

Non-Fermi-Liquid Manifold in a Majorana Device

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We propose and study a setup realizing a stable manifold of non-Fermi-liquid states. The device consists of a mesoscopic superconducting island hosting $N \geq 3$ Majorana bound states tunnel coupled to normal leads, with a Josephson contact to a bulk superconductor. We find a nontrivial interplay between multichannel Kondo and resonant Andreev reflection processes, which results in the fixed point manifold. The scaling dimension of the leading irrelevant perturbation changes continuously within the manifold and determines the power-law scaling of the temperature-dependent conductance.

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Introduction.—Nanoscale devices hosting Majorana bound states are expected to display spectacular nonlocal quantum correlations and long-range entanglement [1–4]. Experimental reports of Majorana fermions [5–10] have so far focused on effectively noninteracting systems, where local resonant Andreev reflection (RAR) physics dominates the transport characteristics [2]. Since interactions tend to suppress RAR, several interesting nonlocal phenomena have been predicted for interacting Majorana devices, such as electron teleportation [11–13], interaction-induced unstable fixed points [14,15], or the topological Kondo effect [16], where strong charging effects cause a multichannel Kondo state. As a general rule, such states display non-Fermi-liquid (NFL) behavior [17–22]. The Kondo and RAR states, respectively, constitute mutually exclusive phases in all settings studied up to now [14–16,23]. In this Letter, we predict that a nontrivial coexistence of Kondo and RAR physics takes place in the device shown in Fig. 1, where a mesoscopic superconducting island is Josephson coupled to a conventional bulk superconductor and hosts $N \geq 3$ Majorana fermions weakly contacted by normal leads. In principle, all ingredients are experimentally available [5–10]. We find that the Kondo-RAR interplay in such a device can result in a continuously tunable *manifold* of NFL states. Although similar physics was proposed before for conventional Kondo systems [24–28], anisotropies destabilize the corresponding NFL fixed points and have prevented their experimental observation. In our proposal, the stability of the NFL manifold is tied to the nonlocal Majorana representation of an effective “quantum impurity,” where Kondo screening and RAR processes both originate from the tunnel coupling between Majorana fermions and lead electrons.

Before entering a detailed discussion, we briefly summarize our main results. The low-energy physics near the ground-state NFL manifold is governed by a leading irrelevant perturbation of scaling dimension

$$y = \min \left\{ 2, \frac{1}{2} \sum_{j=1}^N \left[1 - \frac{2}{\pi} \sin^{-1} \left(\frac{\delta_j}{2(N-1)} \right) \right]^2 \right\}, \quad (1)$$

where the N dimensionless parameters $\delta_j = \sqrt{\Gamma_j/T_K}$ depend on the lead-to-Majorana hybridizations Γ_j and the Kondo temperature T_K being the respective energy scales for RAR and Kondo physics. The $(\delta_1, \dots, \delta_N)$ domain with $y > 1$ corresponds to the NFL manifold, which could be explored experimentally by varying the Γ_j via gate voltages [5]. The NFL character is manifest in the noninteger and continuously tunable scaling dimension

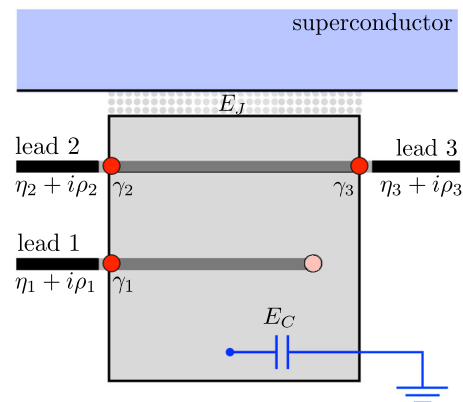


FIG. 1 (color online). Schematic device setup: Several one-dimensional (1D) nanowires with strong spin-orbit coupling are deposited on a floating superconducting island with charging energy E_C . Choosing appropriate system parameters, see Refs. [2–4] for a thorough discussion, each nanowire hosts two spatially separated Majorana bound states. The overhanging parts of the wire act as normal-conducting leads, where only effectively spinless 1D fermions $\Psi_j(x) \sim \eta_j + i\rho_j$ couple to the island. Of the N_{tot} Majorana states on the island, N are connected to leads (here $N = 3$), where the other $N_{\text{tot}} - N$ Majorana fermions have no effect on the physics described here. The island also couples to a bulk superconductor through the Josephson energy E_J .

