



Fitting the Annual Modulation in DAMA with Neutrons from Muons and Neutrinos

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The DAMA/LIBRA experiment searches for evidence of dark matter scattering off nuclei. Data from DAMA show 9.2σ evidence for an annual modulation, consistent with dark matter having a cross section around 2×10^{-40} cm². However, this is excluded by other direct detection experiments. We propose an alternative source of annual modulation in the form of neutrons, which have been liberated from material surrounding the detector by a combination of ⁸B solar neutrinos and atmospheric muons. The phase of the muon modulation lags 30 days behind the data; however, we show that adding the modulated neutrino component shifts the phase of the combined signal forward. In addition, we estimate that neutrinos and muons need ~ 1000 m³ of scattering material in order to generate enough neutrons to constitute the signal. With current data, our model gives as good a fit as dark matter, and we discuss prospects for future experiments to discriminate between the two.

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The DAMA/LIBRA (and formerly DAMA/NaI) experiment searches for keV-energy nuclear recoils, potentially arising from dark matter (DM) originating in the Galactic halo [1–3]. It operates with approximately 250 kg of NaI, located deep underground at Gran Sasso. The DAMA/LIBRA Collaboration claims to have observed a temporal variation in the rate of observed events with a period of roughly one year and at a significance of 9.2σ . This annual modulation is of the order of 2% and is approximately sinusoidal, with a maximum in late May.

Such an annual modulation is consistent with dark matter scattering off nuclei inside the detector, since the relative direction of the incoming DM varies over the course of the year and peaks around June 2nd, similarly to the DAMA data.

However, the annual modulation observed by DAMA requires a cross section of interaction between DM and nucleons of $\sigma \approx 2 \times 10^{-40}$ cm² [4] for a mass $m \approx 10$ GeV (and elastic scattering). Unfortunately, the values of m and σ favored by DAMA are excluded by other direct detection experiments such as CDEX [5], CDMS-II [6,7], EDELWEISS-II [8], LUX [9], SuperCDMS [10], XENON10 [11], and XENON100 [12]. This motivates alternative explanations for the DAMA signal.

One alternative source of an annual modulation is cosmic ray muons [13,14], whose flux is correlated with the temperature of the atmosphere [15,16]. The DAMA signal is then explained as being made up of neutrons which have been liberated by muons interacting in the rock surrounding the detector [13,14]. However, although the period is consistent with the DAMA data, the phase of the muon-induced neutron signal is not; i.e., the muon flux peaks roughly 30 days too late [13,16–18]. Hence, the muon signal is incompatible with DAMA at 5.2σ [16].

In this Letter, we propose a solution in the form of an additional source of neutrons, generated by ⁸B solar neutrinos interacting in the rock or shielding surrounding the DAMA detector. Crucially, the solar neutrino flux varies annually and peaks around January 4th, due to the changing distance between Earth and the Sun. We show that, when combined with the neutrons from cosmic muons, the phase of the signal can be shifted forward by ~ 30 days relative to muons alone, resulting in a fit to the data as good as that from dark matter.

This shift relies upon a degree of cancellation between the two modulated rates and so requires the neutron flux from muons R_μ and ⁸B neutrinos R_ν to be of a similar size. We demonstrate that this is in fact the case for the Gran Sasso lab where $R_\nu/R_\mu \sim 0.1$, as the large neutrino flux compensates for its small cross section relative to muons. We discuss methods of discriminating our model from dark matter using, for example, higher-order modes.

Annual modulation of neutrinos, muons, and dark matter.—In this section, we introduce the cosmic muon and neutrino signals, with the aim of fitting their time variation to the combined data from DAMA/NaI and DAMA/LIBRA and comparing the fit to that from dark matter. We use the full data set over 13 years with a 1.17 ton year exposure in the 2–6 keV bin [1,2].

