Ambitwistor Strings in Four Dimensions

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We develop ambitwistor string theories for four dimensions to obtain new formulas for tree-level gauge and gravity amplitudes with arbitrary amounts of supersymmetry. Ambitwistor space is the space of complex null geodesics in complexified Minkowski space, and in contrast to earlier ambitwistor strings, we use twistors rather than vectors to represent this space. Although superficially similar to the original twistor string theories of Witten, Berkovits, and Skinner, these theories differ in the assignment of world sheet spins of the fields, rely on both twistor and dual twistor representatives for the vertex operators, and use the ambitwistor procedure for calculating correlation functions. Our models are much more flexible, no longer requiring maximal supersymmetry, and the resulting formulas for amplitudes are simpler, having substantially reduced moduli. These are supported on the solutions to the scattering equations refined according to helicity and can be checked by comparison with corresponding formulas of Witten and of Cachazo and Skinner.

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Introduction.—The twistor string theories of Witten, Berkovits, and Skinner [1–3] not only led to remarkable new formulas for tree-level scattering amplitudes but also provided a tantalizing paradigm for how twistor theory might eventually make contact with physics. However, it is still a long way from being fully realized both because of the reliance on maximal supersymmetry of these models and the lack of a clear route to an extension to a critical model in which loop calculations will make sense. More recently, it has become clear that there are many such remarkable tree-level formulas for scattering amplitudes [4–10]. One family [9] was seen to arise naturally from string theories in ambitwistor space, the space of complex null geodesics [11]. Such ambitwistor spaces can be defined in all dimensions, and the string is critical in 10 dimensions. It provides an infinite tension chiral limit of the conventional Ramond-Neveu-Schwarz superstring. An important advantage over the original twistor strings is that it extends naturally to provide a tentative candidate for the full field theory all-loop integrand [12] (albeit one that is likely to reproduce standard field theory divergences).

Ambitwistor strings can be defined almost algorithmically by complexifying the action for a spinning massless particle. While the focus of Ref. [11] was the Ramond-Neveu-Schwarz model, we specialize to four spacetime dimensions in this Letter. Ambitwistor space then has an alternative spinorial representation in which the constraints $P^2 = 0$ are explicitly solved. The resulting ambitwistor string models arise as the complexifications of the four-dimensional Ferber superparticle [13]. Indeed, the original twistor string was similarly interpreted in Refs. [14,15], and the similarity with the ambitwistor approach was also remarked upon in Refs. [11,16]. Using the spinorial representation as the target space, we construct ambitwistor string models for Yang-Mills theory and gravity in four dimensions with any amount of supersymmetry. These models yield remarkably simple new formulas for the tree-level scattering amplitudes that are parity invariant, supported on the scattering equations, and dependent on very few moduli. There is also a reasonably clear route to an extension to a critical theory that will be valid at loops by reduction from a critical model such as that in Ref. [17].

Ambitwistor strings in four dimensions.—Projective ambitwistor space $\mathbb{PA}$ is a supersymmetric extension of the space of complex null geodesics. In four dimensions, ambitwistor space can be expressed as a quadric $Z\cdot W = 0$ inside $\mathbb{PT}\times\mathbb{PT}^*$. Here, we work with $\mathcal{N}$ supersymmetries so that $Z = (\lambda_\alpha, \mu^{\dot\alpha}, \chi^r) \in \mathbb{T} = \mathbb{C}^{4|\mathcal{N}}$, $W = (\tilde\mu, \tilde\lambda, \tilde\chi) \in \mathbb{T}^*$ with $\chi, \tilde\chi$ fermionic, $\alpha = 0, 1$, $\dot\alpha = \dot 0, \dot 1$ chiral spinor indices, and $r = 1, \ldots\mathcal{N}$ R-symmetry indices. Ambitwistor space $\mathbb{A}$ is the set $Z\cdot W = 0$ where

$$Z\cdot W \coloneqq \lambda_\alpha\tilde\mu^\alpha + \mu^{\dot\alpha}\tilde\lambda_{\dot\alpha} + \chi^r\tilde\chi_r,$$

