Nonclassical Paths in Quantum Interference Experiments

Rahul Sawant,[1] Joseph Samuel,[1] Aninda Sinha,[2] Supurna Sinha,[1] and Urbasi Sinha[1,3,*]
[1]Raman Research Institute, Sadashivanagar, Bangalore 560 080, India
[2]Centre for High Energy Physics, Indian Institute of Science, Bangalore 560 012, India
[3]Institute for Quantum Computing, 200 University Avenue West, Waterloo, Ontario N2L 3G1, Canada
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In a double slit interference experiment, the wave function at the screen with both slits open is not exactly equal to the sum of the wave functions with the slits individually open one at a time. The three scenarios represent three different boundary conditions and as such, the superposition principle should not be applicable. However, most well-known text books in quantum mechanics implicitly and/or explicitly use this assumption that is only approximately true. In our present study, we have used the Feynman path integral formalism to quantify contributions from nonclassical paths in quantum interference experiments that provide a measurable deviation from a naive application of the superposition principle. A direct experimental demonstration for the existence of these nonclassical paths is difficult to present. We find that contributions from such paths can be significant and we propose simple three-slit interference experiments to directly confirm their existence.

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Quantum mechanics has been one of the most successful theories of the twentieth century, both in describing fundamental aspects of modern science as well as in pivotal applications. However, in spite of these obvious triumphs, there is universal agreement that there are aspects of the theory which are counterintuitive and perhaps even paradoxical. Furthermore, understanding fundamental problems involving dark matter and dark energy [1,2] in cosmology may need a consistent quantum theory of gravity. Unification of quantum mechanics and general relativity towards a unified theory of quantum gravity [3,4] is the holy grail of modern theoretical physics. Such unification attempts involve modifications of either or both theories. However, all such attempts would rely very strongly on precise knowledge and understanding of the current versions of both theories. This makes precision tests of fundamental aspects of both quantum mechanics and general relativity very important to provide guiding beacons for theoretical development.

The double slit experiment (Fig. 1) is one of the most beautiful experiments in physics. In addition to its pivotal role in optics, it is frequently used in classic textbooks on quantum mechanics [5–7] to illustrate basic principles. Consider a double slit experiment with incident particles (e.g., photons, electrons). The wave function at the detector with slit A open is ψ_A. The wave function with the slit B open is ψ_B. What is the wave function with both slits open? It is usually assumed to be $\psi_{AB} = \psi_A + \psi_B$ [5–7]. This is illustrated in Fig. 1. From the mathematical perspective of solving the Schrödinger equation, this assumption is definitely not true. The three cases described above correspond to three different boundary conditions [8,9] and as such the application of the superposition principle can at best be approximate. Recent numerical simulations of Maxwell's equations using finite difference time domain analysis have shown this to be true in the classical domain [9]. How do we quantify this effect in quantum mechanics?

An intuitive and simple way of understanding this problem is to appeal to Feynman's path integral formalism [10]. The path integral formalism involves an integration over all possible paths that can be taken by the particle through the two slits. This not only includes the nearly straight paths from the source to the detector through either slit (the classical paths) like the green paths in Fig. 2 but also includes paths of the type shown in purple in Fig. 2 (nonclassical paths). These looped paths are expected to make a much smaller contribution to the total intensity at the detector screen as opposed to the contribution from the straight line paths. However, their contribution is finite. Formally, a classical path is one that extremizes the classical action. Any other path is a nonclassical path. This leads to a modification of the wave function at the screen which now becomes

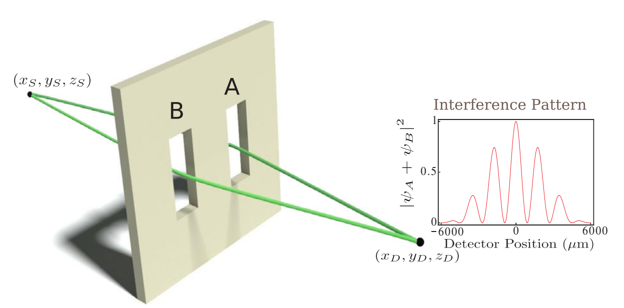

FIG. 1 (color online). Two-slit experiment. Inset shows a typical interference pattern obtained by assuming $\psi_{AB} = \psi_A + \psi_B$.