in Eq. (1). Interestingly, a similar low-energy model has been obtained for the two-channel two-impurity Kondo model, despite a different physical origin, where scaling dimensions and finite-size spectra were derived in Refs. [24,25]. Our predictions can be observed in charge transport, since y governs the power-law scaling of the temperature-dependent conductance tensor at $T \ll T_K$,

$$G_{jk}(T) = \frac{2e^2}{h} \left[\delta_{jk} - A_{jk} \left(\frac{T}{T_K} \right)^{2(y-1)} + \dots \right], \quad (2)$$

with dimensionless numbers $A_{jk}(\delta_1, \dots, \delta_N)$ of order unity. Albeit Eq. (2) coincides with the local RAR result [2] for $T = 0$, it reflects entirely different physics. This difference is readily observable at finite T , where the *nonlocal* conductances $G_{j \neq k}$ in Eq. (2) are finite, in marked contrast to the RAR case.

Device proposal.—We consider the setup in Fig. 1, where a floating mesoscopic superconducting island, with charging energy E_C , is in proximity to at least two nanowires with strong spin-orbit coupling, e.g., InSb or InAs. The island’s superconducting phase, φ , is taken relative to a conventional bulk superconductor, where the Josephson energy E_J denotes their coupling and we assume a large pairing gap such that quasiparticle poisoning is negligible. In the presence of a Zeeman field, Majorana bound states are induced near each end of a superconducting nanowire part [2–10]. We study the case that $N \geq 3$ Majorana fermions, described by operators $\gamma_j = \gamma_j^\dagger$ with anticommutator algebra $\{\gamma_j, \gamma_k\} = \delta_{jk}$, are connected to normal leads. We assume that different Majorana states are well separated; i.e., direct tunnel couplings can be neglected. Note that their distance may exceed the superconducting coherence length since the phase dynamics of the Cooper pair condensate renders transport intrinsically nonlocal in such a device [11]. The island Hamiltonian $H_{\text{island}} = E_C(Q - n_g)^2 - E_J \cos \varphi$ then contains a charging and a Josephson energy contribution, respectively. The total electron number on the island, Q , is due to Cooper pairs and occupied Majorana states [11–13], and the backgate parameter n_g has no effect in the regime studied below. Using units with $\hbar = k_B = 1$, the Hamiltonian $H = H_0 + H_t + H_{\text{island}}$ also contains a lead part $H_0 = -iv_F \sum_j \int_{-\infty}^{\infty} dx \Psi_j^\dagger \partial_x \Psi_j$ with Fermi velocity v_F . In each lead, only an effectively spinless chiral 1D fermion, $\Psi_j(x)$, corresponding to the overhanging wire parts in Fig. 1, connects to the island by tunneling via the Majorana fermion γ_j . This is described by the tunneling Hamiltonian [12] $H_t = \sum_{j=1}^N \lambda_j e^{-i\varphi/2} \Psi_j^\dagger(0) \gamma_j + \text{H.c.}$, where the tunnel couplings λ_j are chosen real positive and $x = 0$ marks the contact. With hybridization parameters $\Gamma_j = 2\pi\nu_0\lambda_j^2$, the lead density of states $\nu_0 = 1/\pi v_F$, and the Josephson plasma frequency $\Omega = \sqrt{8E_C E_J}$, the regime of interest is $\max(\Gamma_j) \ll \Omega \lesssim E_J$. In presently

studied experimental devices [5,29], both the pairing gap and the charging energy of the island are of the order of a few meV. Choosing also the value of E_J —which mainly depends on the interface to the bulk superconductor—within the meV regime, and noting that the hybridizations are also gate tunable with $\Gamma_j \approx 0.01, \dots, 1$ meV [5], the implementation of our proposal seems possible. The observation of the predicted phenomena also requires low temperatures $T \ll T_K$; see below.

Effective low-energy Hamiltonian.—We next show that for $\max(\Gamma_j) \ll \Omega \lesssim E_J$, a simpler effective low-energy theory emerges. In this regime, the phase φ will mostly stay near the minima of the $-E_J \cos \varphi$ term in H_{island} . Phase slips due to tunneling between adjacent minima are exponentially suppressed [30], and it is justified to neglect them. The phase dynamics then consists of fast zero-point oscillations of frequency Ω around a given minimum. Since $\langle (\delta\varphi)^2 \rangle = \Omega/2E_J$, the oscillation amplitude remains small and we may integrate over the φ fluctuations. The resulting effective low-energy Hamiltonian $H_{\text{eff}} = H_0 + H_A + H_K$ is local on time scales above Ω^{-1} . Expressing the lead fermions by pairs of chiral Majorana fields $\Psi_j(x) = [\eta_j(x) + i\rho_j(x)]/\sqrt{2}$ we obtain

$$H_0 = -\frac{iv_F}{2} \sum_{j=1}^N \int dx (\eta_j \partial_x \eta_j + \rho_j \partial_x \rho_j),$$

$$H_A = \sqrt{2}i \sum_j \lambda_j \gamma_j \rho_j(0), \quad H_K = \sum_{j < k} J_{jk} \gamma_j \gamma_k \eta_k(0) \eta_j(0). \quad (3)$$

