pm 3\sigma)^2} - \frac{2k_0 \Delta k_{(1,2)}}{1 \pm 3\sigma} S_{(1,2)} \\ + \gamma_0 K_{(S_1, S_2)} S_{(1,2)} + \gamma_0 F_{(S_1, S_2)} = 0. \end{aligned} \quad (1)$$

$K_U = 2(|P_1|^2 + |P_2|^2 + |S_1|^2 + |S_2|^2) - |U|^2$ is the nonlinear term accounting for both self-phase modulation and XPM for $U = \{P_1, P_2, S_1, S_2\}$. The FWM terms are

$$\begin{aligned} F_{(P_1, P_2)} &= 2P_{(1,2)}^* P_{(2,1)} S_{(2,1)} + S_{(1,2)}^* P_{(2,1)}^2 \\ &\quad + 2P_{(2,1)}^* S_{(1,2)} S_{(2,1)}, \\ F_{(S_1, S_2)} &= P_{(1,2)}^* P_{(2,1)}^2 + 2S_{(2,1)}^* P_{(1,2)} P_{(2,1)}. \end{aligned} \quad (2)$$

We first address a regime of weak idlers, occurring for large phase mismatches $\Delta k_{(1,2)}$, resulting in the idlers $S_{(1,2)} \approx \gamma_0(1 \pm 3\sigma)/(2k_0 \Delta k_{(1,2)}) P_{(1,2)}^* P_{(2,1)}^2$. Correspondingly, from the first of Eqs. (2) we get

$$\begin{aligned} F_{(P_1, P_2)} &= \left[\gamma_0 \frac{1 \mp 3\sigma}{k_0 \Delta k_{(2,1)}} |P_{(1,2)}|^2 |P_{(2,1)}|^2 \right. \\ &\quad \left. + \gamma_0 \frac{1 \pm 3\sigma}{2k_0 \Delta k_{(1,2)}} |P_{(2,1)}|^4 \right] P_{(1,2)}. \end{aligned} \quad (3)$$

Equations (3) show that the weak FWM interaction acts as an equivalent quintic XPM contribution. For a normal dispersion medium $k^{(2)}(\omega_0) = \partial^2 k / \partial \omega^2 > 0$ we obtain $\Delta k_{(1,2)} \approx -4k^{(2)}(\omega_0) \Delta \omega^2$, corresponding to a defocusing action which arrests the collapse and grows when $\Delta \omega$ is decreasing, as long as the cascaded regime holds. Differently from the case of THG, [24] where the phase mismatch cannot be controlled and is usually very large, the frequency difference between the pumps governs the strength of the stabilizing defocusing mechanism.

We then consider the stationary solutions of Eqs. (1). Given a reference spatial waist X_0 and a diffraction length $Z_0 = 2k_0 X_0^2$, we introduce the normalized coordinates $z = Z/Z_0$, $x = y = X/X_0$, the phase mismatches $\Delta_{(1,2)} = \Delta k_{(1,2)} Z_0$, and the amplitudes $p_{(1,2)} = \sqrt{\gamma_0} X_0 P_{(1,2)} e^{i(\beta \mp \Delta \beta)z}$ and $s_{(1,2)} = \sqrt{\gamma_0} X_0 S_{(1,2)} e^{i(\beta \pm 3\Delta \beta)z}$, obtaining

$$\begin{aligned} \frac{i\partial_z p_{(1,2)}}{1 \mp \sigma} + \frac{\nabla_{x,y}^2 p_{(1,2)}}{(1 \mp \sigma)^2} - \frac{\beta \mp \Delta \beta}{1 \mp \sigma} p_{(1,2)} \\ + K_{(p_1, p_2)} p_{(1,2)} + F_{(p_1, p_2)} = 0, \\ \frac{i\partial_z s_{(1,2)}}{1 \pm 3\sigma} + \frac{\nabla_{x,y}^2 s_{(1,2)}}{(1 \pm 3\sigma)^2} - \frac{s_{(1,2)}}{\alpha_{(1,2)}(1 \pm 3\sigma)} \\ + K_{(s_1, s_2)} s_{(1,2)} + F_{(s_1, s_2)} = 0, \end{aligned} \quad (4)$$

with $1/\alpha_{(1,2)} = \beta \pm 3\Delta \beta + \Delta_{(1,2)}$. We are explicitly considering the nonlinear propagation constants $\beta \mp \Delta \beta$ and $\beta \pm 3\Delta \beta$ for the pumps and idlers, respectively. The nonlinear terms K_u and F_u for $u = \{p_1, p_2, s_1, s_2\}$ are defined as for Eqs. (1).

