Dilaton Chiral Perturbation Theory: Determining the Mass and Decay Constant of the Technidilaton on the Lattice

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We propose a scale-invariant chiral perturbation theory of the pseudo-Nambu-Goldstone bosons of chiral symmetry (pion π) as well as the scale symmetry (dilaton ϕ) for large N_f QCD. The resultant dilaton mass M_ϕ reads $M_\phi^2 = m_\phi^2 + \frac{1}{4}(3-\gamma_m)(1+\gamma_m)(2N_f F_\pi^2/F_\phi^2)m_\pi^2 +$ (chiral log corrections), where m_ϕ, m_π, γ_m, F_π, and F_ϕ are the dilaton mass in the chiral limit, the pion mass, the mass anomalous dimension, and the decay constants of π and ϕ, respectively. The chiral extrapolation of the lattice data, when plotted as M_ϕ^2 versus m_π^2, then simultaneously determines (m_ϕ, F_ϕ) of the technidilaton in walking technicolor with $\gamma_m \simeq 1$. The chiral logarithmic corrections are explicitly given.

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Since the Higgs boson was discovered at the LHC [1], the next stage of particle physics will be to elucidate the dynamical origin of the Higgs boson, whose mass and coupling are free parameters within the standard model. One theory beyond the standard model is walking technicolor, which, based on the approximately scale-invariant gauge dynamics, predicted a large anomalous dimension $\gamma_m \simeq 1$ and a pseudo-Nambu-Goldstone (NG) boson of the approximate scale invariance ("technidilaton") as a light composite Higgs boson [2]. The technidilaton was actually shown to be consistent with current LHC data for the Higgs boson [3,4].

A strongly coupled dynamics, walking technicolor would need fully nonperturbative calculations in order to make reliable estimates of the properties of the technidilaton and other composite particles to be compared with the upcoming high statistics data at LHC. There has been much work on the lattice in the search for walking technicolor [5]. Among others, the LatKMI Collaboration [6] observed a flavor-singlet scalar meson lighter than the "pion" (corresponding to the NG boson in the chirally broken phase) in $N_f = 12$ QCD—a theory shown [7] to be consistent with the chirally unbroken (conformal) phase on the same lattice setting. Such a light scalar might be a bound state generated only in the presence of the explicit fermion mass m_f in the conformal phase. Still, it gives a good hint for the technidilaton signature in the walking theory, which should have a similar conformal dynamics, with the role of m_f instead played by the dynamical mass of the fermion generated by spontaneous chiral symmetry breaking.

Amazingly, the LatKMI Collaboration also observed indications of a light flavor-singlet scalar with comparable mass to the pion in $N_f = 8$ QCD [8]—a theory shown [9] to be walking, having both signals of spontaneous chiral symmetry breaking and a remnant of conformality. This should be a candidate for the technidilaton as a light composite Higgs boson in walking technicolor.

However, walking technicolor makes sense only for vanishing fermion mass, $m_f \equiv 0$, and hence the techinidilaton mass should be determined in the chiral limit. We would need an extrapolation formula for the dilaton mass in the same sense as the usual chiral perturbation theory (ChPT) [10] for the lattice data measured at nonzero m_f to be extrapolated to the chiral limit.

In this Letter, we propose a scale-invariant ChPT (sChPT) for the use of chiral extrapolation of the lattice data on the dilaton and the pion in the presence of explicit mass of the fermion m_f. It is a scale-invariant generalization of the usual ChPT [10], based on the nonlinear realization of chiral symmetry in a way to realize the symmetry structure of the underlying walking gauge theory.

The theory consists of the pseudo-NG bosons of the chiral symmetry (pion π, with mass m_π) as well as the scale symmetry (dilaton ϕ, with mass M_ϕ), where both symmetries are broken spontaneously by the fermion-pair condensate, and also explicitly by both the fermion mass m_f and the nonperturbative scale anomaly (induced by the same fermion-pair condensate) [11]. We obtain a tree-level formula in M_ϕ^2 versus m_π^2, Eq. (8), which can be plotted linearly in such a way that the intercept determines the chiral limit dilaton mass m_ϕ^2 (technidilaton mass), while its slope gives the technidilaton decay constant F_ϕ, defined as $\langle 0|D^\mu(0)|\phi(q)\rangle = -iF_\phi q^\mu$, and hence $\langle 0|\partial_\mu D^\mu(0)|\phi(q)\rangle = -F_\phi M_\phi^2$, where $D^\mu(x)$ is the dilatation current. Based on the sChPT we also explicitly calculate one-loop corrections of the chiral logarithm, Eq. (10), which turn out to be negligibly small in current lattice simulations.

