Chiral Mass-Gap in Curved Space

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We discuss a new type of QCD phenomenon induced in curved space. In the QCD vacuum, a mass-gap of Dirac fermions is attributed to the spontaneous breaking of chiral symmetry. If the curvature is positive large, the chiral condensate melts but a chiral invariant mass-gap can still remain, which we name the chiral gap effect in curved space. This leads to decoupling of quark deconfinement which implies a view of black holes surrounded by a first-order QCD phase transition.

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Introduction.—Quantum field theory in curved spacetime is a well-established subject [1] with phenomenological applications to nuclear physics and condensed matter physics now being developed. In the presence of a gravitational background, the metric tensor is naturally distorted from flat Minkowskian space-time. In usual laboratory environments, gravitational effects are negligibly small as compared to typical scales in the considered theory: for example, Λ_{QCD} in quantum chromodynamics (QCD). (See Refs. [2] for a discussion of the possibility that such small effects could explain the QCD origin of dark energy.)

In the Universe, in contrast to laboratory experiments, it may be possible to imagine a situation where curvature can be as large as Λ_{QCD}^2, e.g., black holes [3]. While it is often said that ordinary quantum field theory becomes problematic in the vicinity of the event horizon of a black hole (i.e., the trans-Planckian problem), one can safely consider a region at a specifically chosen distance from the horizon where the standard model should be still valid under non-negligible gravitational effects.

Because of the complicated vacuum structure of QCD, in order to accommodate nonperturbative phenomena like dynamical generation of mass and confinement of quarks and gluons, it is conceivable to regard the black hole as an extended object surrounded by a "media" consisting of QCD vacuum. Such a crustlike content around the black hole is naturally associated with QCD phase transitions (as discussed in Refs. [4]) and should deserve a more attentive study along similar lines to those related to the recent disputes on the black hole complementarity, namely, the so-called firewall hypothesis [5]. At classical level, one may think that no drastic phenomenon should happen in the freely falling frame in which the gravitational field, even if very strong, is locally canceled. In QCD, however, hadron wave functions at rest and those in the infinite momentum frame look totally different [6]. In fact, as we discuss later, the QCD vacuum structure filled with quantum fluctuations should change drastically near, but not too close to, the horizon of the black hole.

Also in laboratory experiments, interestingly enough, descriptions based on quantum field theory in curved backgrounds are, in some cases, important. In relativistic heavy-ion collisions, hot and dense QCD matter (the quark-gluon plasma) is created, and it goes though expansion at the speed of light. Therefore, the space-time evolution has an event horizon [7], and quantum spectra in the expanding geometry (i.e., in the Bjorken coordinates) look analogous to those in Rindler coordinates. [See similarity between Eq. (27) of Ref. [8] and Eq. (5.25) of Ref. [9].] In this context, the speculative scenario that particle production in QCD may be related to a Hawking temperature characterized by the saturation scale of the strong interaction is certainly suggestive [10].

Effects of curved spacetime are also relevant for condensed matter laboratory experiments. There are, for instance, theoretical proposals (awaiting experimental confirmation) of Hawking radiation from acoustic "black holes" in atomic Bose-Einstein condensates [11] with the density correlation being an experimental signature [12]. The formulation and physical implications of the present work may have potential relevance for such systems of ultracold atoms.

In our study, we will focus on the effect of curvature on massless Dirac fermions. This problem should attract general interest not only in relation to the chiral physics of QCD, but also for condensed matter systems. Massless Dirac particles are nowadays known to emerge not only in high-energy physics: in graphene, at the interface on the topological insulators, etc. In principle, deformed materials may realize a nonzero curvature in a controllable way. Then, unlike the case of black holes, it could be sensible to consider a negative curvature as well (with a saddle point shaped deformation), which we will argue later.

The following discussions are based on the observation that Dirac fermions can have a chiral invariant mass gap due to the curvature (and we call this the “chiral gap effect”). On the algebraic level the chiral mass-gap has been partially recognized for many years [1] (See also Ref. [13]), but its application to QCD is not yet mature. Let us look quickly over the calculation scheme to demonstrate how such a chiral invariant mass arises.

