Scalable Boson Sampling with Time-Bin Encoding Using a Loop-Based Architecture

Keith R. Motes,[1] Alexei Gilchrist,[1] Jonathan P. Dowling,[2] and Peter P. Rohde[1,*]
[1]Centre for Engineered Quantum Systems, Department of Physics and Astronomy, Macquarie University, Sydney, New South Wales 2113, Australia
[2]Hearne Institute for Theoretical Physics and Department of Physics and Astronomy, Louisiana State University, Baton Rouge, Louisiana 70803, USA
(Received 19 March 2014; published 18 September 2014)

We present an architecture for arbitrarily scalable boson sampling using two nested fiber loops. The architecture has fixed experimental complexity, irrespective of the size of the desired interferometer, whose scale is limited only by fiber and switch loss rates. The architecture employs time-bin encoding, whereby the incident photons form a pulse train, which enters the loops. Dynamically controlled loop coupling ratios allow the construction of the arbitrary linear optics interferometers required for boson sampling. The architecture employs only a single point of interference and may thus be easier to stabilize than other approaches. The scheme has polynomial complexity and could be realized using demonstrated present-day technologies.

DOI: 10.1103/PhysRevLett.113.120501 PACS numbers: 03.67.Lx, 42.50.Dv, 42.50.Ex, 42.65.Lm

Boson sampling, first presented by Aaronson and Arkhipov (AA) [1], is a nonuniversal approach to linear optics quantum computing (LOQC) [2,3]. While not universal, boson sampling is believed to implement a classically hard algorithm using far fewer physical resources than full LOQC. In this model we prepare p single photons in n modes, which are propagated through a passive linear optics network $\hat{U}$. Finally, we measure the output state using coincidence photodetection. The experiment is repeated many times, thereby building up statistics of the output photon-number distribution. This sampling problem was shown by AA to likely be a classically hard problem and might only require dozens of photons and hundreds of optical elements, which is a significant improvement over universal LOQC. Several elementary experimental demonstrations of boson sampling have recently been performed [4–8]. For a more detailed elementary introduction to boson sampling see Ref. [9].

Presently, various technologies are available for preparing single photon states (e.g., spontaneous parametric down-conversion, which has been shown to be viable for boson sampling [10,11]), and performing photodetection (e.g., avalanche photodiodes). The remaining central challenge in boson sampling is constructing linear optics networks $\hat{U}$. It was shown by Reck *et al.* [12] that arbitrary n-mode linear optics networks can be decomposed into a sequence of $O(n^2)$ beam splitters. In present-day experiments this type of decomposition is implemented using waveguides or discrete optical elements.

In current implementations, the modes in a boson-sampling interferometer are spatial modes, whereby all photons must have simultaneous arrival times. Another alternative is to employ time-bin encoding, whereby n single photons form a "pulse train" within a single spatial mode. In the architecture we present here, we will employ time-bin encoding.

We begin by triggering a single photon source at time intervals τ (the source's repetition rate), which prepares a pulse train of n single photons across a length of fiber. The first step in our architecture is to propagate the pulse train through a fiber loop with dynamically controlled coupling ratios, as shown in Fig. 1(a). The loop's coupling ratio is dynamically controlled by a variable reflectivity beam splitter, implementing the unitary,

$$U_{\mathrm{BS}}(t) = \begin{bmatrix} \gamma_{1,1}(t) & \gamma_{1,2}(t) \\ \gamma_{2,1}(t) & \gamma_{2,2}(t) \end{bmatrix}, \tag{1}$$

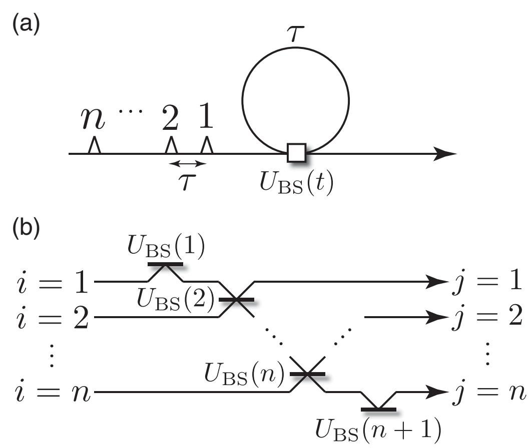

FIG. 1. (a) A fiber loop fed by a pulse train of single photons, each separated in time by τ. The box represents a dynamically controlled, variable reflectivity beam splitter (BS). The switching time of the beam splitter must be less than τ to allow each time bin to be individually addressed. (b) Expansion of the fiber loop architecture into its equivalent beam splitter network.

