Impact of Loss on the Wave Dynamics in Photonic Waveguide Lattices

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We analyze the impact of loss in lattices of coupled optical waveguides and find that, in such a case, the hopping between adjacent waveguides is necessarily complex. This results not only in a transition of the light spreading from ballistic to diffusive, but also in a new kind of diffraction that is caused by loss dispersion. We prove our theoretical results with experimental observations.

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Absorption is an intrinsic feature of photonic systems, arising due to the laws of causality [1]. It results in decoherence and, hence, in a considerable change in the dynamics of optical waves. However, it is generally agreed that, in the particular case of homogeneous and isotropic loss, the impact on the amplitude distribution in the system vanishes, besides a global decay of the integrated power [1]. A very prominent photonic system is arrays of evanescently coupled waveguides [2], where a tailored absorption (or absorption and gain) distribution is the basis for a multitude of unexpected physical phenomena, such as exceptional points [3], unusual beam dynamics [4], spontaneous PT-symmetry breaking [5], nonreciprocal Bloch oscillations [6] and dynamic localization [7], unidirectional cloaking [8], and even tachyonic transport [9]. Owing to the intuition described above, if all lattice sites exhibit exactly the same absorption, its impact vanishes in the evolution equations of these systems. In a more mathematical language, in this case absorption adds to the Hamiltonian as a pure diagonal matrix with identical elements, which can be removed by normalization.

In our work, we show that absorption in coupled waveguide systems does always impact the light dynamics, even if it is homogeneous and isotropic in all lattice sites. Because of the imaginary part of the dielectric function (that describes the absorption), imaginary off-diagonal elements in the Hamiltonian appear that cannot be removed by normalization, causing significant deviations in the light dynamics compared to the Hermitian case. However, our theory holds for all Schrödinger-type systems that can be mapped onto a tight-binding lattice, e.g., paraxial waves in optics or mechanics as well as quantum dynamics in spin chains, population transfer in multilevel systems, and graphene. Our theory supplements the knowledge about the influence of non-Hermiticity to all these systems, in general, including the effect of PT symmetry.

In order to study the impact of absorption in such systems, we consider a one-dimensional array of N identical single mode optical waveguides with width $2w$, intersite spacing d, and the complex relative electric permittivity $\epsilon + i\epsilon'$ at the positions x_n $(n = 1, 2, \ldots, N)$, which is surrounded by a bulk material (with $\epsilon_0 + i\epsilon'_0$). A sketch of this system is shown in Fig. 1.

The dynamics of wave propagating through this system is governed by the Helmholtz wave equation

$$[\nabla^2 + k_0^2\tilde{\varepsilon}(x)]\psi(x, z) = 0, \qquad (1)$$

where $\psi(x, z)$ is the electric field amplitude, $k_0 = \omega/c$ is the propagation constant in free space, and $\tilde{\varepsilon}(x)$ is the relative electric permittivity profile of the system. The relative electric permittivity distribution of the entire structure can be written as a sum of individual waveguide contributions, such that

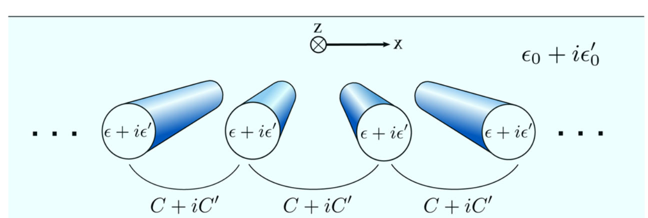

FIG. 1 (color online). One-dimensional array of identical absorbing optical waveguides. The complex relative electric permittivity of all waveguides is $\epsilon + i\epsilon'$, while the surrounding medium is fused silica with relative electric permittivity $\epsilon_0 + i\epsilon'_0$.

$$\tilde{\varepsilon}(x) = \epsilon_0 + i\epsilon_0' + \sum_{n=1}^{N} [(\epsilon - \epsilon_0) + i(\epsilon' - \epsilon_0')]\zeta_n(x). \tag{2}$$

Here, we used $\zeta_n(x) = H(x - x_n + w) - H(x - x_n - w)$ [with $H(x)$ as the Heaviside step function]. In the tight-binding approximation, the full field $\psi(x, z)$ can be written as a superposition of individual waveguide modes:

$$\psi(x, z) = \sum_{n=1}^{N} \phi_n(z) u(x - x_n) e^{i\beta z}, \tag{3}$$

where β is the waveguide's propagation constant $(k_0\sqrt{\epsilon_0} < \beta < k_0\sqrt{\epsilon})$, whereas $u(x - x_n)$ and $\phi_n(z)$ represent the normalized transverse mode profile and the field amplitude in the nth waveguide, respectively. After a somewhat lengthy but straightforward calculation (see Supplemental Material for details [10]), one obtains the coupled-mode equations for the light evolution in the non-Hermitian lattice:

$$-i\frac{d\phi_n}{dz} = i\kappa\phi_n + (C + iC')(\phi_{n+1} + \phi_{n-1}). \tag{4}$$