where we also quotient by the scalings $\Upsilon - \tilde\Upsilon$ where $\Upsilon = Z\cdot\partial/\partial Z$ and $\tilde\Upsilon = W\cdot\partial/\partial W$. We also have the incidence relations

$$\mu^{\dot\alpha} = i(x^{\alpha\dot\alpha} + i\theta^{r\alpha}\tilde\theta_r^{\dot\alpha})\lambda_\alpha, \qquad \chi^r = \theta^{r\alpha}\lambda_\alpha,$$
$$\tilde\mu^\alpha = -i(x^{\alpha\dot\alpha} - i\theta^{r\alpha}\tilde\theta_r^{\dot\alpha})\tilde\lambda_{\dot\alpha}, \qquad \tilde\chi_r = \tilde\theta_r^{\dot\alpha}\tilde\lambda_{\dot\alpha}, \qquad (1)$$

which realize a point $(x, \theta, \tilde\theta)$ in (nonchiral) super-Minkowski space as a quadric $\mathbb{CP}^1\times\mathbb{CP}^1$ parametrized by $(\lambda, \tilde\lambda)$. It is easily seen that these lie inside the set $Z\cdot W = 0$, and indeed, these are the only quadrics in $\mathbb{PA}$ of that degree. To make contact with null geodesics, the momenta can be defined to be $P_{\alpha\dot\alpha} = \lambda_\alpha\tilde\lambda_{\dot\alpha}$, which now

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automatically satisfy the constraint $P^2 = 0$. The symplectic potential is $\Theta = (i/2)(Z{\cdot}dW - W{\cdot}dZ)$.

Our ambitwistor string consists of world sheet fields (Z, W) that are spinors on the world sheet Riemann surface Σ and take values in $\mathbb{T}\times\mathbb{T}^*$. The action is based on the symplectic potential and the constraint $Z{\cdot}W = 0$ imposed by Lagrange multiplier a, a (0,1) form on Σ, so that

$$S = \frac{1}{2\pi}\int_\Sigma W{\cdot}\bar\partial Z - Z{\cdot}\bar\partial W + aZ{\cdot}W.$$

There is a gauge symmetry associated with a that quotients by $\Upsilon - \tilde\Upsilon$. A key difference between this theory and the original Berkovits-Witten theory is that we fix (Z, W) to be spinor fields on the world sheet.

After adding world sheet gravity by starting with the operator $\bar\partial + e\partial$, we gauge fix $e = 0$ leading to a ghost (b, c) system. Similarly, we gauge fix a to obtain the natural $\bar\partial$ operator on the spin bundle and introduce associated ghosts (u, v). We end up with a BRST operator

$$Q = \int cT + uZ{\cdot}W$$

where T is the world sheet stress tensor. This will be anomalous (i.e., $Q^2 \neq 0$) in general, although there should be choices of matter that give an anomaly free theory. At tree level, however, this will not be relevant, and the ghost system will serve to give the $GL(2,\mathbb{C})$ quotients that are needed in the tree-level formulas. Amplitudes will be obtained as correlation functions of vertex operators. In the following, we will just give integrated vertex operators (they simply differ by a factor of c from their unintegrated counterparts) and will instead just divide by the volume of $\mathrm{GL}(2,\mathbb{C})$ in the final formula, understood in the usual Faddeev-Popov sense.

Yang-Mills amplitudes.—Yang-Mills vertex operators arise from general wave functions α that are $\bar\partial$ closed (0,1) forms multiplied by the currents $J{\cdot}t_a$ of some current-algebra J associated to some Lie algebra, of which the t_a are elements (hereon $a = 1, \ldots, n$ indexes particle number), to give $\mathcal{V}_a = \int_\Sigma \alpha J{\cdot}t_a$. In general, such an α corresponds to an off-shell Maxwell field on space-time, but if it extends off $\mathbb{PA}$ into $\mathbb{PT}\times\mathbb{PT}^*$ to third order or beyond, it must be on shell (see for example Ref. [18]), and only when on shell is it manifestly Q closed. On shell, such wave functions are a sum of wave functions pulled back from either twistor space or dual twistor space, thus, leading to two different types of vertex operators. For momentum eigenstates,