at time t, which splits the incident field into a component entering the loop and a component exiting the loop. The component entering the loop takes time τ to transverse the loop such that it coincides with the subsequent pulse. In order for the first photon to interfere with every photon pulse it will traverse this loop n times. The second photon will traverse the loop $n-1$ times and so on. The dynamics of photons propagating through the loop architecture may be "unravelled" into an equivalent series of beam splitters acting on spatial modes, as shown in Fig. 1(b). This elementary network is the basic building block employed by our architecture.

The effective unitary map applied to the time bins through a single such loop is

$$U_{i,j} = \begin{cases} 0 & i > j+1 \\ \gamma_{2,1}(i) & i = j+1 \\ \gamma_{2,2}(i)\gamma_{1,1}(j+1)\prod_{k=i+1}^{j}\gamma_{1,2}(k) & i < j+1 \end{cases}, \tag{2}$$

where i and j represent the input and output modes, respectively. Here we have imposed the boundary condition that $U_{\mathrm{BS}}(1)$ and $U_{\mathrm{BS}}(n+1)$ are completely reflective, coupling all of the first photon into the loop, and ensuring that all of the field remains trapped in a finite time window. We see that the probability of finding a photon in the jth mode decays exponentially with j.

Evidently, the network shown in Fig. 1(b) is not sufficient for universal linear optics networks as it contains many zero elements. To make the scheme universal we must show that the ingredients necessary to perform a full Reck *et al.* type decomposition are available.

To understand the equivalent beam splitter representation of a single loop, consider Fig. 2(a). The pulse train enters the loop, where the numbers on the left represent the corresponding time bin. The first photon is deterministically coupled into the loop as depicted by an open circle. After the first and second photons interact some of the amplitude may escape the loop, which corresponds to the first output time bin. The pulse train continues to interact through the loop via beam splitter operations, which are represented as closed circles. After the nth photon transverses the loop any remaining amplitude deterministically leaves the loop, which corresponds to the nth output time bin.

Now consider Fig. 2(b), which depicts how three consecutive loops in series with three input photons produce an equivalent beam splitter network. The lengths of the black lines represent time in units of τ. The three modes on the left represent the pulse train of photons at the input of the device at the first round-trip. The first photon reaches the first beam splitter at $\tau = 1$, the second photon reaches it at $\tau = 2$, and so on. The photons travel through the fiber loop network interacting arbitrarily, which yields

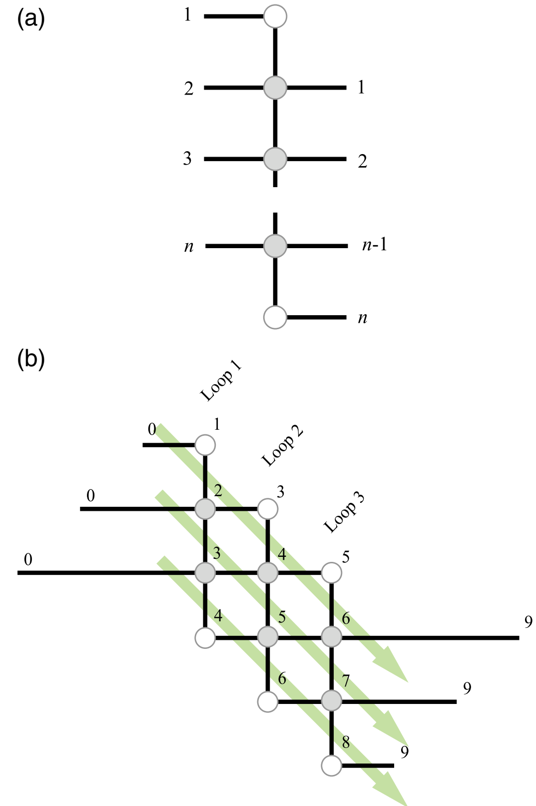

FIG. 2 (color online). The equivalent beam splitter representation for the fiber loop architecture. (a) A single loop is represented for n photons in the pulse train. (b) The equivalent beam splitter network of three consecutive loops with three input modes.

an arbitrary Reck *et al.* style decomposition. Evidently, an n-mode unitary can be built using n modes and $n-1$ loops. In Figs. 3 and 4 we show an alternate proof based on an inductive argument.

