## Further Improvement of the Upper Limit on the Direct $3\alpha$ Decay from the Hoyle State in <sup>12</sup>C

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The direct  $3\alpha$  decay branch from the  $0_2^+$  state at  $E_x = 7.65$  MeV in <sup>12</sup>C, which is known as the Hoyle state, is considered to affect the triple- $\alpha$  reaction rate strongly and to give crucial information on its structure. We have performed a high-precision measurement of the  $3\alpha$  decay from this state using the  ${}^{12}C({}^{12}C, 3\alpha){}^{12}C$  reaction at  $E_{{}^{12}C} = 110$  MeV. The branching ratio of the direct  $3\alpha$  decay was under the detection limit in the present experiment. By comparing with Monte Carlo simulations for three decay mechanisms as the sequential decay through the ground state of <sup>8</sup>Be, the direct decay with equal energies of three  $\alpha$  particles, and the direct decay to the phase space uniformly, we have obtained the upper limit of 0.2% on the direct  $3\alpha$  decay.

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The  $0_2^+$  state at  $E_x = 7.65$  MeV in <sup>12</sup>C plays an important role in the creation of the <sup>12</sup>C nucleus in stellar nucleosynthesis. The <sup>12</sup>C nucleus is produced by the triple- $\alpha$  reaction: two  $\alpha$  particles with the relative kinetic energy of 92 keV form the resonance state of <sup>8</sup>Be at first, and the short lived <sup>8</sup>Be captures a third  $\alpha$  particle before decaying back to two  $\alpha$  particles in the second step. Fred Hoyle claimed that the capture of a third particle in the second step proceeds through a resonant state in <sup>12</sup>C near the  $\alpha$  + <sup>8</sup>Be threshold, thus, enhancing the triple- $\alpha$  reaction rate [1]. This resonant state,  $0_2^+$ , in <sup>12</sup>C was discovered soon after his prediction [2]. For that reason, the  $0_2^+$  state in <sup>12</sup>C is called the Hoyle state.

The structure of the Hoyle state is highly related to the triple- $\alpha$  reaction rate, since it is considered to have a typical  $3\alpha$  cluster structure. In the simple shell model, the  $0^+$  state does not appear at such a low excitation energy as 7.65 MeV in <sup>12</sup>C. According to the microscopic  $\alpha$  cluster models, the Hoyle state has been considered to have a dilute gaslike structure in which the  $\alpha$  clusters are loosely coupled to each other [3,4]. About a decade ago, Tohsaki et al. proposed that this dilute  $\alpha$  gaslike structure was similar to the Bose-Einstein condensation of  $\alpha$  clusters in the nucleus [5–7]. Recently, some *ab initio* calculations have been tried to explain properties of the Hoyle state including the ground state band in <sup>12</sup>C [8–10]. Among them, a lattice approach with chiral effective field theory succeeded in reproducing the excitation energy of the Hoyle state [9,11]. In their result, the Hoyle state is considered to have a "bent-arm" or obtuse triangular configuration.

In the recent progress of experimental studies, the  $2_2^+$  state at 10 MeV has been established [12–14]. It is considered to be the  $2^+$  excitation of the Hoyle state according to the  $\alpha$  cluster and  $\alpha$  condensation models

[3,4,15]. In almost all semimicroscopic and microscopic  $\alpha$  cluster models, this 2<sup>+</sup> state was predicted to appear at around 10 MeV. This fact strongly supports the validity of these models. On the other hand, the *ab initio* lattice calculations also reproduce the excitation energy of the 2<sup>+</sup> state reasonably [11]. Furthermore, the very recently found 5<sup>-</sup> state at 22.4 ± 0.2 MeV in <sup>12</sup>C was explained as a rotational member of the ground state with an equilateral triangular configuration having a  $\mathcal{D}_{3h}$  symmetry [16]. In this model, the Hoyle state is interpreted as the band head of the *A* symmetric stretching vibration or breathing mode of the triangular configuration. Therefore, the structure of the Hoyle state is still controversial.

