Metal-Insulator Transition by Holographic Charge Density Waves

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We construct a gravity dual for charge density waves (CDWs) in which the translational symmetry along one spatial direction is spontaneously broken. Our linear perturbation calculation on the gravity side produces the frequency dependence of the optical conductivity, which exhibits the two familiar features of CDWs, namely, the pinned collective mode and gapped single-particle excitation. These two features indicate that our gravity dual also provides a new mechanism to implement the metal to insulator phase transition by CDWs, which is further confirmed by the fact that dc conductivity decreases with the decreased temperature below the critical temperature.

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Introduction.—In recent years, the holographic correspondence between a gravitational theory and a quantum field theory in condensed-matter physics has been extensively investigated. In particular, inspired by the seminal work in Refs. [1,2], more and more evidence has been accumulated in favor of the consensus that many phenomena related to strongly coupled systems may have a dual description on the gravity side. In this Letter, we shall offer a holographic mechanism to implement the metal-insulator phase transition by charge density waves (CDWs).

A CDW is a novel ground state of the coupled electron-phonon system, which is characterized by a collective mode formed by electron-hole pairs with a wave vector $q = 2k_F$ and a gap in the single-particle excitation spectrum [3]. Ideally, this collective mode would sit at zero frequency, leading to a supercurrent. But because of the inevitable interaction between a CDW and the underlying background, this collective mode is generically shifted to a finite resonance frequency. This pinning effect together with the gapped single-particle excitation suggests a mechanism to induce a metal-insulator phase transition by CDW. With this in mind, it is highly possible to provide a holographic realization of a metal-insulator transition once a holographic CDW is implemented.

It should be stressed that the generation of CDWs in condensed matter corresponds to the *spontaneous* breaking of translational symmetry. Therefore, to implement a holographic description of CDWs, it is essential to introduce some mechanism of the modulated instability of the bulk geometry, which is usually of spatial homogeneity. This issue has been addressed recently, and the examples of spatially modulated unstable modes have been presented [4–16]. However, until now a study on the dynamics of CDWs by holography has been absent. Thus, it is unclear not only whether such holographic CDWs reproduce the observed fundamental features of ordinary CDWs in experiments but also whether the corresponding CDW phase transition is accompanied by the metal-insulator transition. We shall make a first attempt to address these issues and provide an affirmative answer to both of these questions by investigating the optical conductivity of holographic CDWs in a striped black hole background.

Holographic setup and background solutions.—Our model consists of gravity coupled with two gauge fields plus a dilaton field in four dimensions

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\Big[R + \frac{1}{L^2} - \frac{1}{4}t(\Phi)F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}G^{\mu\nu}G_{\mu\nu} - \frac{1}{2}(\partial_\mu\Phi\partial^\mu\Phi + m^2\Phi^2) - \frac{1}{2}u(\Phi)F^{\mu\nu}G_{\mu\nu}\Big], \tag{1}$$

where $F = dA$, $G = dB$, $t(\Phi) = 1 - (\beta/2)L^2\Phi^2$, and $u(\Phi) = (\gamma/\sqrt{2})L\Phi$. The first gauge field A is introduced to form an AdS–Reissner-Nordström (AdS-RN) black hole background with finite temperature and nonvanishing chemical potential, while the second gauge field B as well as the dilaton field will be responsible for the instability of the background, and the CDW phase will be associated with this second $U(1)$ symmetry [16]. Below, we shall set the AdS radius $l^2 = 6L^2 = \frac{1}{4}$ and $m^2 = -(2/l^2) = -8$. In addition, $l^2/2\kappa^2 \gg 1$ is required such that classical gravity is reliable, which corresponds to the large N limit of the dual field theory. Thus, in our setup, the remaining adjustable parameters are β and γ.

Obviously, in the case of $\Phi = 0$ and $B = 0$, the equations of motion always allow the electric AdS-RN black hole solution

$$ds^2 = \frac{1}{z^2}\left(-(1-z)f(z)dt^2 + \frac{dz^2}{(1-z)f(z)} + dx^2 + dy^2\right) \quad (2)$$

with

$$f(z) = 4\left(1 + z + z^2 - \frac{z^3\mu^2}{16}\right), \qquad A_t = \mu(1-z). \quad (3)$$