$$\mathcal{V}'_a = \int \frac{ds_a}{s_a}\bar\delta^2(\lambda_a - s_a\lambda)e^{is_a([\mu\tilde\lambda_a]+\chi^r\tilde\eta_{ar})}J{\cdot}t_a, \qquad (2)$$

$$\tilde{\mathcal{V}}_a = \int \frac{ds_a}{s_a}\bar\delta^2(\tilde\lambda_a - s_a\tilde\lambda)e^{is_a(\langle\tilde\mu\lambda_a\rangle+\tilde\chi_r\eta_a^r)}J{\cdot}t_a, \qquad (3)$$

where for a complex variable z, $\bar\delta(z) = \bar\partial(1/(2\pi i z))$. These can be seen to be straightforwardly Q invariant. However, having the supersymmetry in this form will be inconvenient in what follows. A more convenient representation is obtained by a Fourier transform of the $\tilde\eta$s into ηs in the first type of vertex operator,

$$\mathcal{V}_a = \int \frac{ds_a}{s_a}\bar\delta^{2|\mathcal{N}}(\lambda_a - s_a\lambda|\eta_a - s_a\chi)e^{is_a[\mu\tilde\lambda_a]}J{\cdot}t_a, \qquad (4)$$

where for a fermionic variable χ, $\delta(\chi) = \chi$. We obtain full $\mathcal{N} = 4$ Yang-Mills amplitudes with the above vertex operators for $\mathcal{N} = 3$ (with $r = 1, \ldots, \mathcal{N} = 4$ we would have double the spectrum).

N^{k-2}MHV Yang-Mills amplitudes will be obtained as correlation functions of the above vertex operators taking k from dual twistor space and $n - k$ from twistor space:

$$\mathcal{A} = \langle\tilde{\mathcal{V}}_1\ldots\tilde{\mathcal{V}}_k\mathcal{V}_{k+1}\ldots\mathcal{V}_n\rangle.$$

The current algebra correlator gives the Parke-Taylor denominator (together with some multitrace terms that we will ignore for the purposes of this Letter). As in Ref. [11], rather than attempt to compute the infinite number of contractions required by the exponentials, we instead take the exponentials into the action to provide sources

$$\int_\Sigma \sum_{i=1}^k is_i(\langle\tilde\mu\lambda_i\rangle + \tilde\chi{\cdot}\eta_i)\bar\delta(\sigma-\sigma_i) + \sum_{p=k+1}^n is_p[\mu\tilde\lambda_p]\bar\delta(\sigma-\sigma_p).$$

The equations of motion for Z and W are then

$$\bar\partial_\sigma Z = \bar\partial(\lambda,\mu,\chi) = \sum_{i=1}^k s_i(\lambda_i, 0, \eta_i)\bar\delta(\sigma-\sigma_i),$$
$$\bar\partial_\sigma W = \bar\partial(\tilde\mu,\tilde\lambda,\tilde\chi) = \sum_{p=k+1}^n s_p(0,\tilde\lambda_p,0)\bar\delta(\sigma-\sigma_p). \qquad (5)$$

Since (Z, W) are world sheet spinors, the solutions are uniquely given by

$$Z(\sigma) = (\lambda,\mu,\chi) = \sum_{i=1}^k \frac{s_i(\lambda_i,0,\eta_i)}{\sigma-\sigma_i},$$
$$W(\sigma) = (\tilde\mu,\tilde\lambda,\tilde\chi) = \sum_{p=k+1}^n \frac{s_p(0,\tilde\lambda_p,0)}{\sigma-\sigma_p}. \qquad (6)$$

With this, we are left with the integrals

$$\mathcal{A} = \int \frac{1}{\mathrm{Vol\,GL}(2,\mathbb{C})}\prod_{a=1}^n \frac{ds_a d\sigma_a}{s_a(\sigma_a - \sigma_{a+1})}\prod_{i=1}^k \bar\delta^2(\tilde\lambda_i - s_i\tilde\lambda)$$
$$\times \prod_{p=k+1}^n \bar\delta^{2|\mathcal{N}}(\lambda_p - s_p\lambda(\sigma_p), \eta_p - s_p\chi(\sigma_p)). \qquad (7)$$

