Evidence for Partial Dynamical Symmetries in Atomic Nuclei

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Symmetries in nature offer very simple descriptions of complex systems. Partial Dynamical Symmetries (PDS) can considerably broaden their relevance. To present the first extensive test of a PDS for nuclei, we compare an SU(3) PDS to gamma to ground band $B(E2)$ values for 47 deformed nuclei. The parameter-free PDS is found to be quite successful, but with characteristic discrepancies, suggesting that symmetry remnants are more pervasive than heretofore realized. Furthermore, the SU(3) PDS gives new insights into collective models (e.g., interacting boson approximation). If these reproduce the PDS, they reflect finite size effects, while differences from the PDS point to SU(3) configuration mixing.

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Symmetries in nature are widespread and fundamental to modern science. In complex systems, dynamical symmetries (DSs) [1]—or spectrum-generating algebras—define specific quantum numbers and selection rules and provide analytic, often parameter-free predictions of energies and transition rates. They aim at describing the astonishingly regular and simple patterns exhibited by complex many-body correlated systems. A DS occurs when the Hamiltonian can be written in terms of Casimir operators of a group and its subgroups. Successive terms in the Hamiltonian introduce specific quantum numbers and break a degeneracy of a higher group.

A very successful application of DSs arises in atomic nuclei in the context of the interacting boson approximation (IBA) model [1], which describes collective nuclei in terms of pairs of valence nucleons forming bosons with angular momentum 0-$\hbar$ (s bosons) and 2-$\hbar$ (d bosons) and their interactions. The parent group for the IBA is U(6), and it has nontrivial DSs U(5) (for vibrational nuclei), SU(3) (axially symmetric deformed nuclei), and O(6) (γ soft axially asymmetric deformed nuclei). A few empirical manifestations of nuclei close to each of these have been proposed (see, e.g., Ref. [2]). However, the vast majority of nuclei deviate from any DS. The DSs, thus, serve mainly as idealized benchmarks and as basis states for diagonalizations of model Hamiltonians. Recently, though, the proposal of partial dynamical symmetries (PDSs) [3] and quasidynamical symmetries [4], which break the DSs while preserving important symmetry remnants, suggests a potentially more widespread role of symmetries in nuclei. However, only one empirical manifestation of a PDS based on SU(3) has been identified [5], namely, in ^{168}Er, where analytic, parameter-free, PDS predictions agree very well with both the data and with multiparameter numerical symmetry-breaking calculations. Is this accidental, or does this PDS describe a broad range of nuclei? If so, this could enhance the applicability of symmetries to nuclei and other complex systems such as atoms, molecules, clusters, and crystals [6–9].

It is, therefore, the purpose of this Letter to present the first extensive test of this PDS by studying $E2$ transition rates covering 47 even-even nuclei in the rare earth region. We find that the PDS is quite successful and discuss both the agreement with the data and characteristic discrepancies, the role of finite system size, how the PDS relates to broken-symmetry numerical calculations, and how the latter can now be better understood.

SU(3) is a dynamical symmetry describing a nucleus with axially symmetric prolate quadrupole deformation. The level scheme consists of sequences of rotational bands labeled by quantum numbers: λ, μ, and K, where λ and μ specify the SU(3) representation (family of levels), and K is the projection of the total angular momentum on the symmetry axis. The ground state (g.s.) band has $K = 0$, and the first excited representation has two rotational bands. Except for very small K admixtures due to the Elliott-Vergados [10] transformation, these latter have $K = 0$ (beta, β band) and 2 (gamma, γ band). States in the β and γ bands with equal angular momenta are degenerate [e.g., $E(2^+_\beta) = E(2^+_\gamma)$].

The $E2$ operator that is a generator of SU(3) is given as $T(E2)_{\mathrm{SU(3)}} = (s^\dagger d + d^\dagger s) - \sqrt{7}/2(d^\dagger d)^{(2)}$ in terms of s and d boson creation and destruction operators. This gives the important selection rule $\Delta(\lambda, \mu) = 0$. That is, SU(3) predicts vanishing $B(E2)$ values from the β and γ bands to the g.s. band.

Clearly, such a model cannot describe most deformed nuclei since the β and γ bands are highly nondegenerate and they both (especially the γ band) have collective transitions to the g.s. band. Indeed, these empirical deviations from

SU(3) have long inspired successful symmetry-breaking numerical IBA calculations [11–13].

The SU(3) PDS of Leviatan [5] presents an alternate approach. It is a special realization of the IBA Hamiltonian given by Eq. (2) of Ref. [5], whose key features are that the degeneracy of the β and γ bands is broken, but a strict SU(3) structure is, nevertheless, preserved for the γ and g.s. bands. No such SU(3) structure is preserved for the β band or other states.