The positive “exchange couplings,” $J_{jk} = \lambda_j \lambda_k / 4E_J$ are controlled by E_J . Although E_C does not appear in H_{eff} , it enters the bandwidth given by the plasma frequency Ω . In Eq. (3), H_A couples only to ρ_j and describes RAR [2], while H_K only involves the η_j Majoranas and describes exchange processes between lead electrons and the components $\gamma_j \gamma_k$ of the “impurity spin.” On top of terms $\sim \Psi_j^\dagger(0) \Psi_k(0)$, which also appear in the topological Kondo model of Ref. [16], H_K contains crossed Andreev reflection contributions, e.g., terms $\sim \Psi_j^\dagger(0) \Psi_k^\dagger(0)$, where a Cooper pair splits into two electrons in separate leads. Because of the phase coherence in the superconductor, which is behind the $e^{\mp i\varphi/2}$ phase factors in H_t , both types of exchange processes enter H_K with equal weight. Without the H_A term, H_{eff} is mathematically identical to the $\text{SO}_1(N)$ Kondo model recently proposed for crossed Ising chains, which hosts a NFL Kondo fixed point [31,32] and, for $N = 3$, is equivalent to the conventional two-channel Kondo model because of the group relation $\text{SO}_1(3) \sim \text{SU}_2(2)$.

Renormalization group analysis.—By employing standard energy-shell integration [21], we find the one-loop renormalization group (RG) equations

$$\frac{d\Gamma_j}{dl} = \Gamma_j, \quad \frac{dJ_{j \neq k}}{dl} = 2\nu_0 \sum_{m \neq (j,k)} \frac{J_{jm} J_{mk}}{1 + \Gamma_m / \Omega}. \quad (4)$$

The running couplings $\Gamma_j(l)$ thus approach the strong coupling limit according to the standard RAR equations [2], while the RG flow of the exchange couplings is coupled to the Γ_j . Similar to what happens in the pure Kondo case [31], Eq. (4) implies that anisotropies in the J_{jk} are RG irrelevant, while the isotropic part is marginally relevant. We thus write $J_{jk} = J(1 - \delta_{jk})$, and neglect irrelevant deviations from isotropy from now on. We shall also assume $\Gamma_j = \Gamma$, but return to the role of Γ_j anisotropy later. To one-loop accuracy, we then obtain the estimate $T_K \approx \Omega \exp(-E_J/(N-2)\Gamma)$ for the Kondo temperature. Moreover, Eq. (4) can now be solved analytically. This solution shows that both $\Gamma(l)$ and $J(l)$ flow towards strong coupling for $\Gamma < T_K$. Especially for large N , it is possible to satisfy this condition by choosing $\Omega \approx E_J$ and not too small ratio Γ/E_J . In what follows, we focus on the regime $\Gamma < T_K$, and analyze the physics at low temperatures, $T \ll T_K$. For $\Gamma > T_K$, one instead arrives at the well-known RAR picture [2]. To estimate the Kondo scale for typical parameters, let us put, say, $N = 6$, $\Gamma = 0.2$ meV, and $\Omega = E_J = 2$ meV, where $T_K \approx 0.27$ meV, and $\Gamma < T_K$ is satisfied. The low-temperature regime with $T \ll T_K$ is then also accessible to experiments.

Quantum Brownian motion analogy.—The low-temperature physics within the most interesting regime $\Gamma < T_K$ can be captured from an instructive analogy to quantum Brownian motion in a lattice-periodic potential. To see this, we first bosonize the lead fermions in H_{eff} by writing $\Psi_j(x) = \xi_K^{-1/2} \zeta_j e^{i\phi_j(x)}$ [22] with boson fields $\phi_j(x)$, where $\xi_K = v_F/T_K$ sets the short-distance scale and additional Majorana fermions, ζ_j , represent the Klein factors enforcing fermion anticommutators between different leads [22]. Following Refs. [14,15], each “true” Majorana fermion γ_j is combined with the respective “Klein” Majorana ζ_j to form an auxiliary fermion. The latter has a conserved occupation number and can be gauged away [14]. This yields a purely bosonic action, $S[\Phi] = \sum_j \int (d\omega/2\pi) |\omega| |\tilde{\Phi}_j(\omega)|^2 + \int d\tau V[\Phi(\tau)]$, where $\Phi = (\Phi_1, \dots, \Phi_N)$ with $\Phi_j \equiv \phi_j(x=0)$ and Fourier components $\tilde{\Phi}_j(\omega)$. The Gaussian part describes dissipation by electron-hole pair excitations in the leads, and the RAR-Kondo interplay is encoded by the “pinning potential”