We can write this in terms of homogeneous coordinates on the Riemann sphere $\sigma_\alpha = (1/s)(1,\sigma)$ using the notation $(i\,j) = \sigma_{i\alpha}\sigma_j^\alpha$ (with indices raised and lowered by the usual skew symmetric $\epsilon_{\alpha\beta}$) as follows,

$$Z(\sigma)=\sum_{i=1}^{k}\frac{(\lambda_i,0,\eta_i)}{(\sigma\sigma_i)},\quad W(\sigma)=\sum_{p=k+1}^{n}\frac{(0,\tilde\lambda_p,0)}{(\sigma\sigma_p)} \tag{8}$$

(where we have rescaled W and Z by a factor of $1/s$) and

$$\mathcal{A}=\int\frac{1}{\mathrm{VolGL}(2,\mathbb{C})}\prod_{a=1}^{n}\frac{d^2\sigma_a}{(a\,a+1)}\prod_{i=1}^{k}\bar\delta^2(\tilde\lambda_i-\tilde\lambda(\sigma_i))\times\prod_{p=k+1}^{n}\bar\delta^{2|\mathcal{N}}(\lambda_p-\lambda(\sigma_p),\eta_p-\chi(\sigma_p)). \tag{9}$$

For notational simplicity, we have taken the color order to be $(1,\ldots,n)$; of course, any other choice will just lead to the obvious reordering of the Parke-Taylor denominator. There are $2n$ bosonic delta functions and $2n-4$ integrals, the -4 coming from the $\mathrm{Vol}(\mathrm{GL}(2,\mathbb{C}))$ quotient, yielding four momentum-conserving delta functions. Momentum conservation can be seen from

$$\sum_{p=k+1}^{n}\lambda_p\tilde\lambda_p=\sum_{p=k+1}^{n}\tilde\lambda_p\sum_{j=1}^{k}\frac{\lambda_j}{(p\,j)}=-\sum_{j=1}^{k}\lambda_j\tilde\lambda_j,$$

where we used the first (second) set of delta functions in Eq. (9) to get the first (second) equality. Similarly, $\sum_{a=1}^{n}\tilde\lambda_a\eta_a=0$.

Defining $P(\sigma)=\lambda(\sigma)\tilde\lambda(\sigma)$, the scattering equations $\lambda_a^\alpha\tilde\lambda_a^{\dot\alpha}P_{\alpha\dot\alpha}(\sigma_a)=0$ also follow on the support of the delta functions. Indeed, these are here refined to give just those appropriate to the N^kMHV degree as

$$[\tilde\lambda_i\tilde\lambda(\sigma_i)]=0,\ i=1,\ldots,k,\quad \langle\lambda_p\lambda(\sigma_p)\rangle=0,\ p=k+1,\ldots,n.$$

The formula (9) can be verified at $\mathcal{N}=0$ by comparison with Witten's formula 3.22 in Ref. [4] (or analagous formulas in Ref. [8]) and extended to arbitrary $\mathcal{N}$ by superconformal invariance. This comparison follows by integrating out $2n$ of the $4n$ moduli in Witten's formula against $2n$ of the delta functions leaving just those for the $2n$ homogeneous coordinates u_a. This determines $\lambda(u)$ and $\tilde\lambda(u)$ and leads directly to our formula after identifying $u_i=\sigma_i/\prod_{j=k+1}^{n}(\sigma_j\,\sigma_i)$ for $i=1,\ldots,k$ (see Ref. [19] for more details).

This model will also have vertex operators leading to amplitudes of a nonminimal conformal gravity like that of Berkovits and Witten for the original twistor string.