One would think this would immediately rule out the PDS as a viable description since SU(3) forbids $E2$ transitions between the γ and g.s. bands. However, $T(E2)_{\mathrm{SU(3)}}$ is not the most general $E2$ operator. Indeed, one can write $T(E2) = (s^\dagger d + d^\dagger s) - \sqrt{7}/2(d^\dagger d)^{(2)} + \theta(s^\dagger d + d^\dagger s) = T(E2)_{\mathrm{SU(3)}} + \theta(s^\dagger d + d^\dagger s)$. The first term does give vanishing $E2$ transitions from the γ to the g.s. band, but the second gives finite contributions.

If we consider relative interband $B(E2)$ values—ratios of $B(E2)$ values from a γ band state to two states in the g.s. band—we obtain

$$\frac{B(E2{:}J_\gamma \to J'_{gr})}{B(E2{:}J_\gamma \to J''_{gr})} = \frac{\langle J'_{gr}|\theta(s^\dagger d + d^\dagger s)|J_\gamma\rangle^2}{\langle J''_{gr}|\theta(s^\dagger d + d^\dagger s)|J_\gamma\rangle^2}. \quad (1)$$

Note that θ cancels, and, therefore, these PDS $B(E2)$ ratios are parameter free. Yet the PDS has been shown to give very good agreement with the data for γ to g.s. band $B(E2)$ values in the well-studied nucleus ^{168}Er (see Ref. [5], and Fig. 1 and Table I in the present work).

Our aim is to assess if this intriguing result is an anomaly or widespread. To do this, we investigated 47 rare earth region nuclei from Sm to Hg. The β-γ splitting can always be fit by varying the strengths of two terms in the PDS Hamiltonian without affecting γ to g.s. band $B(E2)$ values. The critical test is, thus, if γ to g.s. band $B(E2)$ values can be reproduced while preserving the SU(3) character for these states. In 22 of the 47 nuclei, there are sufficient $E2$ transition data.

Figure 1 shows a number of examples of the comparisons for a wide range of nuclei from Gd to Os. In the other nuclei, the comparisons are generally similar. For each nucleus, the relative interband γ band to g.s. band $B(E2)$ values are shown. We also give the $R_{4/2}$ value ($R_{4/2} \sim 3.33$ for axial rotor nuclei) and valence nucleon number N_{val} counting to the nearest closed shell using the standard magic numbers 50, 82, and 126. The upper left panel is for ^{168}Er and recapitulates the Leviatan results [5].

Overall, the parameter-free SU(3) PDS predictions account very well for these key data in a wide variety of deformed nuclei [and, with two parameters, also for γ-β degeneracy breaking and intraband $B(E2)$ values [16]]. Note that ^{186}Os lies at the beginning of a transitional region, has an $R_{4/2}$ as low as 3.16, and yet also shows reasonable agreement. Data from the decay of the 5^+_γ level in ^{184}W would be useful.

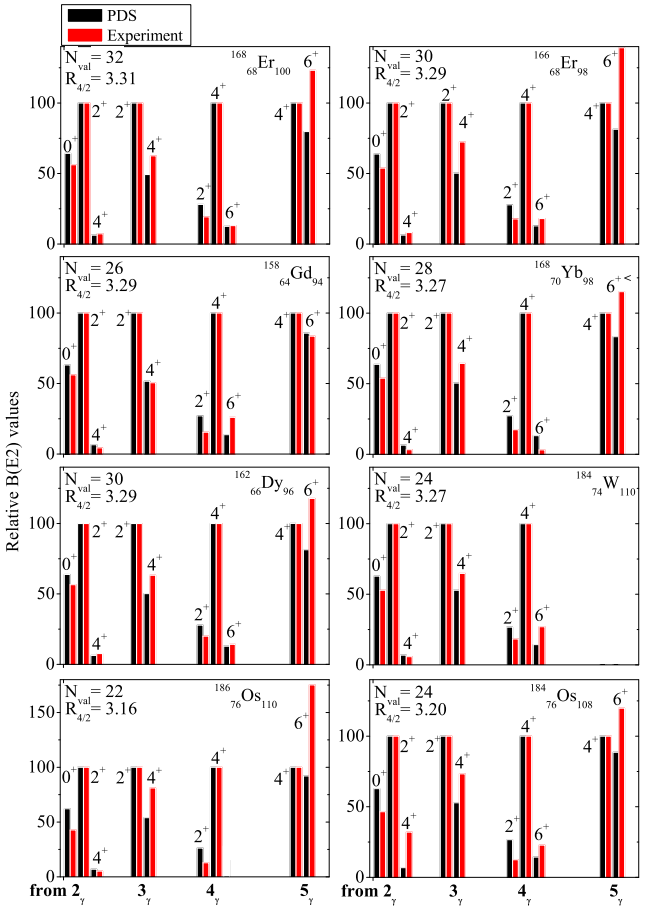

FIG. 1 (color online). Comparison of PDS predictions (calculated for the appropriate boson number) with the data on the relative γ band to g.s. band $E2$ transitions in several deformed nuclei. The red (black) bars are the data [14] (PDS predictions). One transition is normalized to 100 for each initial state. The symbol $<$ on a red bar signifies an upper limit (usually because of unknown $E2/M1$ mixing ratio).