We have shown that a series of consecutive fiber loops can implement an arbitrary sequence of pairwise beam splitter operations. Next, we recognize that each of these fiber loops requires exactly the same physical resources, only differing by the switch's control sequence. We need not physically build each of these identical loops. Rather, we will embed the loop into a larger fiber loop of length $> n\tau$, as shown in Fig. 5. The larger loop is controlled by another two switches, which control the number of round trips in the larger loop. From the result of Reck *et al.* we know that $O(n^2)$ optical elements are required to construct an arbitrary $n \times n$ interferometer. Thus, the number of round trips of the outer loop is $O(n^2)$.

An experimental simplification is when we do not require full dynamic control over the beam splitter ratio. Although this scenario is not universal, it may be possible to construct useful classes of unitaries. We will consider the situation where the beam splitter can be toggled between two settings—completely reflective, or some other arbitrary fixed ratio. The former is required to allow that the time

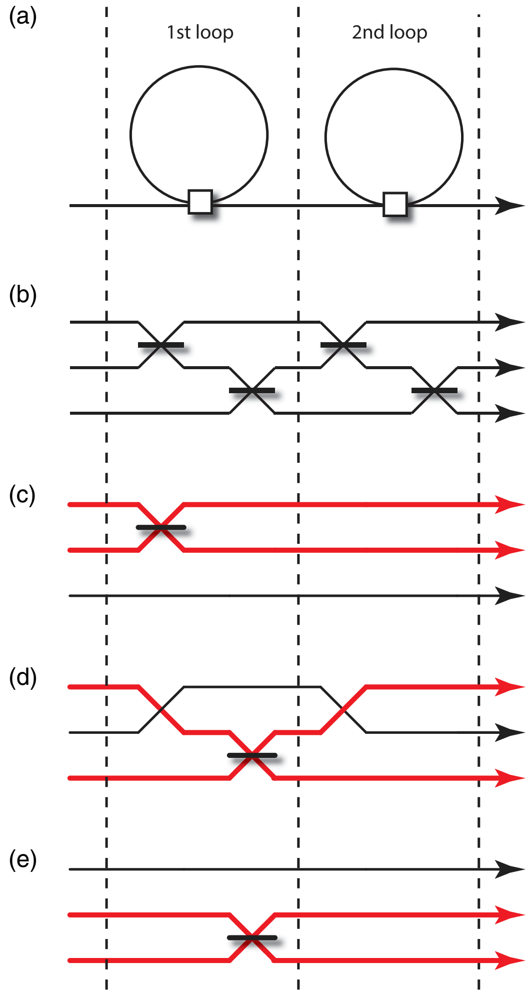

FIG. 3 (color online). (a) Two consecutive fiber loops in series. (b) The equivalent beam splitter expansion for $n = 3$. By setting the beam splitter ratios appropriately the two loops can implement an arbitrary beam splitter between any pair of modes: (c) modes 1 and 2, (d) modes 1 and 3, (e) modes 2 and 3.

bins be restricted to a finite time window, while the latter implements the "useful" beam splitter operations. We may have an arbitrary number of such loops in series, each with a potentially different fixed beam splitter ratio.

Intuitively, we expect that a "maximally mixing" unitary (i.e., one with equal amplitudes between every input to output pair) would implement a classically hard boson-sampling instance, as it maximizes the combinatorics associated with calculating output amplitudes. If, for example, a unitary is heavily biased towards certain output modes, or is sparse, the combinatorics are reduced. Specifically, we define a balanced unitary as $|U'_{i,j}|^2 = 1/n \forall i, j$, such that, up to phase, all amplitudes are equal.