For each wave we define the photon flux $Q_u = 1/(1 + \sigma_u) \int |u|^2 dx dy$, with $u = \{p_1, p_2, s_1, s_2\}$ and $\sigma_u = \{-\sigma, \sigma, 3\sigma, -3\sigma\}$. System (4) is not integrable but possesses the following three integrals of motions [31]. It conserves the Hamiltonian, defined as $H = \int \left[\sum_u (|\nabla u|^2)/(1 + \sigma_u) - \sum_{u,v} (1 - \frac{1}{2} \delta_{u,v} |u|^2 |v|^2) \right] dx dy + H_F$ with u, v summed over $\{p_1, p_2, s_1, s_2\}$. $H_F = -2\text{Re} \int [2p_1 p_2 s_1^* s_2^* + p_1^2 p_2^* s_2^* + p_2^2 p_1^* s_1^*] dx dy$ is the FWM contribution. It possesses the standard Manley Rowe integral for parametric interactions, $Q = Q_{p_1} + Q_{p_2} + Q_{s_1} + Q_{s_2}$, which takes into account the conservation of the total photon number and of the cross photon flux $J = Q_{p_1} - Q_{p_2} + 3Q_{s_2} - 3Q_{s_1}$, regulating the exchange of photons between the pumps and the signals. We look for single hump, radially symmetric self-similar solutions, calculating the nonlinear modes of Eqs. (4) in function of the mismatch parameters. This is equivalent to address the variational problem $\delta(H + \beta N + \Delta \beta J) = 0$: the

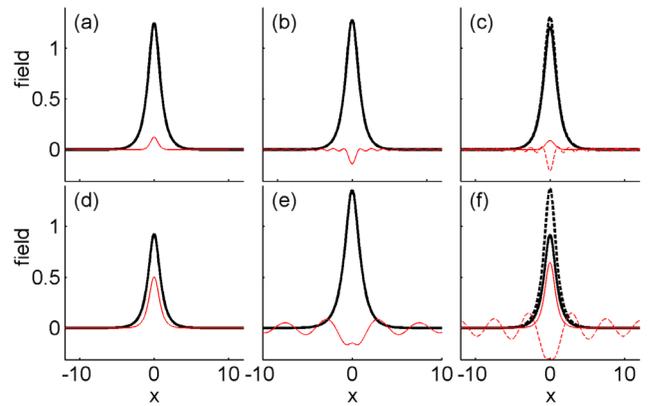


FIG. 1 (color online). Self-localized solutions of the normalized system, Eqs. (4). The nonlinear propagation constants are $\beta = 1$ and $\Delta \beta = 0$. The relative frequency scaling quantity is $\sigma = 0$. The self-localized profiles p_1 (thick line, black), p_2 (thick dashed line, black), s_1 (narrow line, red), s_2 (narrow dashed line, red) are shown in the function of the space coordinate x for several values of the phase mismatches parameters $\alpha_{(1,2)}$. (a) $\alpha_{(1,2)} = 0.0625$, (b) $\alpha_{(1,2)} = -0.0625$, (c) $\alpha_{(1,2)} = \pm 0.0625$, (d) $\alpha_{(1,2)} = 0.5$, (e) $\alpha_{(1,2)} = -0.5$, (f) $\alpha_{(1,2)} = \pm 0.5$.

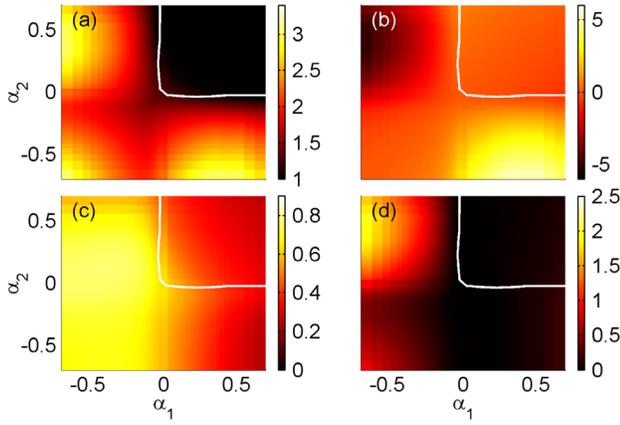


FIG. 2 (color online). (a) Total photon flux $Q = Q_{p1} + Q_{p2} + Q_{s1} + Q_{s2}$, (b) cross photon flux $J = Q_{p1} - Q_{p2} + 3Q_{s2} - 3Q_{s1}$, (c) pump Q_{p1} , and (d) idler Q_{s1} photon fluxes vs the mismatch parameters $\alpha_{(1,2)}$ for nonlinear phase constants $\beta = 1$ and $\Delta\beta = 0$. The white line marks the VK threshold, where the critical collapse takes place. The unstable region is enclosed in the section where $\alpha_1 > 0$ and $\alpha_2 > 0$.