It is difficult to determine the structure of the nuclear excited states, especially the unbound states, experimentally. One possible way is the decay particle measurement. Recently, Raduta *et al.* reported a rather high branching ratio of 17% for the direct  $3\alpha$  decay of the Hoyle state in the  ${}^{40}\text{Ca} + {}^{12}\text{C}$  reaction, and evidence for the  $\alpha$  condensation in the Hoyle state [17]. However, it was in contradiction with the result of the upper limit of less than 4% obtained twenty years ago by Freer *et al.* [18]. After the report by Raduta *et al.*, several experiments have been performed, and they all have obtained results supporting that of Freer *et al.* [19,20]. Very recently, Rana *et al.* succeeded in estimating a nonzero value of the direct  $3\alpha$  decay branch from the complete kinematics measurement of the  ${}^{12}\text{C}(\alpha, \alpha')3\alpha$  reaction [21].

In this Letter, we report further improvement of an upper limit on the direct  $3\alpha$  decay from the Hoyle state via the  ${}^{12}C({}^{12}C, 3\alpha){}^{12}C$  reaction at 110 MeV. Although the reaction is the same as in Ref. [18], the aim of the present Letter is to clarify the direct  $3\alpha$  decay branch of  $0.91 \pm 0.14\%$ reported by Rana *et al.* [21] with more statistics by measuring it at high energy and forward angles.

The experiment was performed at the Cyclotron and Radioisotope Center (CYRIC), Tohoku University. The  $^{12}C^{4+}$  beam was accelerated up to 110 MeV by the K110 AVF cyclotron and bombarded a self-supporting natural carbon foil target with a thickness of 50  $\mu$ g/cm<sup>2</sup> installed in a scattering chamber. The measurement was done by the inverse kinematics method with complete kinematics of the reaction determined. The incident <sup>12</sup>C was inelastically excited to the Hoyle state and the breakup three  $\alpha$  particles were detected in a double-sided Si strip detector (DSSD) with a size of  $50 \times 50$  mm and a thickness of 1500  $\mu$ m. The DSSD consisted of  $16 \times 16$  strips oriented vertically in the front side and horizontally in the rear side with a size of  $3 \times 50$  mm. The positions of three  $\alpha$  particles at DSSD are determined by comparing the energies obtained from the strips in the front and the rear sides. The DSSD, located at 309 mm from the target, covered horizontal and vertical angular ranges from  $3.0^{\circ}$  to  $12.2^{\circ}$ , and from  $-4.6^{\circ}$  to  $+4.6^{\circ}$ , respectively. At such forward angles, not only decay  $\alpha$ particles from the Hoyle state, but also elastically and inelastically scattered <sup>12</sup>C particles would be expected to hit the DSSD. In order to reduce counting rates in the DSSD, an Al plate with a thickness of 200  $\mu$ m was installed in front of the DSSD. Elastically and inelastically scattered <sup>12</sup>C particles stop in the Al plate, while the breakup  $\alpha$ particles can pass through it and stop in the DSSD. The energy of the breakup  $\alpha$  particle before passing through the Al plate was reconstructed using the inverse table of the energy loss calculated by the SRIM2006 package [22]. The deflection angle due to multiple coulomb scattering was also estimated by the SRIM2006 code [22], and was smaller than the geometrical angular resolutions determined by the strip width and the distance from the target to the DSSD.

The recoiling <sup>12</sup>C particles were caught by a silicon detector with a thickness of 150  $\mu$ m at 67°. This angle corresponds to that for the third maximum of the angular distribution of the cross section in the  ${}^{12}C({}^{12}C, {}^{12}C^*(0_2^+)){}^{12}C$ reaction. The recoiling angle was restricted by a copper collimator with an aperture of  $1 \times 10$  mm and a thickness of 2 mm located at 160 mm from the target. The particle identification of <sup>12</sup>C was made by the time of flight method. Figure 1 shows the calculation of the kinematics in the  ${}^{12}C({}^{12}C, {}^{12}C^*){}^{12}C$  reaction and the energy spectrum of the recoiling <sup>12</sup>C. The hatched region indicates the acceptance of the silicon detector for the recoiling <sup>12</sup>C. As shown in Fig. 1, the angle of the recoiling  ${}^{12}C$  from the 3<sup>-</sup> state at 9.64 MeV would not reach 67° and only the Hoyle state could be coincident with the  $3\alpha$  particles detected in the DSSD, which is drawn by a thick solid line in Fig. 1. The kinetic energy of the recoiling <sup>12</sup>C from the Hoyle state, which excitation energy is 7.65 MeV, is about 2.0 MeV, or less, although it can be even lower due to the energy loss in the target foil. In order to reduce the energy loss of the recoiling <sup>12</sup>C, the target was rotated at 60° with respect to the