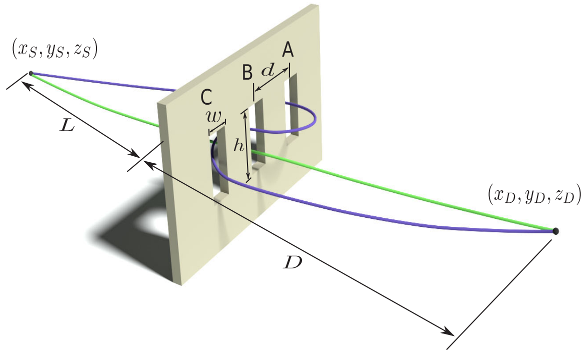

FIG. 2 (color online). Path integrals in a lab. The green line demonstrates a representative classical path. The purple line demonstrates a representative nonclassical path. The various length parameters are marked; d designates the interslit distance, w designates the slit width, h designates the slit height, L designates the distance from the source to the slit plane, and D designates the distance from the slit plane to the detector plane.

$$\psi_{AB} = \psi_A + \psi_B + \psi_L, \tag{1}$$

where ψ_L is the contribution due to the looped. i.e., nonclassical paths. That ψ_L is nonzero was first pointed out in [8] in the nonrelativistic domain where certain unphysical approximations were made in computing ψ_L and hence the results or the methods cannot be used in an experimental situation. Recently, the authors of [9] have reiterated the point that ψ_L can be nonzero without attempting to quantify it in quantum mechanics.

In this paper, we will quantify the effect of such nonclassical paths in interference experiments, thus quantifying the deviation from the common but incorrect application of the superposition principle in different possible experimental conditions. A well-known example of a direct experimental demonstration of such nonclassical paths involves the measurement of the Aharonov-Bohm phase [11]. Berry's "many-whirls" representation [12] provides insight into simple explanations of the Aharonov-Bohm effect in terms of an interference between whirling waves passing around the flux tube. However, in most experimental attempts to measure the Aharonov-Bohm phase, the detection relies on rather complicated experimental architecture and the results are also open to interpretational issues and further discussion [13,14]. In this work, we propose simple triple slit based interference experiments [15] which can be used as tabletop demonstrations of nonclassical paths in the path integral formalism. Nonclassical paths have been used to compute the semiclassical off-diagonal contributions to the two-point correlation function of a quantum system whose classical limit is chaotic [16]. The paths in this case are real. In the Feynman path integral approach, all possible paths going from the initial to final state need to be considered with an appropriate weight. In this sense all paths are real although in a physical quantity the contribution from certain paths may be suppressed.

The triple slit experiment provides a simple way to quantify the effects from nonclassical paths in terms of directly measurable quantities. The triple slit (path) setup has been used as a test bed for testing fundamental aspects of quantum mechanics over the last few years [15,17–21]. Three-state systems are also fast becoming a popular choice for fundamental quantum mechanical tests [22,23]. In order to analyze the effect of nonclassical paths in interference experiments, we have considered the effect of such paths on an experimentally measurable quantity κ. κ (defined below) has been measured in many experiments over the last few years in order to arrive at an experimental bound on possible higher order interference terms in quantum mechanics [24,25] and in effect the Born rule for probabilities [15,18,19]. Investigations of this quantity may also be relevant to theoretical attempts to derive the Born rule [26]. If Born's postulate for a square law for probabilities is true and if $\psi_L = 0$, then the quantity ϵ defined by

$$\epsilon = p_{ABC} - (p_{AB} + p_{BC} + p_{CA}) + (p_A + p_B + p_C) \tag{2}$$

is identically zero in quantum mechanics. Here p_{ABC} is the probability at the detector when all three slits are open, p_{AB} is the probability when slits A and B are open, and so on.