Grand potential in curved space.—In fermionic systems, the effective mass $M_{\rm eff}$ with interaction clouds can differ from the bare one and when the Lagrangian has no explicit mass term (i.e., chiral limit), $M_{\rm eff}$ should be proportional to the scalar (chiral) condensate of the fermion bilinear:

$$M_{\rm eff} = G\langle\bar{\psi}\psi\rangle. \tag{1}$$

Here, G is a coupling that adjusts the proper mass dimension. Such an effective mass should solve the gap equation or, equivalently, should minimize the grand potential $\Omega[M_{\rm eff}]$. Generally speaking, the grand potential consists of two contributions: one from the tree diagrams and the other from the fermion loops. Namely, $\Omega[M_{\rm eff}] = \Omega_{\rm tree}[M_{\rm eff}] + \Omega_{\rm loop}[M_{\rm eff}]$ with

$$\beta\Omega_{\rm loop}[M_{\rm eff}] = -\nu \ln {\rm Det}(i\nabla - M_{\rm eff}), \tag{2}$$

where β is the inverse temperature and ν represents the number of fermionic degrees of freedom. In the chiral limit, thus, $M_{\rm eff}$ is the order parameter for the spontaneous breaking of chiral symmetry.

We adopt the well-known technique of iterating the Dirac operator in Eq. (2) in order to deal with a second order operator [1]. Then, we find the following:

$$\beta\Omega_{\rm loop}[M_{\rm eff}] = -\frac{\nu}{2}\ln {\rm Det}\left[\Box + M_{\rm eff}^2 + \frac{R}{4}\right]. \tag{3}$$

Here, R is the scalar curvature. We note that the d'Alembertian $\Box$ incorporates the spin connection. For maximally symmetric geometries such as (anti–)de Sitter space, it is often possible to take the above determinant exactly (see, for instance, Ref. [14] and references therein), but the final expression is too much involved to guide plain intuition.

To extract the essence of underlying physics, it is convenient to introduce a truncation scheme. Let us first take an ultrastatic metric; i.e., $g_{\tau\tau} = 1$ (in the Euclidean convention). We can actually reach this special form by means of an appropriate conformal transformation. We can then adopt the resummed heat-kernel expansion according to the Jack-Toms-Parker ansatz [15]. In fact, the determinant has ultraviolet singularities and the ζ-function regularization, as first utilized by Hawking [16], Dowker and Critchley [17], is one of the methods most frequently used, particularly in curved space-time (see, however, Ref. [18] for an example that uses cutoff regularization). We can thus put the quantity inside the determinant into the exponential as

$$\begin{aligned}&{\rm Tr}_{\rm space}\, e^{-t[-\partial_\tau^2 - \Delta + M_{\rm eff}^2 + R/4]}\\ &\quad = \frac{1}{(4\pi t)^2}\, e^{-t[-\partial_\tau^2 + M_{\rm eff}^2 + R/4 - R/6]}\sum_k {\rm tr}\, a_k t^k.\end{aligned} \tag{4}$$

Here, ${\rm Tr}_{\rm space}$ means that we take the trace over spatial coordinates only, and a_k's represent the resummed heat-kernel coefficients as listed in Ref. [15]. The crucial point is that Eq. (4) makes a full resummation with respect to R: the coefficients a_k's are functions of $R_{\mu\nu\rho\sigma}$ and $R_{\mu\nu}$ but not of R. The above expression [Eq. (4)] is valid for both constant and nonconstant curvature spacetimes. Notice that in the presence of a cosmological constant, one is led to the former case, and our considerations with constant curvature will be valid. The first few coefficients, for the case of constant curvature, read:

$$\begin{aligned}&a_0 = 1, \qquad a_1 = 0,\\ &a_2 = \frac{1}{180}[R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - R_{\mu\nu}R^{\mu\nu}] + \frac{1}{12}W_{\mu\nu}W^{\mu\nu},\end{aligned} \tag{5}$$

where $R_{\mu\nu\rho\sigma}$ and $R_{\mu\nu}$ are the Riemann and the Ricci curvature tensors, respectively, and $W_{\mu\nu} = [\nabla_\mu, \nabla_\nu]$. Notice that while the property of resummation is maintained also for spacetimes with nonconstant curvature, in this more general case, additional terms (dependent on the derivatives of R and $W_{\mu\nu}$) appear in a_2. Such modifications will not change the main results discussed below. For more calculation details, see Ref. [19]. We note that we do not take the trace over the γ matrices in Eq. (4). This means that a_k's in Eq. (5) should be interpreted as matrices with respect to the Dirac indices.