The solar neutrino flux at Earth depends on the distance between Earth and the Sun according to an inverse-square relation. Since Earth's orbit is slightly eccentric, the distance between Earth and the Sun varies with a period of a year, and therefore so does the neutrino flux. Hence the flux is given by the expression [19]

$$\Phi_\nu = \frac{\mathcal{R}}{4\pi r^2(t)} \approx \frac{\mathcal{R}}{4\pi r_0^2} \left[1 + 2\epsilon \cos\left(\frac{2\pi(t - \phi_\nu)}{T_\nu}\right) \right], \quad (1)$$

where \mathcal{R} is the neutrino production rate in the Sun, t is the time from January 1st, $r(t)$ is the time-dependent distance between Earth and the Sun, r_0 is the average distance, $\epsilon = 0.01671$ is the orbital eccentricity, T_ν is the period, and ϕ_ν is the phase. Earth is closest to the Sun around January 4th (implying $\phi_\nu = 3$ days).

This has been confirmed experimentally. Measurements from Borexino [19] for ${}^7\text{Be}$ neutrinos imply a period of $T_\nu = 1.01 \pm 0.07$ years and a phase of $\phi_\nu = 11.0 \pm 4.0$ days. Additionally, the flux of ${}^8\text{B}$ solar neutrinos has been observed by Super-Kamiokande [20] to be consistent with variation in the Earth-Sun distance.

The muons originate from the decay of cosmic ray particles in the stratosphere. These parent particles can also collide with the air, with more collisions leading to fewer muons being produced. In the winter, the rate of collisions is largest, and so the muon flux is lowest. Hence, the muon flux is correlated with the temperature of the atmosphere [21], giving the expression

$$\Phi_\mu \approx \Phi_\mu^0 + \Delta\Phi_\mu \cos[2\pi(t - \phi_\mu)/T_\mu], \quad (2)$$

where Φ_μ^0 is the average cosmic muon flux, T_μ is the period, and ϕ_μ is the phase.

Measurements of muons by Borexino [15] imply that $T_\mu = 366 \pm 3$ days, $\Delta\Phi_\mu/\Phi_\mu^0 = 0.0129 \pm 0.0007$, and $\phi_\mu = 179 \pm 6$ days. Hence, the phase and period are consistent with an annual modulation of muons peaked on approximately June 21st.

We seek to explain the DAMA annual modulation by using a combination of the neutrino and muon signals. The signal itself is due to neutrons, which are liberated in the rock or shielding by the neutrinos and muons. Our signal therefore takes the form of

$$A_{\mu+\nu} = A_\nu \cos[\omega(t - \phi_\nu)] + A_\mu \cos[\omega(t - \phi_\mu)], \quad (3)$$

where $\omega = 2\pi/T$. The amplitudes A_μ and A_ν correspond to the modulation ‘‘residual’’ which is the relative deviation of the event rate from the time average.

Since there exist no direct measurements of the modulation residuals for low-energy muon- and neutrino-induced neutrons, we leave their amplitudes as free parameters. Indeed, we perform two separate fits of $A_{\mu+\nu}$ to DAMA data: For the first, we leave the amplitudes A_ν and A_μ as free variables and fix the other parameters to be $(T, \phi_\nu, \phi_\mu) = (365, 3, 179)$ days. For the second, we marginalize over ϕ_ν and ϕ_μ with Gaussian priors centered on the best-fit values from Borexino. As such, the phases are not free parameters but are fixed *a priori*.

Shown in Fig. 1 is the result of our first fit to DAMA data, with the phases held fixed. We obtain best-fit amplitudes of $A_\nu = 0.039$ and $A_\mu = 0.047$ with $\chi^2 = 66.74$. For the second fit, we obtain best-fit amplitudes of $A_\nu = 0.022$ and $A_\mu = 0.030$.

We show best-fit contours for the marginalized fit in Fig. 2. The signal fits well for a wide range of amplitudes, and the best fit is given when the peak day matches that of the DAMA data. This depends on the phases of the two signals (which are known *a priori*) and their relative amplitudes, with larger values of A_ν/A_μ shifting the peak day to earlier times. Additionally, the fit gets worse whenever the amplitude of the combined signal is either too small or too large, as indicated by the arrows.