In the experiments reported in the literature, the normalization factor has been chosen to be the sum of the three double slit interference terms called δ given by $\delta = |I_{AB}| + |I_{BC}| + |I_{CA}|$, where $I_{AB} = p_{AB} - p_A - p_B$ and so on. This choice of normalization can sometimes lead to false peaks in the κ as a function of detector position due to the denominator becoming very small at certain positions. We use a somewhat different normalization, $\delta = I_{\max}$, where $I_{\max}$ is the intensity at the central maximum of the triple slit interference pattern to avoid this problem. Then the normalized quantity κ is given by

$$\kappa = \frac{\epsilon}{\delta}. \tag{3}$$

In discussions which invoke the "zeroness" of κ, it is implicitly assumed that only classical paths contribute to the interference. In his seminal work [17], Sorkin had also assumed that the contribution from nonclassical paths was negligible. Now, what is the effect of nonclassical paths on κ? If one can derive a nonzero contribution to κ by taking into account all possible paths in the Feynman path integral formalism, that would mean $\psi_{AB} = \psi_A + \psi_B$ is not strictly true, and experimentalists should not be led to conclude that a measurement of nonzero κ would immediately indicate a falsification of the Born rule for probabilities in quantum mechanics. A measured nonzero κ could also be explained by taking into account the nonclassical paths in the path integral. There is thus a theoretical estimate for a nonzero κ. Of course, the immediate expectation would be a clear domination of the classical contribution and perhaps a very negligible contribution from the nonclassical paths that would in turn imply that $\psi_{AB} = \psi_A + \psi_B$ is true in all "experimentally observable conditions." However, what we go on to discover is that this expectation is not always true. It is possible to have experimental parameter regimes in which κ is measurably large. This in turn leads to a paradigm shift in such precision experiments. Observation of a nonzero κ that is expected from the proposed correction to $\psi_{AB} = \psi_A + \psi_B$ would in fact also serve as an experimental validation of the full scope of the Feynman path integral formalism.

As mentioned before, in calculating κ, one inherently assumes contributions only from the classical straight line paths as shown in green in Fig. 2. In this paper, we have estimated the contribution to κ from nonclassical paths, thus providing the first theoretical estimate for κ.

For simplicity, we will use the free particle propagator in our calculations. For a particle in free space and away from the slits, this is a reasonable approximation. We account for the slits by simply removing from the integral all paths that pass through the opaque metal. An estimate for the error due to this assumption has been worked out in the Supplemental Material [27]. The normalized energy space propagator K [27] for a free particle with wave number k from a position $\vec{r}'$ to $\vec{r}$ is given by

$$K(\vec{r}, \vec{r}') = \frac{k}{2\pi i} \frac{1}{|\vec{r} - \vec{r}'|} e^{ik|\vec{r}-\vec{r}'|}. \tag{4}$$

Although in this paper, we will be mainly focusing on analyzing optics-based experiments using photons, this propagator equation can be used both for the electron and the photon as argued in [27]. We should point out that there are corrections to the propagator due to closed loops in momentum space from quantum field theory considerations. We have explicitly estimated that the effects of such corrections will be negligibly small [32].

Consider the triple slit configuration shown in Fig. 2. According to the path integral prescription, all paths that go from the source to the detector should contribute in the analysis. In the quantity of interest, κ, some important simplifications occur. Only those nonclassical paths that involve propagation between at least two slits would contribute to the leading nonzero value. This is because any nonclassical path that goes through *only* the ith slit can be taken into account in the wave function ψ_i at the detector and hence would cancel out in κ as can be easily checked. In light of the above, the entire set of paths from the source to the detector through the slits can be divided into two classes: (i) paths that cross the slit plane exactly once pertaining to a probability amplitude K_c; a representative path is shown by the green line; and (ii) paths that cross the slit plane more than once at two or more slits pertaining to a probability amplitude K_{nc} [27] as for instance, represented by the purple line.

$$\therefore K = K_c + K_{nc}. \tag{5}$$

We wish to estimate K_{nc} relative to K_c. An example of a representative K_c in our problem is the probability amplitude to go from the source $(-L, 0, 0)$ to the detector $(D, y_D, 0)$ through slit A which we call $K^A(S, D, k)$. This uses the general scheme that a path in Feyman's path integral formalism can be broken into many subpaths and the propagator is the product of the individual propagators [27]. For instance,

$$K_c^A = -\left(\frac{k}{2\pi}\right)^2 \int_{d-\frac{w}{2}}^{d+\frac{w}{2}} \int_{-h}^{h} dy dz \frac{e^{ik(l_1+l_2)}}{l_1 l_2}, \tag{6}$$