Here,

$$\kappa = \frac{k_0^2}{2\beta}\left[\epsilon_0' + (\epsilon' - \epsilon_0')\tanh\left(\frac{w}{\ell}\right)\right] \tag{5}$$

is the loss coefficient (ℓ is the width of the eigenmode), and

$$C = \frac{(\epsilon - \epsilon_0)k_0^2}{2\beta}\frac{w}{\ell}\exp\left(-\frac{d}{\ell}\right), \tag{6}$$

$$C' = \frac{(\epsilon_0' - \epsilon')k_0^2}{\beta}\frac{d}{\ell}\exp\left(-\frac{d}{\ell}\right) \tag{7}$$

represent the real and imaginary part of the intersite hopping rate, respectively. Note that the diagonal term $i\kappa\phi_n$ can be removed by the normalization $\phi_n = E_n e^{-\kappa z}$, whereas the off-diagonal terms $iC'\phi_n$ cannot. It is therefore evident that for *any* absorption present in the waveguides the light dynamics will be affected. Interestingly, for a given absorption profile, one finds the relation

$$C' = \alpha C \tag{8}$$

between the real and the imaginary part of the intersite hopping, with

$$\alpha = 2\frac{(\epsilon_0' - \epsilon')}{(\epsilon - \epsilon_0)}\frac{d}{w} \tag{9}$$

as the absorption discrepancy. Therefore, the imaginary part C' is always in a fixed ratio to the real part C of the hopping. Note that the absorption discrepancy itself is proportional to the intersite spacing d. We note that the absorption discrepancy α vanishes for $\epsilon_0' \to \epsilon'$, i.e., when not only the absorption in the lattice is homogeneous, but the absorption in the entire system (that is, in the lattice and the surrounding bulk material).

There are several important consequences arising from the appearance of an additional imaginary off-diagonal term in the Hamiltonian. First, we find that, for any loss discrepancy (i.e., $\alpha \neq 0$), the light spreading is ballistic for distances $z \ll z_{\mathrm{crit}}$ with

$$z_{\mathrm{crit}} = \frac{1}{4\alpha C} \tag{10}$$

but slows down to diffusive for $z \gg z_{\mathrm{crit}}$ (see Fig. 2). This can be seen by taking into account the Green's function of Eq. (4):

$$E_n(z) = i^n J_n[2(1 + i\alpha)Cz]. \tag{11}$$

The variance of this evolving wave packet is (see Supplemental Material for details on the calculation [10])

$$\sigma^2(z) = \left(\alpha + \frac{1}{\alpha}\right)Cz\frac{I_1(4\alpha Cz)}{I_0(4\alpha Cz)}, \tag{12}$$

which can be approximated as

$$\sigma^2(z) \overset{4\alpha Cz \ll 1}{\to} 2(1 + \alpha^2)C^2z^2 \quad \text{(ballistic)}, \tag{13}$$

$$\sigma^2(z) \overset{4\alpha Cz \gg 1}{\to} \left(\frac{1 + \alpha^2}{\alpha}\right)Cz \quad \text{(diffusive)}. \tag{14}$$

Hence, even for minimal loss decoherence effects impact the wave packet evolution, resulting eventually in a diffusive spreading behavior for sufficiently large propagation distances despite the fact that the lattice exhibits full translational symmetry.

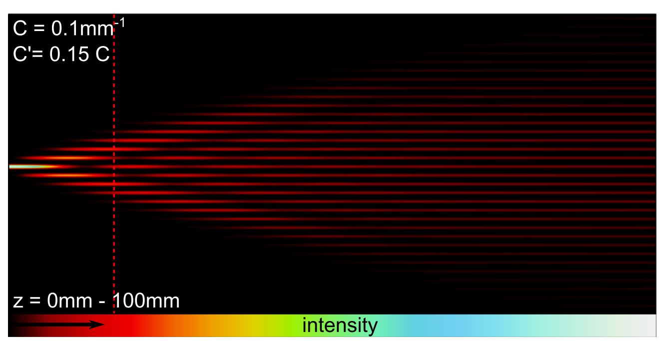

FIG. 2 (color online). Evolution in a waveguide array, where each waveguide exhibits the same loss, resulting in $\alpha = 0.15$. Clearly, after z_{crit} (red dashed line), the light spreading changes from ballistic to diffusive. The power is normalized to 1 at every z.