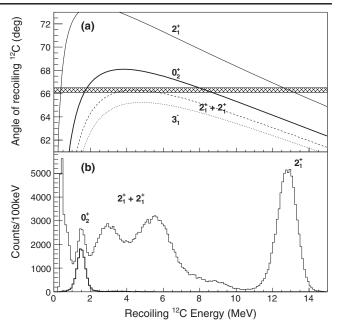


FIG. 1. (a) The kinematics of the  ${}^{12}C({}^{12}C, {}^{12}C^*[3\alpha]){}^{12}C$  reaction. Those of the 4.44 MeV,  $2_1^+$  state, the 7.65 MeV,  $0_2^+$  state, and the 9.64 MeV,  $3^-$  state are drawn by thin solid, thick solid, and dotted lines, respectively. The dashed line shows the kinematics in the case in which both the beam and the target  ${}^{12}C$  are excited to the  $2_1^+$  state. The hatched region indicates the acceptance of the silicon detector for the recoiling  ${}^{12}C$ . (b) Energy spectra of the recoiling  ${}^{12}C$  at 67° are shown. The thin solid and thick solid lines show the energy spectrum of the singles trigger and the coincidence spectrum with the breakup three  $\alpha$  particles, respectively.

beam axis. The events were selected by filtering the recoil energy from 0.7 to 2.4 MeV. Besides the recoil energy filtering, the total energy of the decay  $3\alpha$  particles and the recoiling <sup>12</sup>C was also restricted within the range from 102 to 103.8 MeV, which corresponds to the energy for  $E_{\text{Total}} =$  $E_{\text{Beam}}$  (110 MeV) Q value (7.27 MeV). The kinetic energies of the decay  $3\alpha$  particles in the  ${}^{12}C^*$  rest frame were determined from the energies and angles measured in the DSSD. The total kinetic energy in the  ${}^{12}C^*$  rest frame was also restricted within the range from 0.28 to 0.48 MeV, which corresponds to the energy (~0.38 MeV) of the Hoyle state. The energy resolution of the reconstructed total kinetic energy spectrum in the <sup>12</sup>C\* rest frame was about 45 keV (FWHM), as shown in Fig. 2. The number of decay events from the Hoyle state is about 21 000, which is higher than that in Ref. [21]. The background level is about 0.1%, which is deduced from the total kinetic energy spectrum in Fig. 2. We assumed the background level is the same as the yields at around 0.1 MeV of the total kinetic energy spectrum in the  ${}^{12}C^*$  rest frame, although they are reasonably explained by the sequential decay events of the Monte Carlo simulation, which details are described below. There were no accidental coincidence events which would mimic the Hoyle state by coincidence with  $\alpha$  particles

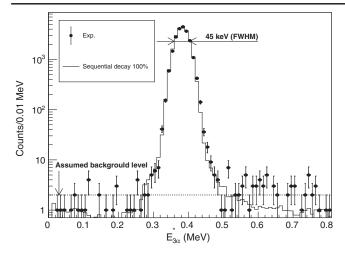


FIG. 2. The total kinetic energy spectrum in the  ${}^{12}C^*$  rest frame. The peak at 0.38 MeV corresponds to the Hoyle state. The result of the Monte Carlo simulation for the 100% sequential decay is drawn by the solid line. The dotted line shows the background level which we assumed.

emitted from the reaction occurring in the next beam bunch from the cyclotron. The beam position was monitored during the measurement by the energy of <sup>12</sup>C at 80° from elastic scattering: For a 1 mm shift in the beam position at the target, the energy of the recoiling <sup>12</sup>C is shifted by about 250 keV.

In order to visualize the energy correlation of the decay  $3\alpha$  particles from the Hoyle state, the Dalitz plot [23] symmetrized in the case of three equal masses is adopted. It is a useful plot to extract the physics information on the decay of the Hoyle state from experimental data, as shown in the <sup>11</sup>B(<sup>3</sup>He, *d*)<sup>12</sup>C<sup>\*</sup>[3 $\alpha$ ] reaction experiment [20]. In the symmetric Dalitz plot, the radial parameter,  $\rho$ , is given by

$$(3\rho)^2 = 3(\varepsilon_j - \varepsilon_k)^2 + (2\varepsilon_i - \varepsilon_j - \varepsilon_k)^2, \qquad (1)$$