We will consider the dual field theory in a grand canonical system; hence, we will use the chemical potential μ as the unit for the system. In this coordinate system, the black hole horizon is located at $z = 1$ and the AdS_4 boundary is at $z = 0$. The Hawking temperature of the black hole is $T/\mu = (48 - \mu^2)/(16\pi\mu)$. The zero temperature limit is reached when $\mu = 4\sqrt{3}$. However, the linear perturbation analysis shows that at low temperature such a black hole will be unstable against the striped phase [16]. To obtain such a resultant striped solution by solving the fully nonlinear bulk dynamics numerically, we assume the following ansatz for the background fields

$$\begin{aligned} ds^2 &= \frac{1}{z^2}\left[-(1-z)f(z)Qdt^2 + \frac{Sdz^2}{(1-z)f(z)} + Vdy^2 \right. \\ &\quad \left. + T(dx + z^2 U dz)^2\right], \\ A &= \mu(1-z)\psi dt, \\ B &= (1-z)\chi dt, \\ \Phi &= z\phi, \end{aligned} \quad (4)$$

where the eight variables involved in the ansatz are functions of x and z. In order to have a spontaneous breaking of the translational symmetry in the dual field theory, the following Dirichlet boundary conditions are imposed:

$$\begin{aligned} &Q[x,0] = S[x,0] = T[x,0] = V[x,0] = \psi[x,0] = 1, \\ &U[x,0] = \chi[x,0] = \phi[x,0] = 0. \end{aligned} \quad (5)$$

Furthermore, we impose the regularity conditions at the horizon such that all the functions have a Taylor expansion in powers of $(1 - z)$. Now the equations of motion reduce to eight partial differential equations with respect to x and z. We solve them numerically with the Einstein-DeTurck method, which has been employed to look for static solutions to Einstein equations [17–22]. We demonstrate the relevant result below, where we focus solely on the case of $\beta = -138$ and $\gamma = 17.1$.

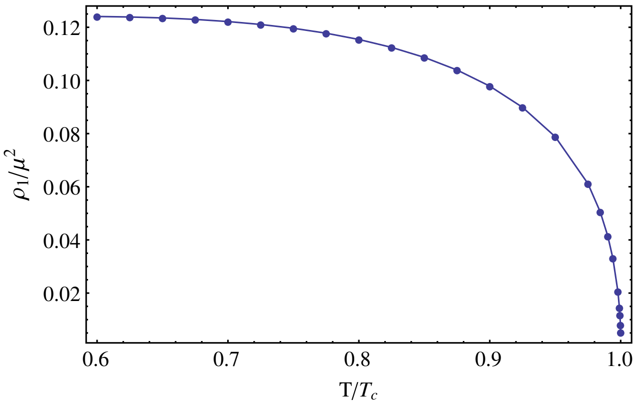

FIG. 1 (color online). The first mode of CDW as a function of temperature.

The corresponding critical temperature for the phase transition to the striped phase is about $T_c = 0.078\mu$, and the critical momentum mode in the x direction is given by $k_c = 0.325\mu$. The onset of CDW can be read off explicitly from the component of the gauge field B_t [23]

$$\begin{aligned} &B_t = -\rho(x)z + O(z^2), \\ &\rho(x) = \rho_0 + \rho_1\cos[k_c x] + \cdots + \rho_n\cos[nk_c x] + \cdots. \end{aligned} \quad (6)$$

We find that the coefficients of even orders in numerical solutions vanishes, such that the charge density for CDWs has the form $\rho(x) = \rho_1\cos[k_c x] + \rho_3\cos[3k_c x] + \cdots$. As shown in Fig. 1, ρ_1 can serve as the order parameter of our system to characterize the phase transition to CDW, as should be the case. Its condensation behavior near the critical temperature indicates that the system undergoes a second-order phase transition to CDW. In Fig. 2 we have plotted the charge density associated with ρ_1 and ρ_3 at various temperatures. From this figure, we notice that near the critical temperature the subleading term ρ_3 is tiny compared with the leading term ρ_1 and can be neglected, while as the temperature goes down, its contribution becomes important.

In Fig. 3 we plot the solutions of the scalar ϕ and the time component of the gauge field χ at the temperature $T = 0.8T_c$. Note that the striped profile increases when one goes deeper into the horizon, which is consistent with the linear perturbation analysis that such a striped phase is triggered by the instability of near-horizon geometry

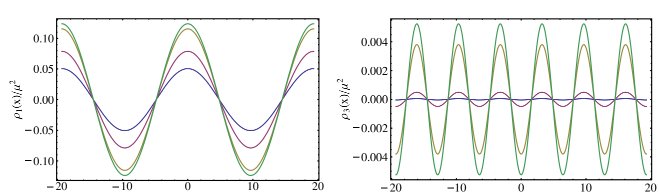

FIG. 2 (color online). The first and third modes of CDW for $T/T_c = 0.6, 0.8, 0.95, 0.98$ from top to bottom.