where d is the interslit distance, w is the slit width, h is the slit height, $l_1^2 = y^2 + L^2 + z^2$ and $l_2^2 = (y_D - y)^2 + D^2 + z^2$ as shown in Fig. 2. For the source and the detector far apart from one another, i.e., in the Fraunhofer regime, $D \gg d$ and $L \gg d$ in the region of integration; therefore, $l_1 \approx L + (y^2 + z^2/2L)$. Similarly $l_2^2 = (y_D - y)^2 + D^2 + z^2$ giving $l_2 \approx D + ((y_D - y)^2 + z^2/2D)$. Thus we have

$$K_c^A = -\gamma \left(\frac{k}{2\pi}\right)^2 \int_{d-\frac{w}{2}}^{d+\frac{w}{2}} \int_{-h}^{h} dy dz e^{ik[\frac{y^2+z^2}{2L} + \frac{(y_D-y)^2+z^2}{2D}]}. \tag{7}$$

Here $\gamma = (1/LD)e^{ik(L+D)}$. These are Fresnel integrals and have been evaluated using *Mathematica*.

Let us now proceed to the probability amplitude for multiple slit crossings, i.e., K_{nc}. An example of a representative K_{nc} in our problem is the probability amplitude to go from the source $(-L, 0, 0)$ to the detector $(D, y_D, 0)$ following the kind of path shown in Fig. 2. In this case, the particle goes from the source to the first slit and then loops around the second and third slits before proceeding to the detector. We represent this by $K_{nc}^A(S, D, k)$. This is approximated by [27]

$$K_{nc}^A = i\left(\frac{k}{2\pi}\right)^3 \int dy_1 dy_2 dz_1 dz_2 \frac{e^{ik(l_1+l_2+l_3)}}{l_1 l_2 l_3}. \tag{8}$$

Here the y_1 integral runs over slit A and y_2 integral runs over slits B and C and where $l_1^2 = (y_1 - y_S)^2 + L^2 + z_1^2$, $l_2^2 = (y_2 - y_1)^2 + (z_2 - z_1)^2$, and $l_3^2 = (y_D - y_2)^2 + D^2 + z_2^2$. Making approximations appropriate to the Fraunhofer regime, using stationary phase approximation [34] for the oscillatory integrals the integral becomes

$$K_{nc}^A = \gamma i^{3/2} \left(\frac{k}{2\pi}\right)^{5/2} \int dy_1 dy_2 dz_1 |y_2 - y_1|^{-1/2} \times e^{ik\left[\frac{y_1^2+z_1^2}{2L} + (y_2-y_1) + \frac{(y_D-y_2)^2+z_1^2}{2D}\right]}. \tag{9}$$

An important simplification occurs at this stage: the z integral in K^A_{nc} is same as in the integral for K^A_c. Since we are just concerned with ratios, the contributions from the z integrals cancel out.

In terms of K_c and K_{nc}, the propagator to go from the source to the detector when all three slits are open is given by

$$K^{ABC} = K^A_c + K^B_c + K^C_c + K^{ABC}_{nc}, \tag{10}$$

where K^{ABC}_{nc} includes nonclassical terms arising when all slits are open. Similarly,

$$K^{AB} = K^A_c + K^B_c + K^{AB}_{nc}. \tag{11}$$

K^{AB}_{nc} are nonclassical terms involving only A and B. Similarly for AC and BC. Thus, in terms of propagators,

$$\epsilon = |K^{ABC}|^2 - |K^{AB}|^2 - |K^{AC}|^2 - |K^{BC}|^2 + |K^A|^2 + |K^B|^2 + |K^C|^2, \tag{12}$$

and the normalization δ is given by $\delta = |K^{ABC}(0)|^2$, where $|K^{ABC}(0)|^2$ is the value of $|K^{ABC}|^2$ at the central maximum. By numerical integration, we find κ at the central maximum of the triple slit interference pattern to be of the order of 10^{-6} for the parameters used in the triple slit experiment reported in Ref. [15]. What would have been expected to be zero considering only straight line paths now turns out to be measurably nonzero having taken the nonclassical ones into account [35]. In Fig. 3, we show κ as a function of detector position. We also show a plot of the triple slit interference pattern as a function of detector position, which gives a clearer understanding of the modulation in the plot for κ.