$$x = \sqrt{3}(\varepsilon_j - \varepsilon_k), \tag{2}$$

$$y = 2\varepsilon_i - \varepsilon_j - \varepsilon_k. \tag{3}$$

where  $\varepsilon_{i,j,k} = E_{i,j,k}/(E_i + E_j + E_k)$  are the  $\alpha$  particle energies in the <sup>12</sup>C<sup>\*</sup> rest frame normalized to the total energy of the decay  $3\alpha$  particles.  $E_{i,j,k}$  are kinetic energies of the decay  $\alpha$  particles in the <sup>12</sup>C<sup>\*</sup> rest frame.  $E_{i,j,k}$  are selected with  $E_i > E_j > E_k$  for simplicity of the symmetry plot. *x* and *y* are the coordinates in the Dalitz plot.

By using the symmetric Dalitz plot, we discuss three decay mechanisms: the sequential decay (SD) through the ground state of <sup>8</sup>Be, the direct decay with an equal energy of three  $\alpha$  particles (DDE), and the direct decay to the phase space uniformly (DD $\Phi$ ). The direct decay of the linear  $3\alpha$  chain (DDL in Refs. [17,20,21]) is not included in our analysis. Figure 3 shows Dalitz plots of experimental data and results of Monte Carlo simulations for three decay

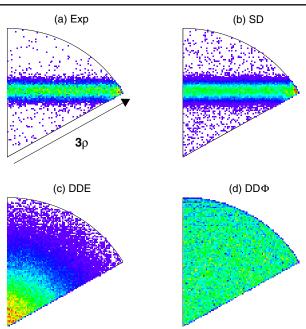
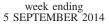


FIG. 3 (color online). Symmetric Dalitz plots for (a) the experimental data, Monte Carlo simulations of (b) the sequential decay (SD), (c) the direct decay with an equal energy of three  $\alpha$  particles (DDE), and (d) the direct decay to the phase space uniformly (DD $\Phi$ ) (see text).

mechanisms (SD, DDE, and DD $\Phi$ ). The Dalitz plot in the DDE mechanism showed a Gaussian distribution with a dispersion of 0.3 in the radial parameter,  $3\rho$ . In the Monte Carlo simulations, experimental conditions, such as beam profiles, detector geometries, energy loss and straggling, effects of multiple Coulomb scattering, position assignment method of the three  $\alpha$  particles, and the event reconstruction algorithm, have all been taken into account. The estimated efficiencies of the  $3\alpha$  detection in coincidence with the recoiling <sup>12</sup>C are as high as 76% for SD, 77% for DDE, and 74% for DD $\Phi$ .

Figure 4 shows the  $\varepsilon_i$  distribution, which is the highest normalized energy among the decay  $3\alpha$  particles. In the case of the SD mechanism, the first decay- $\alpha$  particle takes away two thirds of the released energy of 287.6 keV, i.e., about 192 keV. Then,  $\varepsilon_i$  becomes nearly 0.506. On the other hand, in the case of the direct decay mechanisms, since the released kinetic energy is 379.4 keV, the  $\varepsilon_i$  of the decay  $\alpha$  particles varies from 0.33 to 0.67. The normalized energies of 0.33 and 0.67 correspond to that of three  $\alpha$ particles with an equal energy and that of an  $\alpha$  particle emitted in a direction opposite to that of the other two  $\alpha$ particles, respectively.

In order to obtain the branching ratio for each decay mechanism, we fitted the range from 0.33 to 0.67 of the experimental  $\varepsilon_i$  distribution with those of 10<sup>6</sup> events obtained by Monte Carlo simulations. The branching ratios of DDE and DD $\Phi$  in the best fit are negligibly small with zero-consistent results. The reduced  $\chi^2$  of the fit with three