$$V[\Phi] = -\frac{\lambda}{\sqrt{\xi_K}} \sum_j \sin \Phi_j - \frac{J}{4\xi_K} \sum_{j \neq k} \cos \Phi_j \cos \Phi_k. \quad (5)$$

We thus arrive at the quantum Brownian motion of a fictitious particle with coordinates Φ in the N -dimensional lattice corresponding to $V[\Phi]$; see also Refs. [33,34]. Comparison to the $N = 3$ field theory (see below) shows that, up to an overall prefactor, the renormalized couplings λ and J in Eq. (5) are effectively replaced by $\sqrt{\xi_K}\Gamma$ and $4\xi_K\sqrt{T_K}$, respectively, when approaching the strong-coupling regime. The relative importance of the two terms in Eq. (5) is thus governed by $\delta = \sqrt{\Gamma/T_K}$.

In the ground state, Φ is pinned to one of the minima of $V[\Phi]$. These minima occur for isotropic boson field configurations, $\Phi_j = \Phi_{\text{min}}$, with $\sin(\Phi_{\text{min}}) = \delta/[2(N-1)]$. For $\delta = 0$, the minima at $\Phi_{\text{min}} = 0$ and $\Phi_{\text{min}} = \pi$ correspond to the corner and center points, respectively, of a body-centered hypercubic lattice. These points move in opposite directions when increasing δ , such that we have two interpenetrating cubic lattices. The closest distance between corner and center points, see Fig. 2 for an illustration, is given by $d = \sqrt{N}(\pi - 2\Phi_{\text{min}})$, while the distance between corners (or between centers) remains $d = 2\pi$. Perturbations around the ground state then come from instanton transitions connecting different potential minima. Following the arguments of Yi and Kane [33,34], the scaling dimension y of the perturbation is directly related to the distance d between the potential minima, $y = d^2/(2\pi^2)$. For the leading (nearest-neighbor) term, we arrive at Eq. (1) announced above. This perturbation is RG irrelevant for $\delta < \delta_c$, with

$$\delta_c = 2(N-1) \sin \left[\frac{\pi}{2} \left(1 - \sqrt{\frac{2}{N}} \right) \right]. \quad (6)$$

Since $y(\delta)$ is not an integer, all stable fixed points can be classified as NFL states. As a consequence, we obtain a stable line of NFL fixed points parametrized by $0 \leq \delta < \delta_c$. For $\delta > \delta_c$, the perturbation becomes relevant and destabilizes the fixed point line. Since this corresponds to $\Gamma > T_K$, we conclude that δ_c marks the phase transition to the RAR regime.

Strong coupling approach.—It is reassuring that the above results can be confirmed by an explicit strong-coupling solution for $N = 3$, which we briefly sketch next. Encoding the Majorana triplet $\boldsymbol{\gamma} = (\gamma_1, \gamma_2, \gamma_3)$ by a spin-1/2 operator, $\boldsymbol{S} = -(i/2)\boldsymbol{\gamma} \times \boldsymbol{\gamma}$, plus another Majorana fermion, $b = -2i\gamma_1\gamma_2\gamma_3$ [35], the RAR term in Eq. (3) reads $H_A = 2\sqrt{2}i\lambda b \boldsymbol{S} \cdot \boldsymbol{\rho}(0)$, while the Kondo term becomes $H_K = JS \cdot [-(i/2)\boldsymbol{\eta}(0) \times \boldsymbol{\eta}(0)]$. We now recall that without the RAR term, H_{eff} reduces to the standard

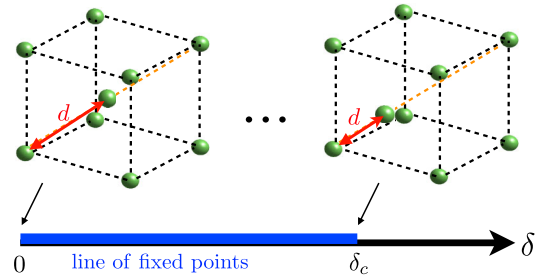


FIG. 2 (color online). Lattice corresponding to the potential minima of $V[\Phi]$ for $N = 3$, with $\delta = 0$ (left) and $\delta = \delta_c$ (right). With increasing δ , the center of the lattice moves along the diagonal towards the corner point. The line of fixed points (corresponding to the non-Fermi-liquid manifold for $\delta_1 = \delta_2 = \delta_3$) terminates at $\delta = \delta_c$.