solutions are the extrema of the Hamiltonian, where β and $\Delta\beta$ are Lagrange multipliers. The system also conserves the transverse and angular momentum [31], although such quantities do not appear in this variational problem. Because of the scaling properties of Eqs. (4) [2] we limit our description to the case $\beta = 1$ without loss of generality; in Figs. 1 and 2 we show representative cases of the profiles and photon fluxes, respectively, for $\Delta\beta = 0$ and $\sigma = 0$. We could find single-hump modes for the pump in the whole exploited range of parameters for $\alpha_{(1,2)}$. For $|\alpha_{(1,2)}| \rightarrow 0$ the solutions collapse to Townes-like modes [2], while for large values of $|\alpha_{(1,2)}|$ the energy is mostly contained in the idlers. In the latter regime, however, additional nonlinear products should be included in the model.

Notably, we could find nonlinear modes also when one or both $\alpha_{(1,2)}$ are negative, corresponding to a defocusing nonlinearity [Eq. (3)]. Here system (4) admits oscillating solutions for the idlers regulated by the parameter $\alpha_{(1,2)}$, consistently with its linear solutions [Figs. 1(c) and 1(f)]. Such oscillations may eventually lead to energy leaking in propagation, but this effect is negligible for small $|\alpha_{(1,2)}|$. These bound states between solitary solutions and radiative components are denoted as quasisolitons and have been previously found in THG systems [24]. The stability properties of the soliton family are studied with the well-known Vakhitov-Kolokolov (VK) criterion [32], which allows discriminating whether or not a perturbation of a solitary solution grows along with propagation. For stability consideration, it rigorously applies to the node-free, lowest energy soliton family branch. For solutions possessing nodes, the VK theorem provides information only on the subspace of perturbations that are zero in the nodes of the solutions, and hence it is not usually sufficient

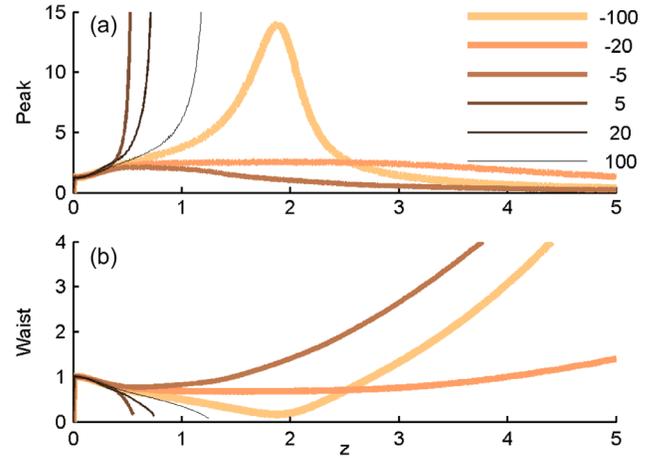


FIG. 3 (color online). Evolution of peak intensity (a) and waist (b) of the pump p_1 for the mismatches parameters $\Delta_{(1,2)}$ as calculated from Eq. (4). $\Delta_1 = \Delta_2$ (values in the legend) and the beam input profiles are $p_1(z=0, x, y) = p_2(z=0, x, y) = 1.078 \exp[-(x^2 + y^2)/1.5^2]$ and $s_1(z=0, x, y) = s_2(z=0, x, y) = 0$.

for stability. This is relevant to the case of the oscillating solutions as in Figs. 1(b), 1(c), 1(e), 1(f): the VK criterion will not give us complete information on the perturbations of the oscillating idlers. However, the VK is sufficient for claiming stability against perturbations of the pumps. This is particularly significant especially when the pumps contain most of the energy. The VK threshold for instability, e.g., for the NLSE, is usually expressed in terms of the derivative of the energy integral with respect to the nonlinear propagation constant, or eigenvalue of the problem. Referring to Eqs. (4), by using a standard approach for two coupled equations as, e.g., in Refs. [22,33], we find that the VK threshold is $\partial_{\beta} Q \partial_{\Delta\beta} J - \partial_{\beta} J \partial_{\Delta\beta} Q = 0$. Such a threshold is reported in Fig. 2 in white, superimposed to the photon fluxes. As expected, the Townes-like modes for $\alpha_{1,2} \rightarrow 0$ are at the threshold; i.e., here the FWM is negligible and a standard critical collapse for the NLSE occurs. For both $\alpha_{1,2} > 0$ the system is unstable, consistently with the self-focusing nature of the FMW interaction