The experiment reported in Ref. [15] was not sensitive to a theoretically expected nonzeroness in κ due to systematic errors. However, in the absence of such systematic errors, it is definitely possible to use a similar setup to measure a nonzero κ. Simulation results indicate that the setup could have measured a much lower value of κ but the presence of the systematic error due to one misaligned opening in the blocking mask set the limitation of the experiment making it possible to only measure a value of κ up to 10^{-2}. There is no reason why this systematic error cannot be removed in a future version of the experiment thus making it a perfect tabletop experiment to test for the presence of nonclassical paths in interference experiments. However, experiments of the kind reported in [18] are not as ideally suited for this purpose. This is because, in our analysis, we have worked in the thin-slit approximation. The effective "slit thickness" in a diffraction grating-based interferometer setup would be quite large and hence the resulting κ would certainly be smaller.

What we go on to also find in our current analysis is that κ is very strongly dependent on certain experimental parameters and one can definitely find a parameter regime where κ would be even bigger, and hence easier to observe. We find that keeping all other experimental parameters fixed, κ increases with an increase in wavelength. Thus, for instance, for an incident beam of wavelength 4 cm (microwave regime) and slit width of 120 cm and interslit distance of 400 cm, a theoretical estimate for κ would be 10^{-3}. This is an experiment that can be performed, for instance, in a radio astronomy lab.

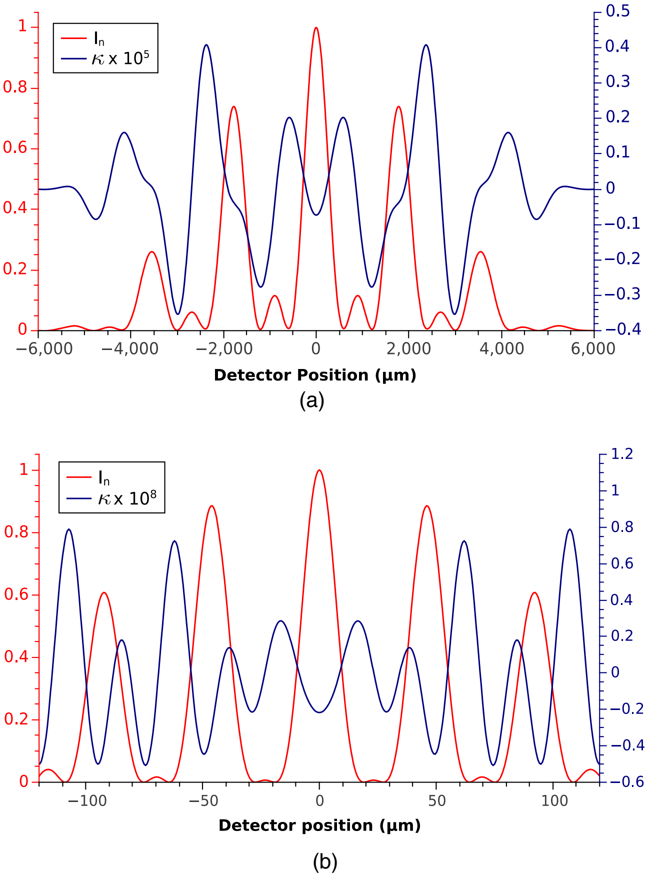

FIG. 3 (color online). Normalized values of κ as a function of detector position. Here $I_n = |K^{ABC}(y)|^2/|K^{ABC}(0)|^2$. (a) This is for incident photons, slit width $= 30\ \mu$m, interslit distance $= 100\ \mu$m, distance between source and slits and slits and detector $= 18$ cm and incident wavelength $= 810$ nm [15]. (b) This is for incident electrons, slit width $= 62$ nm, interslit distance $= 272$ nm, distance between source and slits $=$ 30.5 cm and slits and detector $= 24$ cm and deBroglie wavelength $= 50$ pm [36].

Experiments of this kind where the value of κ due to nonclassical paths can be estimated would definitely be of great interest as they would serve as a simple experimental demonstration of how the basic assumption that a composite wave function is just the sum of component wave functions is not always true. In a sense they would also serve as a direct tabletop demonstration of the complete scope of the Feynman path integral formalism where not only the straight line paths are important but also the looped paths can make a sizeable contribution depending on one's choice of experiment. The effects due to such nonclassical paths may also be used to model possible decoherence mechanisms in interferometer-based quantum computing applications.

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*To whom correspondence should be addressed.
usinha@rri.res.in

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