We now compare our fit to that from a dark matter signal. The differential rate of dark matter interactions with nuclei takes the form of

$$\frac{dR}{dE} = \frac{\rho_{\text{DM}}}{m_N m} \int d^3v \frac{d\sigma}{dE} v f(\mathbf{v} + \mathbf{v}_E(t)), \quad (4)$$

where ρ_{DM} is the DM density, m_N is the mass of the target nucleus, and $d\sigma/dE$ is the differential cross section.

The integral is over the galactic DM velocity distribution $f(\mathbf{v})$ boosted into Earth’s rest frame by $\mathbf{v}_E(t)$. The time dependence enters via this term, expressed as $\mathbf{v}_E(t) = \mathbf{v}_0 + \mathbf{v}_{\text{pec}} + \mathbf{u}_E(t)$, where $\mathbf{v}_0 = (0, 220, 0)$ km s $^{-1}$ and the peculiar velocity $\mathbf{v}_{\text{pec}} = (11.1 \pm 1.2, 12.2 \pm 2.0, 7.3 \pm 0.6)$ km s $^{-1}$ [22]. For the relative velocity between Earth and the Sun $\mathbf{u}_E(t)$, we use the expression from Ref. [23]. We assume a Maxwell-Boltzmann distribution for $f(\mathbf{v})$.

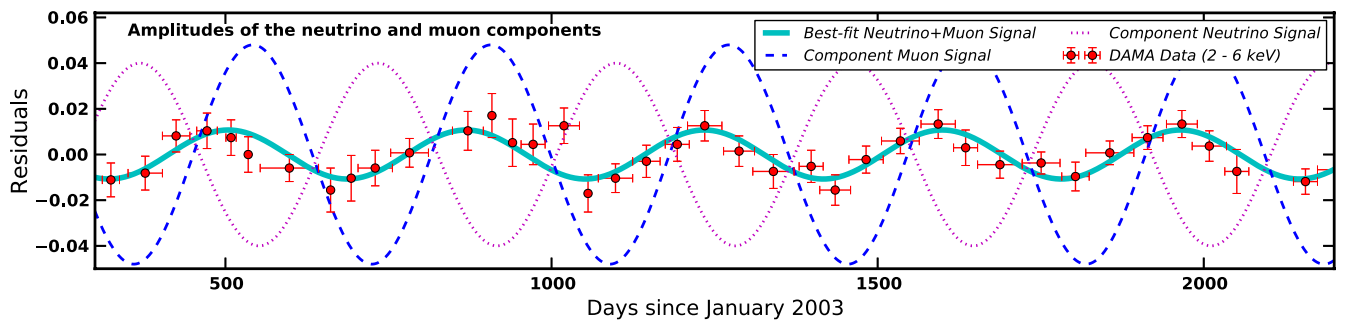


FIG. 1 (color online). The DAMA signal is composed of neutrons liberated in the material surrounding the detector by both solar neutrinos (dotted line) and atmospheric muons (dashed line). Both components have fixed phases, with only their amplitudes as free parameters. Individually, neither of these has the correct phase to fit the data; however, in combination, the fit quality is excellent.

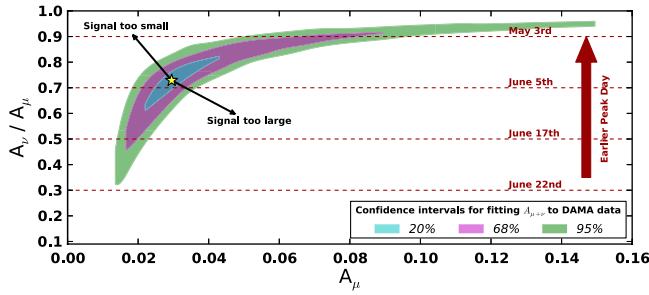


FIG. 2 (color online). Contours (and best-fit point is denoted by star) of the modulation residuals for the muon A_μ and neutrino A_ν induced neutron signal in Eq. (3), for the case where the phases are marginalized over. Shown also are approximate values for the day where the signal peaks for selected values of A_ν/A_μ .