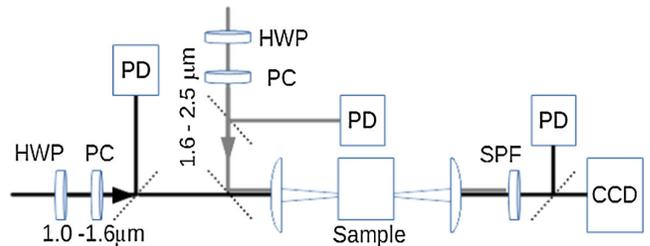


FIG. 4 (color online). Sketch of the experimental setup. The two pumps at 1500 and 1720 nm are obtained from an optical parametric amplifier (OPA). Polarizer crystals (PC), half wave plates (HWP), a short pass filter (SPF) are shown together with the energy monitors photodiodes (PD) and the camera (CCD).

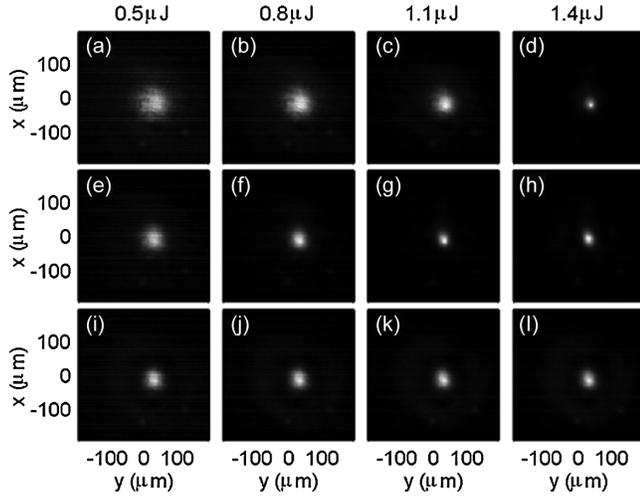


FIG. 5. Images of the 1500 nm beam collected at the glass sample output. (a),(b),(c),(d) Output for $E_{p1} = 0.5, 0.8, 1.1,$ and $1.4 \mu\text{J}$ input beam energies in the absence of the 1720 nm beam. (e),(f),(g),(h) and (i),(j),(k),(l) Same as (a),(b),(c),(d) for a collinear 1720 nm beam with $E_{p2} = 0.3 \mu\text{J}$ and $E_{p2} = 0.6 \mu\text{J}$ input energies, respectively.

in this space of parameters. This analysis also reveals that just one negative α may be sufficient to stabilize the collapse. We verify the stability properties by resorting to simulations of Eq. (4). As an example, we report in Fig. 3 the evolution of the peak intensity (a) and the beam waist (b) for the pump p_1 using as input two Gaussian beams for the pumps and no field for the idlers, as specified in the caption. Little evolution is found for $\Delta_{(1,2)} = -20$, where a solution close to the solitary beam is excited. The collapse is arrested for $\Delta_{(1,2)} < 0$ in all the cases presented here, although a strong self-focusing dynamic is evident for large negative values of $\Delta_{(1,2)}$, where the FWM is weaker. The collapse is enhanced for positive values of $\Delta_{(1,2)}$.

In the experiments, the two pumps are generated by an optical parametric amplifier OPA-800C, driven by a Ti:sapphire laser at 1 kHz repetition rate (Fig. 4). The source produces a signal and an idler of 1 ps duration, respectively, tunable in the ranges 1300–1600 nm and 1600–2100 nm. Our experimental data are recorded at $\lambda_{p1} = 1500$ nm and $\lambda_{p2} = 1720$ nm. We use a nonlinear Schott glass [34] (SF6), featuring normal dispersion around 1600 nm. The propagation length is 6 mm; note that the temporal dispersion length is 20 cm for the 1 ps pulses (the glass second order dispersion coefficient is $\approx 50 \text{ ps}^2/\text{km}$). The beam waists are $\approx 20 \mu\text{m}$ (full details are in the caption of Fig. 6). The output profiles are imaged by an InGaAs CCD camera (Xeva-1.7-640, Xenics). The 1720 nm pump damages the material at the critical energy E_{p2} of $1.8 \mu\text{J}$; at wavelength 1500 nm for energies E_{p1} above $1.4 \mu\text{J}$ the beam collapses in a strong flickering regime that we identify as the beam postcollapse. Such energy is consistent with the reported value of the nonlinear coefficient [34] $n_2 \approx 2 \times 10^{-19} \text{ cm}^2/\text{W}$.