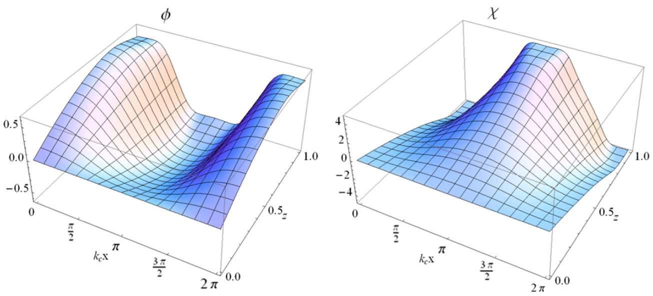

FIG. 3 (color online). Solutions of the scalar field and the time component of the gauge field χ for $T = 0.8T_c$.

$AdS_2 \times R^2$ of an extremal AdS-RN black hole [16]. With this relevant striped deformation, the IR metallic fixed point characterized by $AdS_2 \times R^2$ is driven to another fixed point. As we shall show in the next section, this resultant fixed point corresponds to an insulating phase.

Optical conductivity of holographic CDW and metal-insulator transition.—Now we turn to study the dynamics of holographic CDWs by computing the optical conductivity as a function of frequency. To this end, we separate the variables into the background part and fluctuation part as

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}, \qquad A_\mu = \bar{A}_\mu + a_\mu,$$
$$B_\mu = \bar{B}_\mu + b_\mu, \qquad \Phi = \bar{\Phi} + \varphi. \tag{7}$$

We assume that the fluctuations of all the fields have a time-dependent form as $e^{-i\omega t}$ but independent of the coordinate y. To solve the fluctuation equations, gauge conditions must be imposed for gravity and two gauge fields. Here, we choose the de Donder gauge and Lorentz gauge condition for them, respectively,

$$\bar{\nabla}^\mu \hat{h}_{\mu\nu} = 0, \qquad \bar{\nabla}^\mu a_\mu = 0, \qquad \bar{\nabla}^\mu b_\mu = 0, \tag{8}$$

where $\hat{h}_{\mu\nu} = h_{\mu\nu} - h\bar{g}_{\mu\nu}/2$ is the trace-reversed metric perturbation.

As usual, we adopt ingoing wave boundary conditions at the horizon. While at our AdS boundary $z = 0$, we consider the following consistent boundary condition with

$$b_x(x,0) = 1, \qquad a_x(x,0) = \frac{\partial_z \chi(x,0)}{\mu[1 - \partial_z \psi(x,0)]}$$
$$\text{others}(x,0) = 0. \tag{9}$$

Then by holography, we can extract the homogeneous part of optical conductivity, the quantity we are interested in. Namely, given that $b_x = (1 + j_x(x)z + \ldots)e^{-i\omega t}$ by solving the fluctuation equations, the conductivity associated with the second gauge field reads $\sigma(\omega/\mu) = 4j_x^{(0)}/(i\omega)$, in which a factor of 4 comes from the unusual asymptotic form of the metric in Eq. (2).

One typical plotting for the real and imaginary parts of the optical conductivity at various temperatures is shown in Fig. 4. Two fundamental features of CDWs are observed. One is the pinned collective mode, which is reflected as the first peak appearing in the real part of the conductivity. The second is the gapped single-particle excitation, which corresponds to the occurrence of the second peak in the real part of the conductivity.

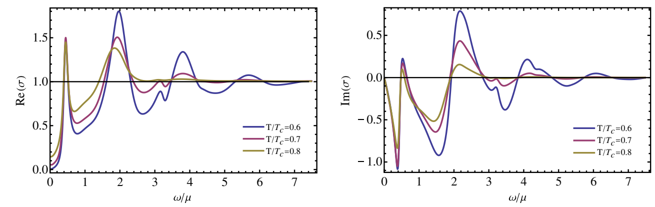

FIG. 4 (color online). The optical conductivity for CDWs, where the black horizontal line denotes the corresponding optical conductivity for a AdS-RN black hole associated with the second gauge field.