Interestingly, the main discrepancies are systematic: In most cases, the PDS significantly underestimates the spin-increasing transitions and overestimates the spin-decreasing transitions (the nucleus ^{168}Yb is an exception for the spin-increasing transitions from the even spin initial states.) Figure 2 shows that the agreement in Fig. 1 is not trivial. It gives the comparisons for shape transitional nuclei, ^{152}Sm and ^{154}Gd at the $N = 90$ spherical-deformed shape transition, and ^{180}Os, and ^{188}Os in a region with gamma softness and decreasing quadrupole deformation. The SU(3) PDS is not expected to work for such nuclei and, indeed, all exhibit large disagreements.

Table I, a compilation and correction of information from Refs. [5,12,13], along with updated data, summarizes the experimental relative $B(E2)$ values for ^{168}Er. It also includes the Alaga rules [15], which assume only a separation of intrinsic and rotational motion, and initial and final states with pure K values of 0 and 2, the PDS

TABLE I. Detailed results for ^{168}Er. The table shows relative γ to g.s. transitions for the data, the Alaga rules [15], the PDS [5], the WCD IBA calculations [12], and the CQF IBA calculations [13]. For each initial state, the transition with the largest Alaga value is set to 100.

$I_i^\pi \to I_f^\pi$	^{168}Er	Alaga	PDS	WCD	CQF
$2_\gamma^+ \to 0^+$	56.2(11)	70	64.3	66	54
$2_\gamma^+ \to 2^+$	100	100	100	100	100
$2_\gamma^+ \to 4^+$	7.3(4)	5	6.3	6	8
$3_\gamma^+ \to 2^+$	100	100	100	100	100
$3_\gamma^+ \to 4^+$	62.6(14)	40	49.3	48	69
$4_\gamma^+ \to 2^+$	19.3(4)	34	28.1	30	18
$4_\gamma^+ \to 4^+$	100	100	100	100	100
$4_\gamma^+ \to 6^+$	13.1(12)	8.64	12.5	12	16
$5_\gamma^+ \to 4^+$	100	100	100	100	100
$5_\gamma^+ \to 6^+$	123(14)	57.1	79.6	72	125
$6_\gamma^+ \to 4^+$	11.2(10)	26.9	20.3	23	9
$6_\gamma^+ \to 6^+$	100	100	100	100	100
$6_\gamma^+ \to 8^+$	37.6(72)	10.6	18.0	17	20

predictions, and results of two numerical IBA calculations that break SU(3). One of these, Warner-Casten-Davidson (WCD) [12], was the first detailed IBA calculation for a deformed nucleus. The other used a revised approach, the consistent Q formalism (CQF) [13], which is simpler, has one fewer parameter, and agrees better with these data than the PDS or the WCD (this is most dramatic for the 5_γ^+ level). Figure 1 and Table I show that the PDS is in very good agreement with widespread data and also very similar to the WCD IBA calculations [12] for ^{168}Er. However, the data are systematically further from the benchmark Alaga rules than the PDS and WCD, and the CQF calculations agree better with the data. These points will turn out to be very instructive.

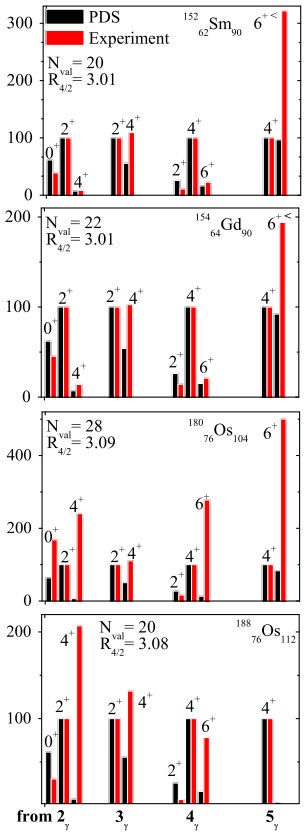

FIG. 2 (color online). Similar to Fig. 1, for transitional nuclei.