In Fig. 6 we take the unitary implemented by a series of m fixed-ratio fiber loops, and compare it with the balanced unitary $\hat{U}'$. We characterize the uniformity of the obtained unitary using the similarity metric,

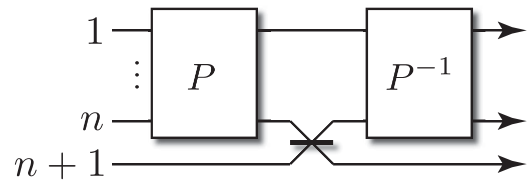

FIG. 4. Generalizing the universality argument presented in Fig. 3 to arbitrary n. We choose a permutation that the first of the desired modes be routed to the beam splitter, which interacts with mode $n + 1$. Then the inverse permutation is applied, leaving us with a network that implements an arbitrary beam splitter operation between one of the first n modes and the $(n + 1)$th mode. It follows inductively that an arbitrary beam splitter operation can be applied between any pair of modes for any n.

$$S = \max_{\hat{U}_{\mathrm{BS}}(t)\forall t}\left[\frac{\left(\sum_{i,j}\sqrt{|U_{i,j}|^2|U'_{i,j}|^2}\right)^2}{(\sum_{i,j}|U_{i,j}|^2)(\sum_{i,j}|U'_{i,j}|^2)}\right]$$
$$= \max_{\hat{U}_{\mathrm{BS}}(t)\forall t}\left[\frac{1}{n^3}\left(\sum_{i,j}|U_{i,j}|\right)^2\right], \tag{3}$$

where we maximize S by performing a Monte Carlo simulation over different beam splitter ratios, $\hat{U}_{\mathrm{BS}}$. That is, S tells us how close $\hat{U}$ is to uniform, with $S = 1$ being completely uniform up to phase. With a sufficient number of loops in series, we obtain very high similarities, suggesting that the simplified architecture may implement hard instances of boson sampling.

The presented universal architecture is in principle arbitrarily scalable, provided the length of the larger loop is sufficiently high ($> n\tau$). However, in practice, fiber is lossy with present-day technology. If we let η_{inner} be the net efficiency of the inner loop (i.e., the probability that an incident photon will reach the output), and η_{outer} be the net efficiency of the outer loop, then the worst case net efficiency of the device is $\eta_{\mathrm{net}} = (\eta_{\mathrm{inner}}{}^n\eta_{\mathrm{outer}})^{O(n^2)}$, which scales exponentially with n. Thus, to construct large interferometers using this architecture will require

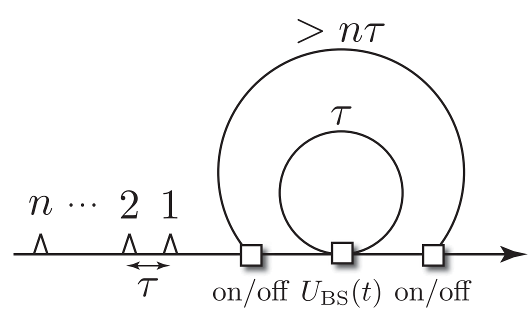

FIG. 5. The complete architecture. The consecutive series of length τ fiber loops is collapsed into a single length τ fiber loop embedded inside a length $> n\tau$ fiber loop. The outer loop allows an arbitrary number of the smaller loops to be applied consecutively.

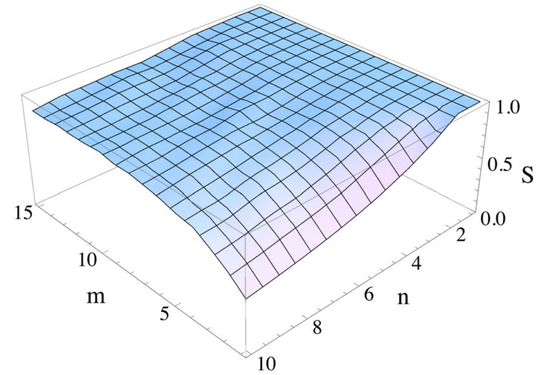

FIG. 6 (color online). The maximum similarity S between $\hat{U}$ and the uniform unitary $\hat{U}'$ after m loops with n input photons. The beam splitter ratio is fixed for each loop but independently, randomly chosen for each loop. This demonstrates that near-uniform unitaries may be constructed with sufficient loops.

exponentially low loss rates. This is also the case for conventional spatially encoded implementations. However, it was shown by Rohde and Ralph [13] that boson sampling might remain a computationally hard problem even in the presence of high loss rates. Other error models, such as dephasing or mode-mismatch [14], exhibit similar scaling characteristics.