Let us start with the chiral and/or scale Ward-Takahashi (WT) identities for the axialvector ($J_5^{a\mu}$) [dilatation (D^μ)] currents in the underlying walking gauge theory with N_f-fermion fields (ψ):

$$\theta^\mu_\mu = \partial_\mu D^\mu = \frac{\beta_{\rm NP}(\alpha)}{4\alpha} G^2_{\mu\nu} + (1+\gamma_m) N_f m_f \bar\psi\psi, \quad \partial_\mu J_5^{a\mu} = 2 m_f \bar\psi i\gamma_5 T^a \psi, \tag{1}$$

where T^a ($a = 1, \ldots, N_f^2 - 1$) are the $\mathrm{SU}(N_f)$ generators and $\beta_{\rm NP}(\alpha)$ is the nonperturbative β function for the nonperturbative running [12] of the gauge coupling α due to the mass scale Λ_χ dynamically generated by the spontaneous breaking of the chiral and scale symmetries through the condensate $\langle(\bar\psi\psi)_{\mu=\Lambda_\chi}\rangle \sim -\Lambda_\chi^3$. $(\beta_{\rm NP}(\alpha)/4\alpha)G^2_{\mu\nu}$ is the nonperturbative trace (scale) anomaly [11,13] defined as a part associated with the nonpertubative running and is induced solely by the chiral condensate with the scale Λ_χ: $\langle(\beta_{\rm NP}(\alpha)/4\alpha)G^2_{\mu\nu}\rangle|_{m_f=0} \sim -\Lambda_\chi^4$.

We now formulate the sChPT so as to reproduce these WT identities. The building blocks $\varphi(x)$ to construct the sChPT are $\varphi(x) = \{U(x), \chi(x), \mathcal{M}(x), S(x)\}$. $U(x) = e^{2i\pi(x)/F_\pi}$, $\pi \equiv \pi^a T^a$, is the usual chiral field with the pion decay constant F_π, and $\chi(x) = e^{\phi(x)/F_\phi}$ with the dilaton field $\phi(x)$ and the decay constant F_ϕ. $\mathcal{M}(x)$ and $S(x)$ are spurion fields introduced so as to incorporate explicit breaking effects of the chiral and scale symmetry, respectively. Under the chiral $\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R$ symmetry, these building blocks transform as $U(x) \to g_L \cdot U(x) \cdot g_R^\dagger$, $\mathcal{M}(x) \to g_L \cdot \mathcal{M}(x) \cdot g_R^\dagger$, $\chi(x) \to \chi(x)$, and $S(x) \to S(x)$, with $g_{L,R} \in \mathrm{SU}(N_f)_{L,R}$. Under the scale symmetry they are infinitesimally transformed as $\delta U(x) = x_\nu \partial^\nu U(x)$, $\delta\mathcal{M}(x) = x_\nu \partial^\nu \mathcal{M}(x)$, $\delta\chi(x) = (1 + x_\nu\partial^\nu)\chi(x)$, and $\delta S(x) = (1 + x_\nu \partial^\nu) S(x)$, with scale dimensions $d_U = d_\mathcal{M} = 0$, $d_\chi = d_S = 1$. The rule of chiral-order counting [10] is thus determined consistently with both the scale and chiral symmetries: $U \sim \chi \sim S \sim \mathcal{O}(p^0)$, $\mathcal{M} \sim m_f \sim \mathcal{O}(p^2)$, $\partial_\mu \sim m_\pi \sim M_\phi \sim \mathcal{O}(p)$, where m_π and M_ϕ are pion and dilaton masses arising from the vacuum expectation values of the spurion fields $\mathcal{M}$ and S, $\langle\mathcal{M}\rangle = m_\pi^2 \times \mathbf{1}_{N_f\times N_f}$, and $\langle S\rangle = 1$.