Einstein gravity amplitudes.—For Einstein gravity, we construct an ambitwistor analogue of David Skinner's model [3] as follows. This model has fields $(Z,\,W)$ that are world sheet spinors with values in $\mathbb{T}\times\mathbb{T}^*$ as before and $(\rho,\,\tilde\rho)$ again in $\mathbb{T}\times\mathbb{T}^*$ but with opposite statistics (i.e., taking values in $\mathbb{C}^{\mathcal{N}|4}$ rather than $\mathbb{C}^{4|\mathcal{N}}$). In order to break conformal invariance we introduce infinity twistors I_{IJ} and I^{IJ} that in general can encode a cosmological constant and a gauging of R symmetry but are rank 2 in the simplest zero cosmological constant ungauged case that we will work with here setting $I_{IJ}Z_1^IZ_2^J=\langle\lambda_1\lambda_2\rangle=:\langle Z_1,Z_2\rangle$ and $I^{IJ}W_{1I}W_{2J}=[\tilde\lambda_1\tilde\lambda_2]=:[W_1,W_2]$. We, furthermore, gauge the following currents

$$K_a=(Z\cdot W,\rho\cdot\tilde\rho,W\cdot\rho,[W,\tilde\rho],Z\cdot\tilde\rho,\langle Z,\rho\rangle,\langle\rho,\rho\rangle,[\tilde\rho,\tilde\rho]),$$

which leads to the introduction of the corresponding weight zero ghosts $(\beta_a,\,\gamma^a)$, together with the fermionic $(b,\,c)$ ghosts as before [20]. These lead to a BRST Q operator

$$Q=\int cT+\gamma^aK_a-\frac{i}{2}\beta_a\gamma^b\gamma^cC^a_{bc}, \tag{10}$$

where C^a_{bc} are the structure constants of the current algebra K_a.

In this Einstein gravity model, Q invariance implies that vertex operators are built from $\bar\partial$ closed (0,1) forms h of weight 2 on twistor space and similarly $\tilde h$ from dual twistor space. For momentum eigenstates, h and $\tilde h$ are given by

$$h_a=\int\frac{ds_a}{s_a^3}\bar\delta^{2|\mathcal{N}}(\lambda_a-s_a\lambda|\eta_a-s_a\chi)e^{is_a[\mu\tilde\lambda_a]}, \tag{11}$$

$$\tilde h_a=\int\frac{ds_a}{s_a^3}\bar\delta^2(\tilde\lambda_a-s_a\tilde\lambda)e^{is_a(\langle\tilde\mu\lambda_a\rangle+\tilde\chi_r\eta_a^r)}. \tag{12}$$

These yield two types of vertex operators, appearing in integrated or unintegrated form, here integration being with respect to ghost zero modes. The ghosts $\gamma=(\gamma^3,\gamma^4)$, $\nu=(\gamma^5,\gamma^6)$ each have one zero mode, and these can be fixed by the insertion of one each of the unintegrated vertex operators

$$V_h=\int_\Sigma\delta^2(\gamma)h,\qquad \tilde V_{\tilde h}=\int_\Sigma\delta^2(\nu)\tilde h.$$

As usual, the remaining states are represented by integrated vertex operators

$$\mathcal{V}_h=\int\left[W,\frac{\partial h}{\partial Z}\right]+\left[\tilde\rho,\frac{\partial}{\partial Z}\right]\rho\cdot\frac{\partial h}{\partial Z}, \tag{13}$$

$$\tilde{\mathcal{V}}_{\tilde h}=\int\left\langle Z,\frac{\partial\tilde h}{\partial W}\right\rangle+\left\langle\rho,\frac{\partial}{\partial W}\right\rangle\tilde\rho\cdot\frac{\partial\tilde h}{\partial W}. \tag{14}$$

For full $\mathcal{N}=8$ supergravity, we can again use the above vertex operators for $\mathcal{N}=7$. This suggests an interesting connection with Hodges' $\mathcal{N}=7$ formalism [21].