Allowing the amplitude to vary freely, we obtain a best-fit chi square of $\chi^2 = 69.76$.

We show in Fig. 3 the neutrino + muon signal from our first fit (with $A_\nu = 0.039$ and $A_\mu = 0.047$) compared with a dark matter signal and the best-fit signal from muons alone. The neutrino + muon and dark matter signals are very close together in phase, and both fit well to the DAMA data. As expected, the muon-only model provides the worst fit, as it has a phase which lags ~ 30 days behind the data. This is confirmed by the χ^2 values, which we show in Table I.

We present two additional metrics in Table I, which account for the different numbers of free parameters. For the Akaike information criterion (AIC) [24] the neutrino + muon model gives the best fit, and for the Bayesian information criterion (BIC) dark matter has the lowest value, but only by a difference of $\Delta\text{BIC} = 1.36$, which is not significant. We conclude that our neutrino+muon model fits as well to the DAMA modulation as a dark matter signal.

Rates of cosmogenic neutrons.—We have modeled the DAMA annual modulation by using neutrons produced by solar neutrinos and atmospheric muons. Indeed, the DAMA events cannot be due *directly* to muon or neutrino scattering, due to statistical arguments for the former [13] and too

small a rate for the latter [25]. In this section, we discuss whether these muons and neutrinos can produce enough neutrons to constitute the DAMA signal.

Muons produce neutrons via scattering in either the rock or potentially the lead shielding around the detector [13,14,26]. Likewise, neutrons from neutrino neutral-current scattering have been proposed as a detection method for supernovae neutrinos using ^9Be , ^{23}Na , ^{35}Cl , ^{56}Fe , and ^{208}Pb targets [27–30]. For ^{208}Pb the neutron emission threshold for the neutrino is $E_\nu > 7.37$ MeV [27], and so ^8B solar neutrinos could stimulate neutron spallation, since these have energies up to 14 MeV [19,20].

We now calculate the amount of target needed for cosmogenic neutrons to explain the DAMA signal. We estimate the rate of neutrons by using $R \sim \Phi\sigma nV$, where Φ is the flux, σ is the interaction cross section, n is the number density of the target, and V is its volume.

For ^8B solar neutrinos, the flux is of the order $\Phi_\nu \sim 10^6 \text{ cm}^{-2} \text{ s}^{-1}$ [20]. Assuming a ^{208}Pb target, the cross section for neutrino-induced neutron spallation is $\sigma \sim 10^{-41} \text{ cm}^2$ [27]. Hence, the rate of neutrino-induced neutron emission is of the order of $R_\nu \sim 10^{-35} nV$ neutrons/sec. For muons, we assume a flux at the Gran Sasso lab of $\Phi_\mu \sim 10^{-8} \text{ cm}^{-2} \text{ s}^{-1}$ and a cross section for neutron production $\sigma \sim 10^{-26} \text{ cm}^2$ [26]. This gives a muon-induced neutron rate of $R_\mu \sim 10^{-34} nV$ neutrons/sec. Hence, our estimates imply $R_\nu/R_\mu \sim 0.1$, which is encouraging given that we required for the modulation residuals $A_\nu/A_\mu \approx 0.5$ to provide a good fit to DAMA data.