Importantly, for any given initial condition, the field evolution is completely controlled by the dispersion relation $k_z(k_x)$. It relates the longitudinal wave number k_z to the transverse wave number k_x (which we normalized by the lattice spacing) and determines how the individual Fourier components dephase during propagation. Following the coupled-mode equations for the normalized amplitudes E_n, the complex dispersion relation reads as

$$k_z(k_x) = 2C\cos(k_x) + i2C'\cos(k_x). \quad (15)$$

In order to study the impact of this dispersion relation on the light evolution, we follow the analysis performed in Ref. [11] and apply it to our complex dispersion. When a broad beam is launched into the lattice around a fixed central wave number $k_{x,0}$, the dispersion relation (15) can be expanded into a Taylor series:

$$k_z(k_x) \approx k_{z,0} + \gamma(k_x - k_{x,0}) + \frac{\delta}{2}(k_x - k_{x,0})^2, \quad (16)$$

with

$$\begin{aligned} k_{z,0} &= k_z(k_{x,0}) = 2C\cos(k_{x,0}) + i2C'\cos(k_{x,0}) \\ &= k_{z,r} + ik_{z,i}, \end{aligned} \quad (17)$$

$$\begin{aligned} \gamma &= \left.\frac{dk_z}{dk_x}\right|_{k_{x,0}} = -2C\sin(k_{x,0}) - i2C'\sin(k_{x,0}) \\ &= \gamma_r + i\gamma_i, \end{aligned} \quad (18)$$

$$\begin{aligned} \delta &= \left.\frac{d^2k_z}{dk_x^2}\right|_{k_{x,0}} = -2C\cos(k_{x,0}) - i2C'\cos(k_{x,0}) \\ &= \delta_r + i\delta_i. \end{aligned} \quad (19)$$

A plot of these quantities is shown in Fig. 3. As the formal solution of Eq. (4) is given by Fourier decomposition, inserting Eq. (16) into this solution shows that the evolution of broad beams can be described by the partial differential equation

$$\left[i\frac{\partial}{\partial z} - (i\gamma_r - \gamma_i)\frac{\partial}{\partial n} - \left(\frac{\delta_r}{2} + i\frac{\delta_i}{2}\right)\frac{\partial^2}{\partial n^2}\right]a(n,z) = 0 \quad (20)$$

of the distributed amplitude function

$$a(n,z) = \exp\{-i([k_{z,r} + ik_{z,i}]z + k_{x,0}n)\}E_n(z). \quad (21)$$

However, it is very important to note that the validity of Eq. (20) depends strongly on the approximation of the dispersion relation Eq. (16). If we assume that the center of mass of the normalized amplitudes $E_n(n,z)$ in the k_x space moves along $k_{x,c}(z)$ and has a variance $\Delta k_x^2(z)$, then our approximation of the dispersion relation limits the entire analysis to cases where

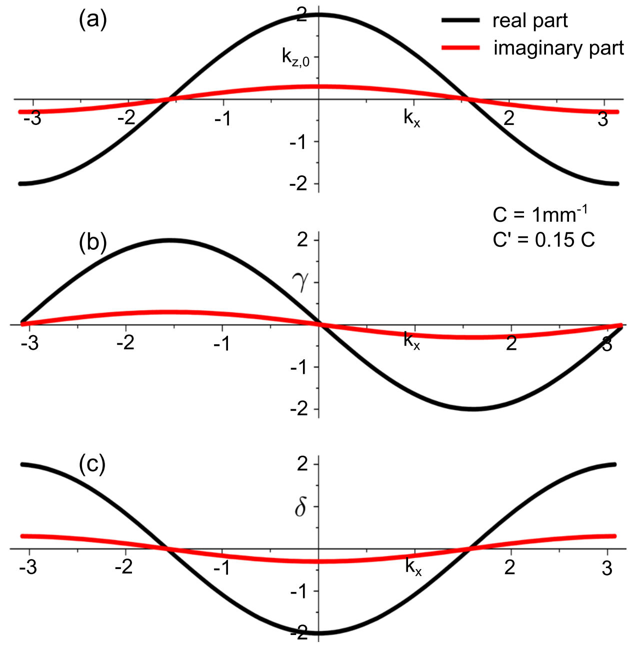

FIG. 3 (color online). (a) The real and imaginary parts of $k_z(k_{x,0})$. (b) The real and imaginary parts of $\gamma(k_{x,0})$. (c) The real and imaginary parts of $\delta(k_{x,0})$.

$$|k_{x,c}(z) - k_{x,0}| \ll 1 \quad \text{and} \quad \frac{1}{3!}[\Delta k_x(z)]^3 \ll 1 \quad (22)$$

(for a detailed argumentation of these requirements, see Supplemental Material [10]).