For the time being, we neglect a_k with $k > 0$ to capture the qualitative features of fermions in curved spacetime and come back to these corrections later. We note that such a truncation is justifiable if the number of dimensions, D, is large enough. Although we do not assume any specific form of the geometry except that the scalar curvature is constant, let us take simple concrete examples. For maximally symmetric geometries such as (anti–)de Sitter space, indeed, the Riemann and the Ricci tensors are as suppressed by D as

$$\frac{R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}}{R^2} = \frac{2}{D(D-1)}, \qquad \frac{R_{\mu\nu}R^{\mu\nu}}{R^2} = \frac{1}{D}. \tag{6}$$

For more general geometries, one may come to the same conclusion by using the Weyl decomposition [see formula (228) in Ref. [20]].

Once we admit the $k = 0$ dominance, we eventually come by a very simple picture, where the effect of the scalar curvature replaces the effective mass as seen in Eq. (4) as

$$M_{\rm eff}^2 \to M_{\rm eff}^2 + \frac{R}{12}. \tag{7}$$

This is a profound observation, and at the same time, seems to be puzzling at a glance. In the chiral symmetric phase, we have $M_{\rm eff} = 0$, but fermions are still gapped with $R/12$. How can we reconcile chiral symmetry and such gapped fermions?

Similarity and dissimilarity to the thermal mass.—This kind of chiral invariant mass is common in thermal field theory. In the hard thermal loop resummation, the self-energy should be inserted in the fermion propagator which produces a thermal mass [21]; nevertheless, it does not affect the commutativity between the propagator and γ_5. In the limit of vanishing spatial momenta ($\boldsymbol{p} = 0$), the fermion propagator inverse reads:

$$iS^{-1}(p_0) = p_0\gamma^0 - \frac{m_T^2}{p_0}\gamma^0 - M_{\rm eff}, \tag{8}$$

as a function of the energy p_0, where $m_T^2 = (1/6)g^2T^2$ represents the one-loop thermal mass squared. The pole is located at $p_0 = \pm m_T \neq 0$, even when $M_{\rm eff} = 0$ in the chiral symmetric phase. Although m_T^2 must be a function of T, the authors of Ref. [22] have calculated $\Omega[M_{\rm eff}]$ as a function of m_T as if m_T is an independent variable and they found the chiral transition temperature lowered with increasing m_T.

We see that the correction to the fermion mass due to R is quite similar to m_T^2. As a matter of fact, the interpretation of Eq. (7) is even simpler. If we naïvely combine it with Eq. (1), a shift in $G^2\langle\bar\psi\psi\rangle^2$ would break chiral symmetry. We should, however, identify this shift in a chiral invariant $G^2\rho$ with $\rho \equiv \langle\bar\psi\psi\rangle^2 + \langle\bar\psi i\gamma_5\boldsymbol{\tau}\psi\rangle^2$. Hence, the fermion mass gap can be consistent with chiral symmetry.

This mechanism to generate a mass-gap helps us to understand how the curvature would affect the chiral phase transition. Let us consider a spacetime of constant curvature and parametrize the ρ dependence of the grand potential as

$$\beta\Omega[\rho] = a(T - T_c)\rho + \lambda\rho^2 + \cdots \tag{9}$$

near the second-order critical point at $T = T_c$. It should be noted that we can introduce the temperature T independently from R by using the metric tensors depending on the real-time t and compactifying the manifold along the imaginary-time τ. In the presence of a finite curvature, Eq. (7) shifts ρ as $\rho + R/(12G^2)$, and so T_c is also modified as

$$\beta\Omega[\rho] \to \left[a(T - T_c) + \frac{\lambda R}{6G^2}\right]\rho + \rho^2 + \cdots, \tag{10}$$

from which the critical temperature shifts to

$$T_c^* = T_c - \frac{\lambda R}{6G^2 a}. \tag{11}$$

This is a result reminiscent of the effect of m_T on the chiral phase transition as argued in Ref. [22]. We note that this shift successfully reproduces the qualitative behavior found in solvable examples [14].