Taking the number density to be $n = 10^{29} \text{ m}^{-3}$, a volume of $V \sim 1000 \text{ m}^3$ is enough to generate ~ 100 neutrons per day, which is similar to the rate observed in DAMA. For the muon-induced neutrons, the mean free path (MFP) is $\lambda \approx 2.6 \text{ m}$ [31]. Hence, we estimate the effective volume over which these neutrons are produced and still reach the detector to be $V_{\text{eff}} = 4\pi \int dr r^2 \exp[-r/\lambda] \approx 450 \text{ m}^3$, which is close to the volume V needed to explain the DAMA signal. The neutrino-induced neutrons will be of lower energies, resulting in a shorter MFP and a smaller V_{eff} . However,

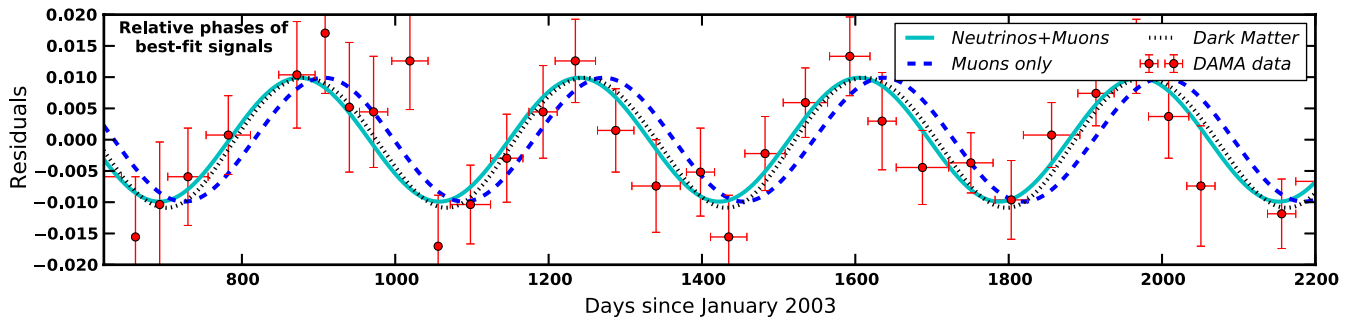


FIG. 3 (color online). Comparison of models for the DAMA data. The model proposed in this Letter is shown as the solid cyan line, composed of neutrons produced by solar neutrinos and atmospheric muons [with fixed phases $(\phi_\nu, \phi_\mu) = (3, 179)$ days]. Adding the solar neutrino contribution to that from muons shifts the phase forward by ~ 30 days, markedly improving the fit to the data.

TABLE I. Compatibility of three models with DAMA data, where $AIC = \chi^2 + 2k$ and $BIC = \chi^2 + k \ln n$, with k as the number of parameters and $n = 80$ the number of data points.

	χ^2	AIC	BIC
Muons and neutrinos	66.74	70.74	75.50
Dark matter	69.76	71.76	74.14
Muons only	90.39	92.39	94.77

we have neglected the production of multiple neutrons per neutrino (through cascade interactions of the neutrons) which may compensate for the smaller volume.

We have yet to comment on why only DAMA (and perhaps CoGeNT [32,33]) sees a modulation signal [7]. This could be due to a combination of several factors, most notably shielding and thresholds. For the former, other experiments employ different, and possibly stronger, neutron shields [34,35]. The geometry of the shielding may also be important: For example, at the KIMS experiment the polyethylene shield is between the lead shield and the detector [36].

For the latter, it is known that muons produce neutrons with a spectrum which rises at low energies, and so the majority of the muon-induced neutrons have a kinetic energy of the order of 10–100 keV [26]. The neutrino-induced spectrum will be similar but also includes a population of low-energy neutrons from neutrino scatters near the detector. Hence, if the neutrons are scattering off Na in DAMA, then the recoil energies fall into the 2–6 keV bin. However, for heavier targets such as xenon or germanium, the recoil energies would likely be below threshold.

Higher-order modes and future tests.—Based on annual modulation alone, the muon + neutrino and dark matter models provide equally valid fits to the DAMA data. In this section, we discuss methods for discriminating between these two models.