The impact of the dispersion relation on the evolution of broad beams is best illustrated when in Eq. (20) each term is individually analyzed, i.e., when only one quantity from the set $[\gamma_r, \gamma_i, \delta_r, \delta_i]$ is taken into account and the others are set to zero. Moreover, we would like to illustrate the new dynamics for an initially tilted Gaussian beam

$$E(n, z=0) = a_0 \exp\left(-\frac{n^2}{w_0^2} + ik_{x,0}n\right), \quad (23)$$

where w_0 is the initial beam width. In this case, the two conditions of Eq. (22) are equivalent to

$$\left|\frac{2\gamma_i z}{w_0^2 + 2\delta_i z}\right| \ll 1, \qquad w_0^2 + 2\delta_i z \gg 1. \quad (24)$$

For only $\delta_r \neq 0$, these two conditions simplify to $w_0 \gg 1$, such that Eq. (20) reduces to

$$i\frac{\partial}{\partial z}a(n,z) = \frac{\delta_r}{2}\frac{\partial^2}{\partial n^2}a(n,z), \quad (25)$$

which is the paraxial wave equation. Therefore, δ_r represents the *diffraction strength* and can be positive or negative, depending on the transverse wave number $k_{x,0}$, i.e., the initial tilt of the beam. Importantly, the beam width always increases for both $\delta_r > 0$ and $\delta_r < 0$ and stays constant for $\delta_r = 0$. The term $i\gamma_r \partial/\partial n$ in Eq. (20) can be removed by the coordinate transformation $n \to n + \gamma_r z$, suggesting that γ_r is a *group velocity* (which can be also positive or negative, depending on $k_{x,0}$). This is consistent with the Hermitian case [11]. However, in the non-Hermitian case, there are two more quantities [γ_i and δ_i]. Interestingly, when taking into account only δ_i, Eq. (20) reduces to

$$\frac{\partial}{\partial z} a(n,z) = \frac{\delta_i}{2} \frac{\partial^2}{\partial n^2} a(n,z), \tag{26}$$

which is a diffusion equation. Therefore, the quantity δ_i can be associated with a *diffusion coefficient*. The solution of this equation for the initial condition (23) reads as

$$a(n,z) = a_0 \frac{w_0}{w(z)} \exp\left(-\frac{n^2}{w^2(z)}\right), \tag{27}$$

with the beam width

$$w(z) = \sqrt{w_0^2 + 2\delta_i z}. \tag{28}$$

According to Eq. (24), this result is valid if

$$w_0^2 + 2\delta_i z \gg 1. \tag{29}$$

Also δ_i can be positive or negative; however, the beam behaves differently in both cases (in contrast to δ_r). For $\delta_i > 0$, condition (29) is always satisfied for broad input beams, and one finds that

$$w(z)|_{z\to\infty} \to \sqrt{2\delta_i z}, \tag{30}$$

which indeed characterizes diffusive broadening. For $\delta_i < 0$, in contrast, condition (29) is satisfied only for $z \ll w_0^2/2|\delta_i|$. At larger distances, the expansion Eq. (16) is not valid anymore, and standard discrete diffraction [2] dominates the light evolution. Finally, when only the quantity γ_i is taken into account, the conditions for the validity of Eq. (20) are $w_0 \gg 1$ and $z \ll w_0^2/2|\gamma_i|$. The evolution of the Gaussian input beam is then described by

$$\frac{\partial}{\partial z} a(n,z) = i\gamma_i \frac{\partial}{\partial n} a(n,z), \tag{31}$$

yielding the solution

$$a(n,z) = a_0 \exp\left(-\frac{2i\gamma_i z n - (\gamma_i z)^2 + n^2}{w_0^2}\right). \tag{32}$$

Hence, one can clearly see that γ_i causes only a deformation of the phase front but leaves the general intensity profile of the beam unchanged. Both δ_i as well as γ_i are intrinsic features of the appearance of a complex coupling coefficient. Consequently, the existence of a diffusive mobility regime does not rely on a $\mathcal{PT}$-symmetric loss distribution expressed on the diagonal of the Hamiltonian [12]. Even the homogeneous loss, before thought to cause only a global, exponential decay, will eventually force the wave function to diffuse due to the so far not considered imaginary part of the off-diagonal elements.