We first consider the chiral limit $m_f \to 0$. To the leading order $\mathcal{O}(p^2)$ of sChPT, the chiral Lagrangian for the scale-invariant action is uniquely determined as [14]

$$\mathcal{L}^{\rm inv}_{(2)} = \frac{F_\phi^2}{2}(\partial_\mu\chi)^2 + \frac{F_\pi^2}{4}\chi^2 \mathrm{tr}[\partial_\mu U^\dagger \partial^\mu U]. \tag{2}$$

As noted above, even in the chiral limit, the scale symmetry is explicitly broken by the dynamical generation of the fermion mass itself in the underlying walking gauge theory ("hard-scale anomaly," or scale violation by the marginal operator) characteristic to the conformal phase transition [15]. Hence, we have $4E = \langle\theta^\mu_\mu\rangle_{m_f=0} = F_\phi/d_\theta \langle 0|\theta^\mu_\mu|\phi\rangle_{m_f=0} = F_\phi/4\langle 0|\partial_\mu D^\mu|\phi\rangle_{m_f=0} = -(F_\phi^2 m_\phi^2/4) < 0$ [partially conserved dilatation current (PCDC) relation] [16], where m_ϕ denotes the chiral-limit dilaton mass and we understand that the scale dimension of θ^μ_μ is equal to the canonical dimension, $d_\theta = 4$, for $m_f = 0$.

We may incorporate the corresponding explicit breaking terms, involving the spurion field S, to make the action formally scale invariant [17]:

$$\mathcal{L}^S_{(2)\rm hard} = -\frac{F_\phi^2}{4} m_\phi^2 \chi^4 \left(\log\frac{\chi}{S} - \frac{1}{4}\right). \tag{3}$$

This is a unique form having scale dimension four, which correctly reproduces the underlying nonperturbative scale anomaly $(\beta_{\rm NP}(\alpha)/4\alpha)\langle G^2_{\mu\nu}\rangle$ in the scale WT identity, Eq. (1), in the chiral limit $m_f \to 0$. In fact, when $\langle S\rangle = 1$, a noninvariant term arises from $\log\chi$ to yield the scale anomaly $\langle\theta^\mu_\mu\rangle = \langle\partial_\mu D^\mu\rangle = \langle\delta\mathcal{L}^S_{(2)\rm hard}\rangle = -F_\phi^2 m_\phi^2 \langle\chi^4\rangle/4$, in accord with the PCDC relation. The last factor, $-1/4$, yields a correct vacuum energy $E = \langle -\mathcal{L}^S_{(2)\rm hard}\rangle = -F_\phi^2 m_\phi^2/16 = \langle\theta^\mu_\mu\rangle/4$.

As was discussed in Ref. [18], the explicit breaking terms due to the fermion current mass m_f may also be introduced so as to reproduce the chiral WT identity in Eq. (1):

$$\mathcal{L}^S_{(2)\rm soft} = \frac{F_\pi^2}{4}\left(\frac{\chi}{S}\right)^{3-\gamma_m} S^4 \mathrm{tr}[\mathcal{M}^\dagger U + U^\dagger \mathcal{M}] - \frac{(3-\gamma_m)F_\pi^2}{8}\chi^4 (N_f \mathrm{tr}[\mathcal{M}^\dagger\mathcal{M}])^{1/2}. \tag{4}$$

The factor $(3-\gamma_m)$ in the first line reflects the full dimension of the fermion bilinear operator $\bar\psi\psi$ in the underlying gauge theory. The scale-invariant term in line two, having no contributions to θ^μ_μ, was introduced in the case without the hard-scale anomaly term $\mathcal{L}^S_{(2)\rm hard}$ [18,19] in order to stabilize the dilaton potential so as to make the otherwise tachyonic dilaton mass term positive, $M_\phi^2 > 0$.