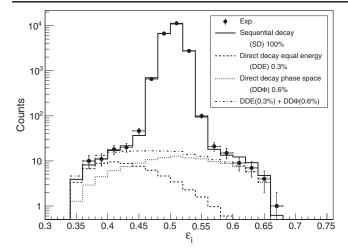


FIG. 4.  $\varepsilon_i$  distribution.  $\varepsilon_i$  is the highest energy among the decay  $3\alpha$  particles from the Hoyle state (see text). The solid circles show the experimental data. The solid line shows the  $\varepsilon_i$  distribution obtained by the fit with the result of the Monte Carlo simulation for the sequential decay (SD). The dashed, dotted, and dashed-dotted lines indicate the distributions for DDE, DD $\Phi$ , and the sum of these two direct decays reported in Ref. [21]

decay mechanisms is 1.24 per degree of freedom. In the case of the fit with only the SD mechanism, the reduced  $\chi^2$  becomes 1.10 per degree of freedom. Therefore, we conclude that the Hoyle state decays to three  $\alpha$  particles through the ground state of <sup>8</sup>Be almost 100%, and that the direct  $3\alpha$  decay from the Hoyle state is under the detection limit of the present experiment. We have estimated the upper limits on the branching ratios of the direct  $3\alpha$  decays for 2 standard deviations, i.e., 95% C.L., by finding the limits of the ratios for DDE and DD $\Phi$ , in which  $\chi^2$  increases by 4 comparing to the  $\chi^2$  minimum ( $\Delta \chi^2 = 4$ ). The upper limits on DDE and DD $\Phi$  are obtained as 0.08% and 0.2%, respectively, which are smaller than the previous best limits obtained in Ref. [20].

These upper limits are incompatible not only with the branching ratios of 7.5% for DDE and 9.5% for DD $\Phi$  reported by Raduta *et al.* [17], but also with recently reported values of 0.3% for DDE and 0.6% for DD $\Phi$  by Rana *et al.* [21]. The direct decay branches obtained in various experiments are listed in Table I for comparison. The dashed and dotted lines in Fig. 4 show the simulated  $\varepsilon_i$  distributions for a 0.3% DDE and a 0.6% DD $\Phi$ 

contribution, respectively. The sum of DDE and DD $\Phi$  contributions drawn by the dashed-dotted line appears to reproduce the shoulders of the experimental  $\varepsilon_i$  distribution. However, as described in Ref. [18], SD events misassigned the positions of three  $\alpha$  particles in the DSSD due to the finite energy resolution can reproduce the shoulders of the normalized energy spectrum as well as the direct decay events. Consequently, the experimental  $\varepsilon_i$  distribution is well reproduced with only that of the SD mechanism.

As described in Refs. [20,21], the DDE and DD $\Phi$  mechanisms do not directly link to the structure of three  $\alpha$  clusters, because the branching ratios and the energy distribution of the decay particles are strongly affected by tunneling through the Coulomb barrier. Practically, advanced three-body decay calculations such as those presented in Refs. [24,25] are inevitably needed in order to connect the branching ratio and the energy distribution to some specific structure. Such a combination analysis of the precise three-body calculation and the Monte Carlo simulation of the experiment has been applied in the studies of the three-body decay [26,27].

In summary, we have measured the  $\alpha$  decay from the Hoyle state with complete kinematics in the  ${}^{12}C({}^{12}C, {}^{12}C^*[3\alpha]){}^{12}C$  reaction. Using Dalitz plots obtained from Monte Carlo simulations and the difference of the decay kinematics between the direct  $3\alpha$  decay and the sequential decay, the direct  $3\alpha$  decay from the Hoyle state has been investigated. The highest normalized energy distribution,  $\varepsilon_i$ , was well reproduced with only that of the sequential decay through the ground state of <sup>8</sup>Be. From the  $\chi^2$  distribution of the fit, we obtained an upper limit of 0.2% on the direct  $3\alpha$  decay branch, which is about a half smaller than the previous upper limit of 0.5%. The present upper limit does not support the direct  $3\alpha$  decay branch of  $0.91 \pm 0.14\%$  obtained in Ref. [21]. Precise three-body decay calculations connecting the branching ratio of each decay mode and the structure of the Hoyle state are highly desired.

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Experiment	DDE (%)	DD4 (%)	DDL (%)	DD (%)	C.L. (%)
Ref. [18]				< 4	99.5
Ref. [17]	$7.5 \pm 4.0$		$9.5 \pm 4.0$	$17.0\pm5.0$	
Ref. [19]	< 0.45	$1.3 \pm 0.9 \ (< 3.9)$			99.75
Ref. [20]	< 0.09	< 0.5	< 0.09	< 0.5	95
Ref. [21]	$0.3 \pm 0.1$	$0.60\pm0.09$	< 0.1	$0.91\pm0.14$	99.75
Present	< 0.08	< 0.2		< 0.2	95

TABLE I. Comparison of various experimental results on the direct decay branches of the Hoyle state.

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