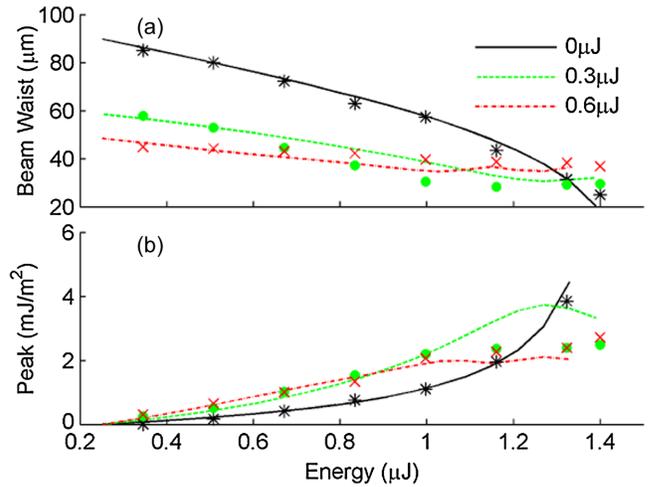


FIG. 6 (color online). Evolution of the 1500 nm output beam size vs input energy. Markers: experimental data for $E_{p2} = 0$ (stars, black), $E_{p2} = 0.3 \mu\text{J}$ (points, green) and $E_{p2} = 0.6 \mu\text{J}$ (crosses, red) input energies of a collinearly propagating 1720 nm beam. The beam size is extracted with a Gaussian fit of the output images; the accuracy is within the scale of the marker size. Lines: simulations of Eqs. (1) including the temporal dynamics of the pulse. For the simulation, we used Gaussian beams with waists $w_{p1} = 20 \mu\text{m}$ and $w_{p2} = 15 \mu\text{m}$ focused at 0 and 2.54 mm from the entrance, respectively, as measured by beam profiling. The pulse duration is 1 ps as measured by the autocorrelation. The propagation length is 6 mm, $\Delta k_1 = 0.7 \text{ mm}^{-1}$, and $\Delta k_2 = 4 \text{ mm}^{-1}$. The refractive index is $n = 1.76$ and the dispersion coefficient is $50 \text{ ps}^2/\text{km}$. The critical energy is $1.2 \mu\text{J}$, calculated as a fitting parameter of the single collapse beam curve. The energy values also include 20% of injection losses.

Figure 5 shows the output spatial intensities profiles of the 1500 nm beam at several input energies E_{p1} , for $E_{p2} = 0, 0.3$ and $0.6 \mu\text{J}$ energies of the 1720 nm beam (top-bottom). Figure 6 shows the output beam size vs the input pulse energy. The superimposed beam clearly enhances the self-focusing at low E_{p1} energies [Figs. 5(e)–5(g) and 5(i)–5(j)], where the XPM plays a major role, but at higher energies [Figs. 5(h) and 5(k)–5(l)] the self-focusing is clearly reduced, as the beam waist never reaches the minimum size measured close to the collapse threshold in the case of single-beam propagation. We could not detect a substantial drop in the beam output energy, indicating negligible nonlinear absorption. At larger energy values the beam manifests an unstable dynamic. These observations are consistent with the previous analysis. The experimental data are fitted by solving an extended version of Eqs. (1). We included the temporal dynamic propagation of the pulse by adding a second order dispersion term and solving the resulting 3D + 1 model in radial symmetry, with numerical parameters extracted by the Schott tables [34] and reported in the caption of Fig. 6.

In conclusion, we demonstrated a set of stable 2D + 1 quasisolitons in a pure Kerr media sustained by a FWM

interaction. The system involves the interaction of two pumps of different colors in a weak FWM regime with two idlers, and we have shown, theoretically and experimentally, collapse-free propagation in instantaneous Kerr media. The stabilization mechanism is given by higher order cascaded nonlinearities in normal dispersion. Notably, collapse-free propagation in a nonlinear Kerr material with normal dispersion also allows the generation of X waves [20,35,36], optical pulses with a characteristic X shape in the space-time or angle-wavelength domain. Although the study of X waves is beyond the scope of this Letter, the interpretation of the nonlinear phase exchange among the components in terms of higher order defocusing nonlinearity opens up new perspectives for understanding such waves.

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