two-channel Kondo model, where the results of Refs. [24,25,36–41] imply: (i) The $\boldsymbol{\eta}$ triplet of lead Majoranas obeys twisted boundary conditions, $\boldsymbol{\eta}(x) \rightarrow \text{sgn}(x)\boldsymbol{\eta}(x)$. The sign change when passing the impurity implies that an incoming electron is effectively reflected as a hole with unit probability. This resembles the RAR mechanism and rationalizes why the $T = 0$ conductance in Eq. (2) coincides with the RAR result. (ii) Screening processes, entangling the impurity spin with $\boldsymbol{\eta}$, are effectively described by writing $S = i\sqrt{\xi_K}a\boldsymbol{\eta}(0)$, where a is a new Majorana fermion capturing the remaining unscreened degree of freedom. (iii) The leading irrelevant operator corresponds to $H'_K = 2\pi T_K \xi_K^{3/2} a\eta_1(0)\eta_2(0)\eta_3(0)$.

Including now the RAR term, $\lambda \neq 0$, we combine the a and b Majoranas to a conventional fermion, $d = (a + ib)/\sqrt{2}$. Using (ii) and bosonizing the lead fermions as above, the low-energy form of the RAR contribution is $H'_A = (\sqrt{6}v_F\delta/\pi)[d^\dagger d - 1/2]\partial_x\phi_0(0)$, with $\phi_0 = (\phi_1 + \phi_2 + \phi_3)/\sqrt{3}$. This expression is reminiscent of the X-ray edge singularity problem [22], suggesting that the marginal perturbation H'_A can be nonperturbatively included into H_0 by a unitary transformation. Indeed, with $U = e^{i(2\sqrt{6}\delta/\pi)(d^\dagger d - 1/2)\phi_0(0)}$, this is the case, where UH'_KU^\dagger generates eight different operators. The smallest scaling dimension, $y(\delta) = (3/2)[1 - \delta/(2\pi)]^2$, then identifies the leading irrelevant operator [24,25]. This result is exact for $\delta \ll 1$, where it matches Eq. (1). Stability requires $\delta < \delta_c = 2\pi(1 - \sqrt{2/3}) \approx 1.153$, in good agreement to the value predicted by Eq. (6), $\delta_c \approx 1.137$.

Discussion.—So far we have studied the isotropic setup with $\Gamma_j = \Gamma$. While anisotropic deviations in the exchange couplings J_{jk} are RG irrelevant, deviations in the Γ_j convert the fixed point line into an N -dimensional manifold parametrized by the $\delta_j = \sqrt{\Gamma_j/T_K}$. In the quantum Brownian motion approach, the Φ potential minima then move away from isotropic configurations, and $y = y(\delta_1, \dots, \delta_N)$ in Eq. (1) has been obtained by computing the distance d between nearest neighbor minima. The resulting NFL can be probed in charge transport experiments. The conductance tensor is defined by $G_{jk}(T) = -e\partial I_j/\partial\mu_k$, where the j th lead has chemical potential μ_j , and the charge currents I_j are oriented towards the island. Closely following the technical steps detailed in Ref. [42], their steady-state expectation values can be obtained from a Keldysh functional integral, since the fixed point theory is represented by a Gaussian action for the *dual* boson fields. Perturbation theory in the leading irrelevant perturbation, of scaling dimension (1), then determines the linear conductance tensor for $T \ll T_K$ as stated in Eq. (2). For $\delta_j = \delta$, all matrix elements $A_{jk}(\delta)$ in Eq. (2) are equal, and hence the finite- T conductance corrections are completely isotropic. Remarkably, all nonlocal conductances $G_{j\neq k}$ in Eq. (2) exhibit the same power-law temperature dependence and vanish at $T = 0$, thereby providing a highly characteristic

signature to look for in experiments. Indeed, the RAR scenario predicts $G_{j\neq k} = 0$ at all T , while the NFL manifold can be identified by a finite- T nonlocal conductance exhibiting power-law scaling.

Conclusions.—In this work we have proposed a (challenging but realistic) device hosting a stable manifold of NFL states. By Josephson coupling a Majorana fermion system to a superconductor, this suggests a novel route to a first realization of this elusive behavior. Future theoretical work should also study the full crossover from high to low temperatures, e.g., using numerical RG simulations [43].

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