The Lagrangian for the scale-invariant and chirally invariant action at leading order $\mathcal{O}(p^2)$ is thus constructed from terms in Eqs. (2)–(4):

$$\mathcal{L}_{(2)} = \mathcal{L}^{\rm inv}_{(2)} + \mathcal{L}^S_{(2)\rm hard} + \mathcal{L}^S_{(2)\rm soft}. \tag{5}$$

From this we finally read off the dilaton mass term ϕ^2 as [20]

$$M_\phi^2 = m_\phi^2 + (1+\gamma_m)(3-\gamma_m)\frac{N_f F_\pi^2 m_\pi^2}{2F_\phi^2}. \tag{6}$$

Our result can also be derived directly from the underlying gauge theory through Eq. (1) as [21]

$$\langle 0|\theta^\mu_\mu|\phi\rangle = \langle 0|\frac{\beta_{\rm NP}(\alpha)}{4\alpha}G^2_{\mu\nu}|\phi\rangle + (1+\gamma_m)N_f m_f\langle 0|\bar\psi\psi|\phi\rangle. \tag{7}$$

We may further rewrite the dilaton mass, Eq. (6), in a form convenient for lattice simulations:

$$M^2_\phi = m^2_\phi + s m^2_\pi,$$
$$s \equiv \frac{(3-\gamma_m)(1+\gamma_m)}{4}\frac{2N_f F^2_\pi}{F^2_\phi} \simeq \frac{2N_f F^2_\pi}{F^2_\phi} \equiv r, \tag{8}$$

where the prefactor $(3-\gamma_m)(1+\gamma_m)/4 = 1-(\delta/2)^2 \simeq 1$ $[\delta \equiv 1-\gamma_m;\ (\delta/2)^2 \ll 1]$ is very insensitive to the exact value of γ_m as long as $\gamma_m \simeq 1$ in walking gauge theory.

This is our main result. It is useful for determining simultaneously the chiral limit values of both the mass m_ϕ and the decay constant F_ϕ of the flavor-singlet scalar meson as the technidilaton of walking technicolor on the lattice. Simultaneously fitting the intercept and the slope of a plot of M^2_ϕ versus m^2_π from the lattice data would give m^2_ϕ (intercept) and F_ϕ through the slope parameter $s \simeq r \equiv 2N_f F^2_\pi/F^2_\phi$ [22]. For a given N_f all the quantities γ_m, F_π, F_ϕ, and m_π in the expression of the slope parameter s can be measured separately in lattice simulations on the same set up. Hence, measuring s would be a self-consistency check of the simulations as a dilaton observation, when compared with the value of F_ϕ determined by some other way. In Fig. 1 we present plots $(x,y) = (m^2_\pi, M^2_\phi)$ of mock-up data for the general case $s \simeq r = (0.2, 0.5, 1.0)$ in the one-family model, $N_f = 8$ (4 weak doublets) with the electroweak (EW) scale related to the pion decay constant as $F_\pi = v_{\rm EW}/\sqrt{4} \simeq 123$ GeV, by normalizing the masses to a chiral breaking scale $\Lambda_\chi = 4\pi F_\pi/\sqrt{N_f}$. The first number ($s = 0.2$) corresponds to a phenomenologically favorable value [3,4], $F_\phi \simeq \sqrt{2N_f}F_\pi/0.44 \simeq 1.1$ TeV, consistent with the current Higgs boson data at LHC. The third one ($s = 1.0$) is the holographic estimate in the large N_c limit [4]. The second value ($s = 0.5$) is just a sample number in between. The close-up window on the top left-hand panel in the figure shows that the dilaton mass gets larger than m_π when the ChPT expansion parameter $\mathcal{X} \equiv m^2_\pi/\Lambda^2_\chi = N_f m^2_\pi/(4\pi F_\pi)^2 \lesssim 0.06(0.1)$ for $s = 0.2(0.5)$. Note also that for $s < 1$, there exists a crossing point where $M^2_\phi < m^2_\pi$ changes to $M^2_\phi > m^2_\pi$ near the chiral limit, as noted in Ref. [8].