These comparisons of the PDS with the data, the Alaga rules, and numerical IBA calculations pose two fundamental questions: Why do the PDS predictions, which also have pure K values for the γ and g.s. bands, differ from the Alaga rules, and how can such seemingly different descriptions as the SU(3) PDS and broken-symmetry (WCD) numerical IBA calculations be simultaneously successful and so similar? Understanding this will give insights into the nature of the PDS predictions and into the effects of symmetry breaking in collective models which have been the backbone of successful treatments of collective even-even nuclei for decades.

The answer to the first question lies in the nature of the IBA model (and, hence, the PDS) as a valence space model in which the number of valence nucleons is conserved and whose predictions are valence nucleon-number dependent [1]. The differences between the PDS and the Alaga rules are, in fact, solely due to valence nucleon-number-dependent effects, as can be seen from the SU(3) matrix elements of the θ term in $T(E2)$ for γ to g.s. band transitions (see Ref. [1], p. 55). The success of the PDS is perhaps the most striking systematic evidence for such effects so far discovered and leads to a new understanding of IBA calculations.

To address this second issue, we consider the discrepancies that do occur. These latter, as noted, are systematic. Consider the γ to ground Alaga rules. They are always small for spin-increasing transitions. The reason is simple. Consider 2_γ^+ to 4_1^+ transitions. The 4_1^+ state has zero intrinsic angular momentum and $4\hbar$ units of rotational angular momentum. The 2_γ^+ level is dominated by $2\hbar$ units of intrinsic (vibrational) angular momentum and zero units of rotational angular momentum. An $E2$ transition connecting them primarily changes the rotational angular momentum by $4\hbar$ and is highly suppressed.

Simple models can often be improved by introducing configuration mixing. In deformed nuclei, γ-ground band mixing is a well-established phenomenon [17]. Such mixing adds coherent components to the transition matrix elements that can be either constructive or destructive. However, in the special case of unperturbed transitions that are forbidden, an added component, regardless of sign, can only increase the $B(E2)$ value. Hence, the disagreements between the PDS and the data in Fig. 1 and Table I for spin-increasing transitions are clear

signatures that such a simple model cannot account for missing perturbations.

Thus, if numerical IBA calculations differ from the PDS (that is, differ from what would be expected due solely to finite number effects), those differences are a measure of mixing effects. Any of the nuclei we have studied can show this, but we use the results for ^{168}Er in Table I as a convenient example since it has been a standard test bench for collective models of deformed nuclei. Thus, we conclude that the numerical WCD IBA results (which are almost identical to the PDS) must contain quite weak $\Delta K = 2$ mixing effects for these interband transitions. Though noted previously [5,12,13], this weak mixing has not been generally recognized.

Now consider the later CQF calculations (Table I), which deviate further from the Alaga rules and which are in better agreement with the data than the PDS and the WCD calculations. From the previous discussion, this implies that they contain stronger symmetry breaking, that is, stronger mixing of SU(3) configurations [this is also evident from an expansion of the wave functions in an SU(3) basis and a band mixing analysis (see Refs. [13,18])] and establishes that this mixing plays a complementary role to finite nucleon-number effects in actual nuclei. Thus, we see that a comparison of Alaga rules, PDS, and numerical calculations gives us a tool to disentangle the effects of finite valence nucleon-number and symmetry-breaking contributions in calculations of axially deformed atomic nuclei. The upshot is a better understanding of decades of collective model calculations that highlights the balance of valence nucleon-number effects and mixing in manifestations of collectivity.

To summarize, we have presented the first extensive test of a partial dynamical symmetry and have shown that it accounts very well for an abundance of data [degeneracy breaking of the γ and β bands and γ to g.s. $B(E2)$ values] in a widespread survey of axially deformed atomic nuclei, although it does not completely account for the differences between interband $B(E2)$ values and geometrical models of pure intrinsic states (Alaga rules). The differences of the PDS from geometrical expectations stem from finite nucleon-number effects, and are, in fact, the most direct evidence for their importance in collective nuclei. Further, numerical IBA calculations can be dissected in terms of nucleon-number and specific band mixing effects, the latter directly reflected in deviations of predicted γ to g.s. band $B(E2)$ values from the PDS. The better agreement with these data of the CQF calculations than the PDS or earlier calculations, especially in spin-increasing γ to g.s. band transitions, signals the need for mixing effects that are absent from the PDS.

The present results suggest a much wider applicability of dynamical symmetries, that the IBA triangle is suffused with important elements of symmetry, and encourage detailed tests of other PDSs (see Ref. [19] for examples). They also point more generally to the complementary roles of constituent number and configuration mixing in the rise of collectivity in finite interacting systems.

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