Our architecture has the same efficiency scaling as conventional bulk-optics or waveguide implementations. If the worst-case photon efficiency (combining state preparation, evolution, and photodetection) is ζ, then with n photons the net efficiency is lower bounded by ζ^n.

Because there is only a single point of interference, this architecture may be significantly easier to stabilize and mode match than conventional approaches, where $O(n^2)$ independent beam splitters must be simultaneously aligned and stabilized. At this point of interference, the dominant source of error will be temporal mode mismatch [15], which is caused by errors in the lengths of the fiber loops, or time jitter in the photon sources. Temporal mismatch may be regarded as a displacement in the temporal wave packet of the photons [16]. Let us assume that at each round trip the photon exiting the inner loop is mismatched by time Δ. Over short time scales this yields dephasing [17], and over longer time scales, ambiguity as to which time bin the photon resides in. The worst case is that a given photon undergoes temporal mismatch of magnitude $n\Delta$. Time-bin ambiguity occurs when $n\Delta \geq \tau$, which yields the requirement that $n < \tau/\Delta$. Over shorter time scales, temporal mode mismatch is equivalent to dephasing as mismatched photons yield which-path information. This leads to the constraint that $n\Delta \ll \sigma$, where σ is the width of the photons' wave packets. Thus, time jitter or temporal mode mismatch must be kept small relative to the scale of the photons' wave packets. The current switching rates of state-of-the-art dynamically controlled switches is on the order of GHz [18–21] and the temporal spacing of photons is on the order of nanoseconds. While these switches are fast enough, they require additional coupling that involves high loss. This will encourage further development of these type of technologies which is also required for LOQC architectures.

In principle, the fiber loops could be replaced by any quantum memory or delay line such as propagation in free space, which would be significantly less lossy. In this case, the dominant source of loss would be in the dynamic switches, which, using present-day technology, have high loss rates.

Integrated waveguides are gaining popularity in photonics as they are inherently very stable. However, although interferometrically stable after the fabrication process, there are nonetheless $O(n^2)$ points of interference, which must be carefully aligned. On the other hand, the fiber-loop architecture has only a single point of interference that needs to be aligned. Another advantage of our architecture is that only one photon source (such as a quantum dot or SPDC source with high repetition rate) could be employed, whereas bulk optics or waveguide implementations would require an array of sources operating in parallel, further reducing the experimental overhead.

The experimental viability of loop-based photonic architectures was validated by recent quantum walk [22] experiments by Schreiber *et al.* [23,24], where quantum memories were implemented via delay lines in free space. It was also shown by Donohue *et al.* [25] that transmitting time-bin encoded photons in optical fibers is a robust form of optical quantum information given that the separation of time bins is larger than the time resolution of the detector. It was pointed out that the unitary may be simplified since Boson Sampling requires only n photons amongst $O(n^2)$ modes. With a Reck *et al.* unitary decomposition and input photons at the apex of the beamsplitter triangle, a subset of beamsplitters may be omitted as they have vacuum inputs [26,27].

We have presented an arbitrarily scalable architecture for universal boson sampling based on two nested fiber loops. The complexity of the architecture is constant, independent of the size of the interferometer being implemented. Scalability is limited only by fiber and switch transmission efficiencies. There is only one point of interference in the architecture, which suggests that it may be significantly easier to stabilize than traditional approaches based on waveguides or discrete elements. We also considered an experimental simplification where full dynamic control is not required and showed that, while not universal, with sufficient loops the unitary approximates a maximally mixing unitary. While we have specifically considered this architecture in the context of boson sampling, the same scheme, or variations on it, may lend themselves to other linear optics applications, such as interferometry, metrology, or full-fledged LOQC.

This research was conducted by the Australian Research Council Centre of Excellence for Engineered Quantum

Systems (Project No. CE110001013). J. P. D. would like to acknowledge the Air Force Office of Scientific Research, the Army Research Office, and the National Science Foundation. We would like to thank Michael Steel for helpful discussions.