We shall point out two important differences between the roles played by m_T and R. The first is that, unlike m_T, R is an independent variable and is not constrained by T. Curvature and temperature effects are in sharp contrast in the way they manifest in realistic systems: we cannot address a quantum phase transition induced by m_T, but it would be sensible to do so for R. In fact, Eq. (11) implies that a phase transition takes place at $R = 6G^2aT_c/\lambda$ even for $T = 0$.

The second difference is that a negative shift with $R < 0$ is also possible in Eq. (7) (see Ref. [23] for an analysis of dynamical symmetry breaking in spaces of negative curvature). Then the effective mass increases by $|R|/12$, so that T_c^* should increase. This may lead us to an interesting conjecture that the chiral phase transition and the quark deconfinement could become completely distinct if R is negative large. To confirm this, however, we need to address the Yang-Mills dynamics in curved spacetime using a first-principle approach [24], which is beyond our current scope. Instead, in this work, we shall focus on the effect of fermion excitations on the quark deconfinement and find that the decoupling tendency is common also for $R > 0$.

Thermal excitation and quark deconfinement.—At finite T, we can characterize the quark deconfinement using an order parameter:

$$\Phi = \frac{1}{N_{\rm c}}{\rm tr}L, \tag{12}$$

which is called the (traced) Polyakov loop (see Ref. [25] and references therein). We can rigorously define the quark deconfinement only in the pure Yang-Mills theory that has center symmetry. In QCD with light flavors, the deconfinement crossover turns out to be smooth due to fermion interactions. As discussed in Ref. [26], the chiral phase transition controls the fermion mass, and the deconfinement would be more favored with lighter quarks after the chiral phase transition. This is a qualitative picture to understand the simultaneous crossover of quark deconfinement and chiral restoration as observed in the lattice QCD simulation.

Thermally excited fermions on the gluonic background generate terms that break center symmetry. They concretely arise from

$$\beta\Omega_{\rm loop}[M_{\rm eff}] = -N_{\rm f}\sum_{i=1}^{N_{\rm c}} \ln{\rm Det}(i\nabla - M_{\rm eff} + i\phi_i\gamma^t), \tag{13}$$

where ν is specified as the flavor number $N_{\rm f}$, and we also take the trace over color up to $N_{\rm c}$. In the determinant, a new variable ϕ_i's appear to represent the eigenvalues of the Polyakov loop matrix: $L = {\rm diag}(e^{i\phi_i})(i = 1, 2, \ldots, N_{\rm c})$. In a special gauge called the Polyakov gauge, ϕ_i's correspond to the diagonal components of the temporal gauge potential A_τ.

As we already saw, the effective mass is shifted by $R/12$, and a straightforward summation over the Matsubara frequencies yields $\Omega_{\rm loop}^{T=0} + \Omega_{\rm loop}^{T}$ with

$$\beta\Omega_{\rm loop}^{T} = -2N_{\rm f}V\int\frac{d^3p}{(2\pi)^3}\mathrm{Tr}\{\ln[1+Le^{-\beta(\varepsilon_p-\mu)}] + \ln[1+L^\dagger e^{-\beta(\varepsilon_p+\mu)}]\}, \tag{14}$$

where the quasiparticle energy dispersion is $\varepsilon_p \equiv \sqrt{p^2+M_{\rm eff}^2+R/12}$. This is a simple expression but it encompasses the essence of all complicated calculations as done in Refs. [27,28]. In flat space, usually, $M_{\rm eff}$ controls the explicit breaking of center symmetry. As soon as a nonzero R is turned on, thermally excited fermions are suppressed by not only $M_{\rm eff}$ but also R. Therefore, even in the chiral symmetric limit, if R is larger than T, fermion excitations are almost absent, so that center symmetry can be an approximate symmetry.

To quantify this speculation, let us plot the (dimensionless) magnitude of the center symmetry breaking for $\mu = 0$; i.e., $\Delta \equiv (\beta^4/V)(\Omega_{\rm loop}^{T}[\Phi=1] - \Omega_{\rm loop}^{T}[\Phi=-1])$. As we see in Fig. 1, because of a small coefficient $1/12$, we need to have hundreds times as large R as T^2 to realize decoupling.

Once the decoupling happens, the gluonic sector should behave as pure Yang-Mills theory. Thus, the quark deconfinement transition should be of first order rather than smooth crossover. One may think that the deconfinement is also eased by large R, and indeed, we remark that the infrared singularity is weakened in curved spacetime [29].