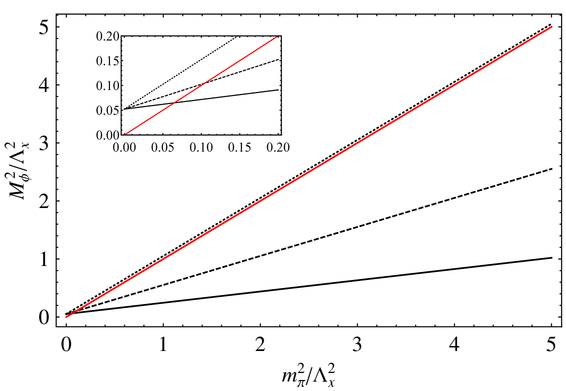

FIG. 1 (color online). A plot of M^2_ϕ/Λ^2_χ with respect to $m^2_\pi/\Lambda^2_\chi(\equiv\mathcal{X})$ obtained from Eq. (8), with $N_f = 8$ and $F_\pi = 123$ GeV and the chiral-limit dilaton mass $m_\phi = 125$ GeV. The slope $s \simeq r = 2N_f F^2_\pi/F^2_\phi$ in Eq. (8) has been taken to be 0.2 (solid black line), 0.5 (dashed black line), and 1.0 (dotted black line). The solid red line corresponds to $M^2_\phi = m^2_\pi$. The inset figure just denotes the close-up window for the small pion mass region.

As in the case of the usual ChPT [10], chiral logarithmic corrections at the loop level would modify the chiral scaling of the dilaton mass formula in Eq. (8). Since the dilaton remains massive in the chiral limit due to the nonperturbative scale anomaly, only the pion loop corrections become significant for the chiral scaling of the dilaton mass. Such chiral logarithmic corrections will be operative in the soft-pion region $m_\pi \lesssim M_\phi$ (corresponding to the region where ChPT is valid: $\mathcal{X} \equiv m^2_\pi/\Lambda^2_\chi \lesssim 0.1$ in Fig. 1). We shall compute the chiral logarithmic corrections coming from the pion loops arising from the vertices at the leading $\mathcal{O}(p^2)$ Lagrangian Eq. (5). Those corrections softly break the scale symmetry by the form $\sim(1,r)\cdot\mathcal{X}\log\mathcal{X}$ when the cutoff Λ is identified with Λ_χ, which will be renormalized by the soft-breaking $\mathcal{O}(p^4)$ counterterms proportional to $m^2_\pi \sim \mathcal{M}$.

Using dimensional regularization [23], we thus find the $D = 4$ pole (logarithmically divergent) contributions to the terms in quadratic order of dilaton fields:

$$\frac{1}{2}Z_{F_\phi}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\tilde m^2_\phi\phi^2, \tag{9}$$

where

$$Z_{F_\phi} = 1 + r\frac{N^2_f-1}{2N^2_f}\mathcal{X}\log\frac{\Lambda^2}{m^2_\pi},$$
$$\tilde m^2_\phi = \left[m^2_\phi - r m^2_\pi\frac{2(N^2_f-1)}{N^2_f}\mathcal{X}\log\frac{\Lambda^2}{m^2_\pi} + \frac{(3-\gamma_m)(1+\gamma_m)}{4}r m^2_\pi Z_{F_\phi}Z^{-1}_{F_\pi}Z^{-1}_{m_\pi}\right],$$

with $Z_{(i=F_\pi,m_\pi)} = 1+(\Gamma_i/N_f)\mathcal{X}\log(\Lambda^2/m^2_\pi)$ and $\Gamma_{F_\pi} = N_f/4$ and $\Gamma_{m_\pi} = -1/N_f$. After renormalizing the divergent parts at the renormalization scale μ [24] and defining the renormalized dilaton field $\phi_r = \sqrt{Z_{F_\phi}}\phi$, we find the renormalized ϕ^2 terms, $\frac{1}{2}\partial_\mu\phi_r\partial^\mu\phi_r - \frac{1}{2}M^2_\phi\phi^2_r$, with the dilaton mass including the chiral logarithmic corrections of $\mathcal{O}(p^4)$:

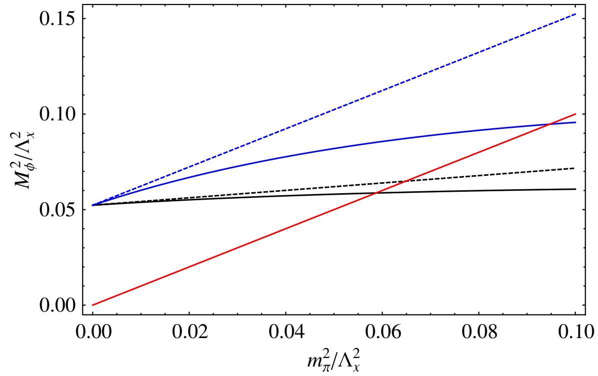

FIG. 2 (color online). A plot of M_ϕ^2/Λ_χ^2 with respect to $m_\pi^2/\Lambda_\chi^2(\equiv \mathcal{X})$ including the chiral logarithmic corrections in Eq. (10) for the one-family model with $N_f = 8$, $F_\pi = 123$ GeV, the chiral-limit mass $m_\phi = 125$ GeV, and the factor $s \simeq r = 2N_f F_\pi^2/F_\phi^2 = (0.2, 1.0)$ (solid black and blue curves). The leading-order scalings in Eq. (8) are also depicted for $s = 0.2$ and 1.0 by dashed black and blue lines, respectively. The solid red line corresponds to $M_\phi^2 = m_\pi^2$.

$$M_\phi^2 = m_\phi^2\left[1 + r\frac{N_f^2-1}{2N_f^2}\mathcal{X}\log\frac{m_\pi^2}{\mu^2}\right] + rm_\pi^2\left[\frac{2(N_f^2-1)}{N_f^2}\mathcal{X}\log\frac{m_\pi^2}{\mu^2}\right] + sm_\pi^2\left[1 + \frac{N_f^2-4}{4N_f^2}\mathcal{X}\log\frac{m_\pi^2}{\mu^2}\right] + (\text{counterterms renormalized at}\,\mu). \tag{10}$$

We may assume that all the counterterms in Eq. (10) vanish at $\mu = \Lambda_\chi$, so that they are induced only by the pion loops in the sChPT. As a concrete example, we again consider the one-family model with $N_f = 8$ and $F_\pi = 123$ GeV, and take the factor $s \simeq r = 2N_f F_\pi^2/F_\phi^2 = 0.2, 1.0$ and the chiral-limit dilaton mass $m_\phi = 125$ GeV in the light of the LHC. In Fig. 2 we plot the chiral scaling behavior of the dilaton mass for a small pion mass region $\mathcal{X} \equiv m_\pi^2/\Lambda_\chi^2 \lesssim 0.1$, including the chiral logarithmic corrections from the pions at the one-loop level for $s = 0.2$ and 1.0 (solid black and blue curves). Also plotted is the leading-order formula in Eq. (8) (dashed black and blue curves). The figure implies that the chiral logarithmic effect may be appreciable for the soft-pion mass region. However, such chiral logarithmic effects are negligibly small for the current status of $N_f = 8$ QCD on the lattice, where simulations have been performed for a larger pion mass region $3 \lesssim \mathcal{X} \lesssim 5$ [9].

In conclusion, we have established a scale-invariant chiral perturbation theory for the pseudo-NG bosons, the pion (π), and the dilaton (ϕ), which will be useful in its own right in various situations. It is straightforward [25] to include the vector mesons into this framework via hidden local symmetry [26]. As its prominent consequence, we obtained a formula relating the masses M_ϕ^2 versus m_π^2, Eq. (8) (tree), or Eq. (10) (one loop), which we believe plays a vital role for making chiral extrapolations of lattice data of the flavor-singlet scalar meson, thereby obtaining the mass (m_ϕ) and decay constant (F_ϕ) of the technidilaton as a composite Higgs boson in walking technicolor.

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Note added.—Recently, the LatKMI Collaboration published a paper [27] (follow-up of Ref. [8]) finding a light flavor-singlet scalar in $N_f = 8$ QCD, with the data analyzed based on Eq. (8) to be roughly consistent with 125 GeV Higgs boson as the technidilaton.