In order to prove the existence of the diffusive spreading in waveguide lattices with a homogeneous loss distribution (i.e., full translational symmetry), we perform experiments in laser-written waveguide arrays in fused silica glass [13]. For the fabrication of the waveguides, we tightly focus ultrashort laser pulses (wavelength 515 nm, pulse duration 308 fs, average power 222 mW, repetition rate 100 kHz) by using a 40× objective into a 10 cm long fused silica glass wafer, which is transversely translated with 250 mm/ min by using a high-precision positioning system. Each waveguide lattice consists of 45 waveguides, and the spacing between the waveguides is 17 μm, which corresponds to $C = 0.1\ \mathrm{mm}^{-1}$. We analyze the light evolution in the structure by launching light at $\lambda = 633$ nm into the central guide by using fiber butt coupling and observe the light evolution by a fluorescence microscope technique [14]. The light evolution in the lossless array is shown in Fig. 4(a), exhibiting clearly ballistic spreading. The situation changes when strong loss is introduced to the waveguides. This is done by writing the waveguides in a sinusoidal fashion [12] with an amplitude (perpendicular

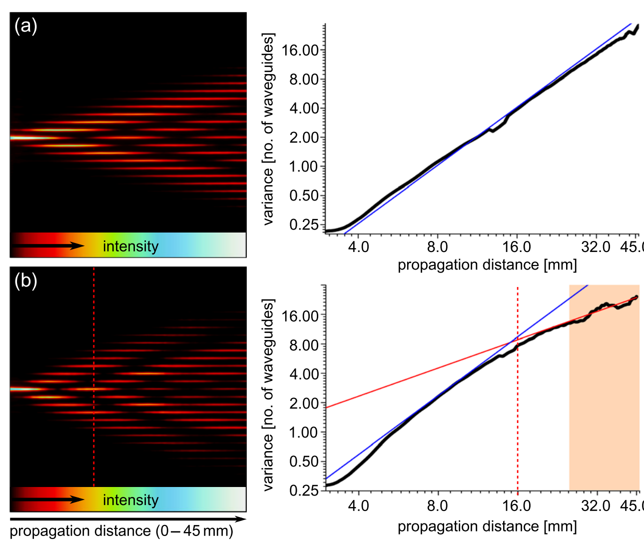

FIG. 4 (color online). (a) Experimental light evolution in a waveguide lattice with negligible loss (left panel). Plotting the extracted variance as a function of the propagation distance z in a double-logarithmic plot results in a straight line, which has in the ballistic case a slope of 2, represented by the blue line (right panel). (b) Experimental light evolution in a waveguide lattice with loss $\alpha = 0.16$ (left panel). In the double-logarithmic plot of the extracted variance, one sees a transition from slope 2 (ballistic, blue line) to slope 1 (diffusive, red line). The latter was fitted by using data from the orange area, resulting in a slope of 0.96, which is very close to the theoretical value of 1. In both panels, z_{crit} is indicated by a red dashed line.

to x) of 3 μm and a period of 3 mm, which enhances the radiation losses of the guides. In this case, it is $C' \neq 0$, and, hence, the spreading of the light field should change from ballistic to diffusive after a particular propagation distance. This is exactly what we observe in the experiment, which is shown in Fig. 4(b). The transition occurs after $z_{\mathrm{crit}} \approx 16$ mm, which implies $\alpha = 0.16$ [according to Eq. (10)]. This is the experimental proof that, although all waveguides exhibit the same loss, in Eq. (4) not only the on-diagonal loss term κ has to be taken into account, but also the off-diagonal imaginary coupling C' that cannot be removed by normalization.

In conclusion, we have shown that if losses are present in a photonic waveguide lattice exhibiting translational symmetry, the intersite coupling is complex. This results in a modified dispersion relation with an additional band due to the complex coupling. As a further consequence, the light spreading slows down from ballistic to diffusive after a characteristic propagation distance that is determined by the loss. We believe that our findings have a fundamental impact on the understanding of light evolution in non-Hermitian lattices, in particular, those with space-time reflection (PT) symmetry [15,16]. Consequently, the loss effect on transport [17], which could lead to regimes such as sub- or super-diffusive or even superballistic ones [18], in addition to the changes on the band structure as a result of higher order couplings [19] are open questions for further investigations [12]. It is also interesting to study the impact of our results on the two-dimensional array of waveguides as candidates for ultrahigh-capacity optical communications [20] or a spatio-temporal vortex soliton (a result of the nonlinear Kerr effect) [21,22] and their dynamical properties.

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