Amplitudes are now given by the world sheet correlation function

$$\mathcal{M} = \left\langle \tilde{V}_{\tilde{h}_1} \prod_{i=2}^{k} \tilde{\mathcal{V}}_{\tilde{h}_i} \prod_{p=k+1}^{n-1} \mathcal{V}_{h_p} V_{h_n} \right\rangle. \qquad (15)$$

A correlator of a fermion system, here $(\rho, \tilde{\rho})$, is the determinant of a matrix of possible contractions. Here, we extend this to the following $n \times n$ matrix:

$$\mathcal{H} = \begin{pmatrix} \mathbb{H} & 0 \\ 0 & \tilde{\mathbb{H}} \end{pmatrix},$$

where, for $i, j \in \{1, \ldots, k\}$ and $p, q \in \{k+1, \ldots, n\}$

$$\mathbb{H}_{ij} = \frac{\langle i\, j \rangle}{(i\, j)}, \quad i \neq j, \qquad \mathbb{H}_{ii} = -\sum_{j=1, j\neq i}^{k} \mathbb{H}_{ij}, \qquad (16)$$

$$\tilde{\mathbb{H}}_{pq} = \frac{[p\, q]}{(p\, q)}, \quad p \neq q, \qquad \tilde{\mathbb{H}}_{pp} = -\sum_{q=k+1, q\neq p}^{n} \tilde{\mathbb{H}}_{pq}. \quad (17)$$

The off-diagonal element $\mathcal{H}_{ij}$ is the contraction of the ρ term in the ith vertex operator with the $\tilde{\rho}$ term in the jth, and the diagonal elements of $\mathcal{H}$ come from the remaining terms in the integrated vertex operators. Repeating the steps for Yang-Mills theory, we obtain for gravity amplitudes

$$\mathcal{M} = \int \frac{\prod_{a=1}^{n} d^2\sigma_a}{\mathrm{Vol\, GL}(2, \mathbb{C})} \det{}'(\mathcal{H}) \prod_{i=1}^{k} \bar{\delta}^2(\tilde{\lambda}_i - \tilde{\lambda}(\sigma_i))$$
$$\times \prod_{p=k+1}^{n} \bar{\delta}^{2|\mathcal{N}}(\lambda_p - \lambda(\sigma_p), \eta_p - \chi(\sigma_p)), \qquad (18)$$

where $\det{}'\mathcal{H}$ is the determinant omitting a row and column from each of $\tilde{\mathbb{H}}$ and $\mathbb{H}$ corresponding to the unintegrated vertex operators; the answer is independent of this choice because each has kernel $(1, \ldots, 1)$.

The equivalence to the formula of Ref. [6] is seen by following the Yang-Mills strategy and making judicious identifications of reference spinors with the given σ_a [19].

Conclusion.—Ambitwistor strings provide a chiral infinite tension limit of conventional strings. Here, we have formulated them in four space-time dimensions in terms of twistors and dual twistors, leading to remarkably simple new formulas for tree amplitudes for (super) Yang-Mills and (super) gravity. These are nontrivially related to previous twistor string formulas, as we describe in detail in Ref. [19]. Our gravitational formula is similar to the link representation of Ref. [22], and so one can regard ambitwistor strings as providing the theory underlying such representations.

There are many directions for future exploration. One important question regards the representation of loop amplitudes. Although our model is sufficient for computing tree-level amplitudes, in general it is noncritical and anomalous (the gauge anomalies require $\mathcal{N} = 4$ for the first model and $\mathcal{N} = 8$ for the Einstein gravity model, which suggest a doubling of the spectrum in our context). On the other hand, it is likely that a critical, anomaly-free theory can be obtained by coupling to appropriate matter as, for example, that obtained by reduction from an anomaly-free theory in 10 dimensions [11,12,17]. It might then be possible to represent loop amplitudes as integrals over higher genus moduli spaces of maps.

An issue raised by the gauging associated with a is the validity of our imposition of the choice of degree of the line bundles on Σ in which (Z, W) take their values. Often one would sum over the degrees of the line bundle spanned by $\Upsilon - \tilde{\Upsilon}$ as will be discussed in more detail in Ref. [19] where it will be seen that different choices give the same answer or zero, so the answer is only changed by an overall constant.

Another direction is the generalization of our formulas to nonzero cosmological constant; our model already allows for this, leading to nonzero entries in the off-diagonal blocks of $\mathcal{H}$, and could provide an efficient method for computing tree-level correlation functions in AdS_4 and dS_4. These can be compared to the formulas of Refs. [23,24] and may in turn have applications to the AdS_4/CFT_3 correspondence and cosmology.

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