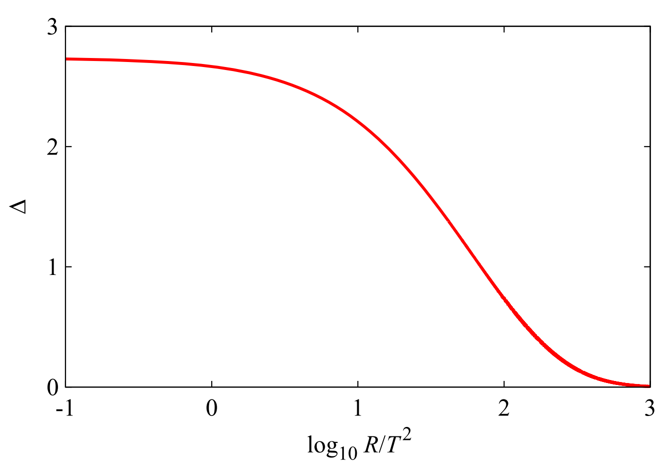

FIG. 1 (color online). Coupling strength between dynamical quarks and the gluonic sector quantified through the center symmetry breaking Δ as a function of dimensionless curvature R/T^2.

Higher-order corrections.—Higher-order corrections from the heat kernel expansion can be easily taken into account in our scheme. These terms involve combinations of Riemann and Ricci curvature tensors and correct the grand potential by $\delta\Omega_{\rm loop}$. We can utilize Eq. (5) to show that it is related to the leading-order term as

$$\delta\Omega_{\rm loop} = a_2\left(\frac{\partial}{\partial M_{\rm eff}^2}\right)^2\Omega_{\rm loop}. \tag{15}$$

It is a nontrivial finding that the correction terms take a form of mass derivatives. Then, we notice that a mass shift can reproduce the above result as $\Omega_{\rm loop}[M_{\rm eff}^2+\delta M_{\rm eff}^2] \approx \Omega_{\rm loop}[M_{\rm eff}^2] + \delta\Omega_{\rm loop}[M_{\rm eff}^2]$. We can show that the coefficient a_2 is negative in general, and then the mass squared correction turns out to be purely imaginary:

$$\delta M_{\rm eff}^2 = i\sqrt{|a_2|}, \tag{16}$$

or the self-energy has an imaginary part. Actually, one can confirm $a_2 < 0$ by plugging Eq. (6) into a_2 in Eq. (5). The appearance of complex energy dispersion indicates that the vacuum is not stable. In fact, the curvature induced particle production, as observed in Ref. [24], suggests an alteration of the vacuum persistence. Further investigations to deepen our understanding on the physical interpretation of these complex corrections is necessary.

Finally, let us also point out that Eq. (15) is proportional to the chiral susceptibility χ. Since χ is enhanced at the chiral phase transition, there may be an interesting interplay between the chiral dynamics and the curvature effect at critical point.

Discussions and summary.—Our analysis indicates that the predominant effect on fermions in curved space is the appearance of a chiral symmetric mass-gap due to the scalar curvature R, which we call the chiral mass-gap. We have shown that the mass shift is systematically formulated in a form of the resummed expansion with respect to the Riemann and the Ricci curvature tensors. The chiral mass-gap gives an intuitive explanation for the nature of the chiral phase transition in curved space; chiral symmetry tends to get restored with $R > 0$, while the chiral condensate and the chiral transition temperature becomes larger with $R < 0$. Importantly, the chiral gap effect also suggests decoupling between the chiral dynamics and the quark deconfinement.

In principle, lattice QCD simulations can verify our conjecture. So far, however, it is not easy to formulate the problem numerically for a geometry that constantly curves in space. The difficulty originates from the singularity associated with polar coordinates that are most convenient to describe curved geometries. In this sense, therefore, our analysis may be useful to guide future attempts to simulate QCD in curved space or specifically in the Schwarzchild metric.

In the future, it will be indispensable to study the gluonic sector more carefully in curved space, for which lattice simulations are the most powerful tool, but not necessarily a unique choice. One might utilize the strong-coupling expansion or employ a description based on the inverted Weiss potential [30] with nonflat metric tensors, which would be an intriguing research subject to pursue.

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