The Sun goes through cycles of activity with a period of approximately 11 years. Indeed, atmospheric muons possess a significant additional modulated component with a period of 10.7 ± 0.3 years [16,17]. However, no correlation between solar activity and ^8B solar neutrinos has been observed by Super-Kamiokande [20] or for ^7Be neutrinos in Borexino [19] (though there may be evidence of a quasibiennial mode [37]). Hence, it is not possible for the two signals to interfere at the 11 year mode as they do for the annual mode (i.e., $A_\nu/A_\mu|_{T=11 \text{ year}} < A_\nu/A_\mu|_{T=1 \text{ year}}$), and so a lack of power for this harmonic in the data would exclude our model.

We do not have the full DAMA data set; however, it is possible to partially reconstruct this information by using the annual averages [2,17]. Indeed, the annually averaged rates appear consistent with the $\sim 1\%$ modulation expected from the 11 year mode. However, a statistical analysis of this data is difficult, since the earliest data come from

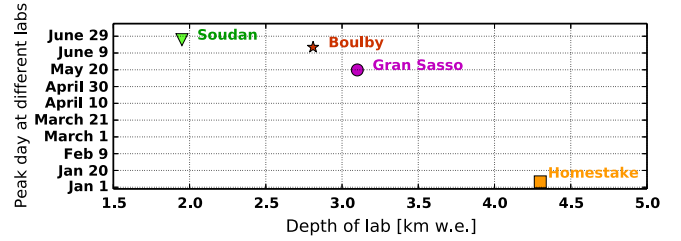


FIG. 4 (color online). Approximate peak day of the neutrino + muon signal at four different labs. Deeper labs have a lower muon flux [42] and so a phase closer to that of the solar neutrinos.

DAMA/NaI (with larger error bars) while the later part comes from DAMA/LIBRA [2].

There are further tests which may be able to distinguish our model from dark matter. For example, we used only the 2–6 keV energy bin. However, the DAMA Collaboration have shown that the best-fit phase shifts forwards by ~ 10 days when fitting to events with energies between 2 and 4 keV [1]. Since A_ν/A_μ could increase with lower energy, such a shift may be expected from our combined model.

Additionally, our model predicts a modulation in double-scatter neutrons. However, this will likely be washed out by other more numerous unmodulated double-scatter events, such as gammas.

Finally, future experiments such as DM-Ice [38], KIMS [39], SABRE [40], or ANAIS [41] will be able to exploit a location-dependent phase change, e.g., due to the depreciation of muon flux with depth. We show in Fig. 4 the expected peak day of the neutrino + muon model at four labs. For example, if $A_\nu/A_\mu = 0.7$ at Gran Sasso, it will be 0.44 at Boulby, since the muon flux is 1.6 times larger [42], leading to a peak day ~ 20 days later.

Conclusion.—In this Letter, we have proposed a new model for the DAMA annual modulation, which is a sum of two annually modulating components with different phases. More specifically, the events are composed of neutrons, which are liberated in the material surrounding the detector by a combination of ^8B solar neutrinos and atmospheric muons. The model is shown in Fig. 1.

The muons alone cannot explain the DAMA annual modulation, as has been remarked upon before [13,16,17], since they peak approximately ~ 30 days too late. Inclusion of the solar neutrinos solves this issue, as they also modulate and peak around January 4th, effectively shifting the phase of the combined model forward. This is shown in Fig. 3. Because of this phase shift, we found that our model fits as well to the DAMA annual modulation as dark matter.

We have shown that both the muon and neutrino signals can produce enough neutrons provided they scatter in a volume approximately $\sim 1000 \text{ m}^3$ in size around the DAMA detector. A detailed Monte Carlo simulation of the detector is required in order to verify if our determination of the neutron rate is realistic.

However, this degeneracy between the muon + neutrino model and dark matter extends only to modulation with a period of one year. A search for an 11 year mode or an energy-dependent phase may break the degeneracy, and future experiments will additionally be able to study if the modulation phase varies with location.

Hence, it is premature to disregard cosmogenic neutrons as an explanation for the DAMA modulation based on the phase, and our model presents a testable alternative for future experiments aiming to look for an annual modulation due to dark matter.

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