The pinning is a common phenomenon for CDWs on account of the various interactions with the other components of the system, which can be described by a damped harmonic oscillator with Lorentz resonance

$$\sigma_{\mathrm{CDW}}(\omega) = \frac{K\tau}{1 - i\omega\tau(1 - \omega_0^2/\omega^2)}, \tag{10}$$

where τ is the relaxation constant, K is proportional to the number density of the CDW, and ω_0 is the average pinning resonance frequency [3]. This formula has been widely employed in the analysis of CDW optical response experiments. Here, we also use it to fit our data. In consistence with the fact that our holographic CDW is always generated with multiple wave vectors, we find that, in general, our data in the low-frequency region of the conductivity can be well fit with multiple Lorentz oscillators. In particular, as the temperature is not quite low, for instance $T \geq 0.6T_c$, it can be fit with only two oscillators, namely, $\sigma_{\mathrm{tot}}(\omega) = \sigma_{\mathrm{CDW1}}(\omega) + \sigma_{\mathrm{CDW2}}(\omega)$, because in this case the contribution from those CDWs with higher wave vectors is negligible. Figure 5 shows such a fit to this formula for $T/T_c = 0.6$. The parameters in the Lorentz formula for various temperatures are listed in Table I.

Although this pinned collective mode is gapless, our single-particle excitation is gapped, as clearly evident from

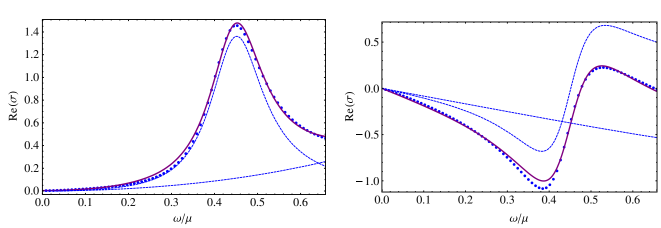

FIG. 5 (color online). The fit of optical conductivity with two Lorentz oscillators in the low-frequency regime for $T = 0.6T_c$. The contributions from the individual oscillator are also plotted with dashed lines.

TABLE I. The fit parameters in the Lorentz formula at various temperatures.

T/T_c	K_1/μ	$\tau_1\mu$	ω_{01}/μ	K_2/μ	$\tau_2\mu$	ω_{02}/μ
0.6	0.207	6.593	0.452	2.225	0.609	1.629
0.7	0.207	6.327	0.449	2.512	0.498	1.653
0.8	0.188	5.870	0.442	1.587	0.802	1.354

Fig. 4. In particular, the magnitude of gap is estimated as $2\Delta/T_c \approx 20.51$ by locating the position of the second minimum in the imaginary part of the conductivity, which is obviously much larger than the mean-field BCS value $2\Delta/T_c \approx 3.52$. This large gap ratio associated with this gap is indicative of a strongly coupled CDW phase transition in our system, as should be the case by holography. On the other hand, remarkably, this large gap ratio turns out to be comparable to that of some CDW materials. For example, it follows from the experimental data on the optical conductivity that the gap ratio is given by $2\Delta/T_c \approx 15.80$ for the single crystalline $TbTe_3$ compound [24].

Now with the spectral weight transferred to our pinned collective mode and gapped single-particle excitation, the resultant CDW can be identified as an insulator, as further evidenced by the decreasing behavior of conductivity at zero frequency with the decreased temperature. Thus, our holographic CDW provides an alternative implementation of metal-insulator transition [25].

We conclude this section with the remark that in the high-frequency regime, as the temperature goes down, the contribution from higher-order CDWs will become relevant such that more peaks and gaps emerge in this regime, which have been observed in Fig. 4 when $T \leq 0.7T_c$. Describing these new resonances quantitatively requires one to go to much lower temperature, which involves heavier numerical computation and is beyond the scope of our Letter.

Discussion.—We have constructed a new type of striped black hole solution that is characterized by the condensation of CDWs. Two fundamental features of CDWs have been precisely reproduced by investigating the optical conductivity of our holographic CDWs. Together with the behavior of dc conductivity, we are successfully led to a new mechanism of the metal-insulator transition by holographic CDWs.

In addition, taking into account that the significantly large gap ratio of our holographic CDW is comparable to the experimental data on some CDW materials, our work opens a promising window for understanding the related phenomena of CDWs in condensed-matter physics by holography.

We would like to end this Letter with one important reminder. As mentioned before, we have worked in the large N limit; therefore, the dangerous infrared thermal or quantum fluctuations are parametrically suppressed as $1/N$ corrections [26,27], which explains how we can have the spontaneous breaking of translational symmetry along one dimension in a $2+1$ dimensional system, in apparent contradiction to the Landau-Peierls theorem [28–30]. Note that the two- or three-dimensional CDW phase is generically robust against fluctuations; thus, it is significant to explicitly check whether our main results obtained here are carried over onto higher-dimensional holographic CDWs, although it should be the case. But the involved numerical computation is extremally nontrivial and expected to be reported in the future.

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