*dr.rohde@gmail.com; http://www.peterrohde.org

[1] S. Aaronson and A. Arkhipov, The computational complexity of linear optics, *Proceedings of the 43rd annual ACM symposium on Theory of computing, 2011, San Jose* (ACM, New York, 2011), p. 333.
[2] P. Kok and B. W. Lovett, *Introduction to Optical Quantum Information Processing* (Cambridge University Press, Cambridge, England, 2010).
[3] E. Knill, R. Laflamme, and G. Milburn, Nature (London) **409**, 46 (2001).
[4] A. Peruzzo, M. Lobino, J. C. F. Matthews, N. Matsuda, A. Politi, K. Poulios, X.-Q. Zhou, Y. Lahini, N. Ismail, K. Wrhoff *et al.*, Science **329**, 1500 (2010).
[5] M. A. Broome, A. Fedrizzi, S. Rahimi-Keshari, J. Dove, S. Aaronson, T. C. Ralph, and A. G. White, Science **339**, 794 (2013).
[6] M. Tillmann, B. Daki, R. Heilmann, S. Nolte, A. Szameit, and P. Walther, Nat. Photonics **7**, 540 (2013).
[7] A. Crespi, R. Osellame, R. Ramponi, D. J. Brod, E. F. Galvao, N. Spagnolo, C. Vitelli, E. Maiorino, P. Mataloni, and F. Sciarrino, Nat. Photonics **7**, 545 (2013).
[8] J. B. Spring, B. J. Metcalf, P. C. Humphreys, W. S. Kolthammer, X.-M. Jin, M. Barbieri, A. Datta, N. Thomas-Peter, N. K. Langford, D. Kundys *et al.*, Science **339**, 798 (2013).
[9] P. P. Rohde and T. C. Ralph, Phys. Rev. A **85**, 022332 (2012).
[10] K. R. Motes, J. P. Dowling, and P. P. Rohde, Phys. Rev. A **88**, 063822 (2013).
[11] A. P. Lund, A. Laing, S. Rahimi-Keshari, T. Rudolph, J. L. O'Brien, and T. C. Ralph, arXiv:1305.4346 [Phys. Rev. Lett. (to be published)].
[12] M. Reck, A. Zeilinger, H. J. Bernstein, and P. Bertani, Phys. Rev. Lett. **73**, 58 (1994).
[13] P. P. Rohde and T. C. Ralph, Phys. Rev. A **85**, 022332 (2012).
[14] P. P. Rohde, Phys. Rev. A **86**, 052321 (2012).
[15] P. P. Rohde and T. C. Ralph, Phys. Rev. A **71**, 032320 (2005).
[16] P. P. Rohde, W. Mauerer, and C. Silberhorn, New J. Phys. **9**, 91 (2007).
[17] P. P. Rohde and T. C. Ralph, Phys. Rev. A **73**, 062312 (2006).
[18] P. Winzer, A. Gnauck, C. Doerr, M. Magarini, and L. Buhl, J. Lightwave Technol. **28**, 547 (2010).
[19] P. Schindler, D. Korn, C. Stamatiadis, M. OKeefe, L. Stampoulidis, R. Schmogrow, P. Zakynthinos, R. Palmer, N. Cameron, Y. Zhou *et al.*, J. Lightwave Technol. **32**, 760 (2014).
[20] K. Prosyk, A. Ait-Ouali, C. Bornholdt, T. Brast, M. Gruner, M. Hamacher, D. Hoffmann, R. Kaiser, R. Millett, K.-O. Velthaus *et al.*, in *Optical Fiber Communication Conference* (Optical Society of America, Washington, DC, 2012), p. OW4F–7.
[21] M. Steel (private communication).
[22] Y. Aharonov, L. Davidovich, and N. Zagury, Phys. Rev. A **48**, 1687 (1993).
[23] A. Schreiber, A. Gábris, P. P. Rohde, K. Laiho, M. Štefaňák, V. Potoček, I. Jex, and C. Silberhorn, Science **336**, 55 (2012).
[24] A. Schreiber, K. N. Cassemiro, V. Potoček, A. Gábris, P. J. Mosley, E. Andersson, I. Jex, and C. Silberhorn, Phys. Rev. Lett. **104**, 050502 (2010).
[25] J. M. Donohue, M. Agnew, J. Lavoie, and K. J. Resch, Phys. Rev. Lett. **111**, 153602 (2013).
[26] G. K. Brennen, S. S. Bullock, D. P. O'Leary, Quantum Inf. Comput. **6**, 436 (2006).
[27] D. Berry, (private communication).