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anomaly, $\langle\theta_\mu^\mu\rangle_{\rm perturbation}$ ($\sim -\Lambda_{\rm QCD}^4$), is characterized by the intrinsic scale $\Lambda_{\rm QCD}$, a scale responsible for the asymptotically free (perturbative) running of the coupling in the ultraviolet region. Hence, $\langle\theta_\mu^\mu\rangle$ is nonzero only in the broken phase, $\langle\theta_\mu^\mu\rangle \sim -\Lambda_\chi^4$, where Λ_χ, in sharp contrast to the usual QCD, where $\Lambda_{\rm QCD} \sim \Lambda_\chi$. For details see, e.g., the third reference in Ref. [11].

[14] The Lagrangian is constructed uniquely (up to total derivatives) by the requirement that the action $S[\varphi(x)] = \int d^4x\mathcal{L}[\varphi(x)]$ be invariant under the scale (and also chiral) transformation $\delta\varphi(x) = (d_\varphi + x_\mu\partial^\mu)\varphi(x)$, $\delta x_\mu = -x_\mu$. This implies that $\delta S[\varphi(x)] = \int(\delta d^4x\mathcal{L} + d^4x\delta\mathcal{L}) = \int d^4x(-4\mathcal{L} + d_\mathcal{L}\mathcal{L}) = 0$ namely, the scale dimension of $\mathcal{L}$ must be 4, $d_\mathcal{L} = 4$. Equation (2) is a unique chirally invariant Lagrangian having scale dimension four and a correct kinetic term of $\varphi = (\phi, \pi)$. Note that the dimensionless field $\chi(x)$ transforms as an operator with the scale dimension one, with $\langle\chi\rangle = 1$. Similarly, the Lagrangians, Eqs. (2)–(4) are unique in a way to have the scale dimension four and to satisfy the anomalous WT identities as well as the stable vacuum. See also Ref. [19].

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[19] The coefficient of the scale-invariant term χ^4 is uniquely determined as in Eq. (4) by the vacuum stability requirement to be proportional to the fermion mass m_f, and hence does not exist in the chiral limit Lagrangian Eq. (2) with $m_f \to 0$. See Ref. [18] for details. Thus, our Lagrangian, Eq. (5), as well as Eq. (2), is unique at order $\mathcal{O}(p^2)$.

[20] A similar formula was also discussed in a completely different context, i.e., hadron physics, which we believe has no approximate scale symmetry and is irrelevant to our discussions. See, e.g., J. Ellis, Nucl. Phys. **B22**, 478 (1970); R. J. Crewther and L. C. Tunstall, arXiv:1203.1321.

[21] Note that $\langle 0|\theta_\mu^\mu|\phi\rangle_{m_f\neq 0} = \langle 0|\partial_\mu D^\mu|\phi\rangle_{m_f\neq 0} = -F_\phi M_\phi^2$, $\langle 0|(\beta_{\rm NP}(\alpha)/4\alpha)G_{\mu\nu}^2|\phi\rangle = \langle 0|\partial_\mu D^\mu|\phi\rangle_{m_f=0} = -F_\phi m_\phi^2$, while the second term on the right-hand side of Eq. (6) comes from the scale-WT identity (see [16]): $\langle 0|m_f\bar\psi\psi|\phi\rangle = (3-\gamma_m)2m_f\langle\bar\psi\psi\rangle/(2F_\phi)$, when combined with the chiral-WT identity (Gell-Mann-Oakes-Renner relation); $2m_f\langle\bar\psi\psi\rangle\delta^{\alpha\beta} = \langle[iQ_5^\beta, 2m_f\bar\psi i\gamma_5 T^\alpha\psi]\rangle = F_\pi\langle 0|\partial_\mu J_5^{\alpha,\mu}|\pi^\beta\rangle = -F_\pi^2 m_\pi^2\delta^{\alpha\beta}$, where $Q_5^\alpha = \int d^3xJ_5^{\alpha,0}(x)$. [Note that θ_μ^μ in Eq. (1) need not be a subtracted one in this derivation, since the constant subtraction $\langle\theta_\mu^\mu\rangle_{\rm perturbation}$ sandwiched by $\langle 0|$ and $|\phi\rangle$ is zero.]

[22] Note that r has no explicit N_f independence, since F_ϕ^2 is associated with the flavor-singlet operator having a sum of N_f-flavor contributions; namely, F_ϕ^2 is proportional to N_f, while F_π^2 is not.

[23] One might suspect that typical regularization schemes such as dimensional regularization with a scale $\mu \sim \Lambda_\chi = 4\pi F_\pi/\sqrt{N_f}$ would explicitly break the scale symmetry even in the chiral limit $m_f \sim m_\pi^2 \to 0$. However, such hard-breaking terms correspond to the nonperturbative scale anomaly due to the dynamical generation of the fermion mass of order $\mathcal{O}(\Lambda_\chi)$, and should be absorbed by redefining the nonperturbative contributions to the scale anomaly: $\theta_\mu^\mu = \theta_{\mu{\rm full}}^\mu - \langle\theta_\mu^\mu\rangle_{\rm perturbation}$ [11]. Thus, such a scale breaking by the regularization of order Λ_χ is irrelevant to our chiral log effects, which are the far infrared physics of near massless pion loops. We can thus safely perform the loop calculation by focusing only on the soft-scale breaking terms arising as the soft-pion effects.

[24] The logarithmically divergent parts can be absorbed by introducing the $\mathcal{O}(p^4)$ counterterms in the chiral and scale invariant form along with the spurion fields $\mathcal{M}$ and S:

$$
\begin{aligned}
\mathcal{L}_{(4)}^{\rm counterterm} = &L_4{\rm tr}[\partial_\mu U^\dagger\partial^\mu U]{\rm tr}[\mathcal{M}^\dagger U + U^\dagger\mathcal{M}]S^2 \\
&+ L_5{\rm tr}[\partial_\mu U\partial^\mu U^\dagger(\mathcal{M}^\dagger U + U^\dagger\mathcal{M})]S^2 \\
&+ L_6({\rm tr}[\mathcal{M}^\dagger U + U^\dagger\mathcal{M}])^2 S^4 \\
&+ L_8{\rm tr}[\mathcal{M}^\dagger U\mathcal{M}^\dagger U + \mathcal{M}U^\dagger\mathcal{M}U^\dagger]S^4 \\
&+ H_2{\rm tr}[\mathcal{M}^\dagger\mathcal{M}]S^4 + L_4^\chi\partial_\mu\chi\partial^\mu\chi{\rm tr}[\mathcal{M}^\dagger U + U^\dagger\mathcal{M}] \\
&+ L_6^\chi({\rm tr}[\mathcal{M}^\dagger U + U^\dagger\mathcal{M}])^2\left[\left(\frac{\chi}{S}\right)^4 - 1\right]S^4 \\
&+ L_8^\chi{\rm tr}[\mathcal{M}^\dagger U\mathcal{M}^\dagger U + \mathcal{M}U^\dagger\mathcal{M}U^\dagger]\left[\left(\frac{\chi}{S}\right)^4 - 1\right]S^4 \\
&+ H_2^\chi{\rm tr}[\mathcal{M}^\dagger\mathcal{M}]\left[\left(\frac{\chi}{S}\right)^4 - 1\right]S^4,
\end{aligned}
$$

where the chiral coefficients $L_{(i=4,5,6,8)}$ and H_2 absorb the divergences in Z_{F_π} and Z_{m_π} in Eq. (9) (which get the same renormalization effects as those in the usual chiral perturbation theory [10]), while the others $L_{(i=4,6,8)}^\chi$ and H_2^χ renormalize the log terms having F_ϕ^2 in Z_{F_ϕ} and $\tilde m_\phi^2$.

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[26] M. Bando, T. Kugo, and K. Yamawaki, Phys. Rep. **164**, 217 (1988); M. Harada and K. Yamawaki, Phys. Rep. **381**, 1 (2003).

[27] Y. Aoki, T. Aoyama, M. Kurachi, T. Maskawa, K. Miura, K. I. Nagai, H. Ohki, E. Rinaldi, A. Shibata, K. Yamawaki, and T. Yamazaki, Phys. Rev. D **89**, 111502(R) (2014).