Backaction-Driven Transport of Bloch Oscillating Atoms in Ring Cavities

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We predict that an atomic Bose-Einstein condensate strongly coupled to an intracavity optical lattice can undergo resonant tunneling and directed transport when a constant and uniform bias force is applied. The bias force induces Bloch oscillations, causing amplitude and phase modulation of the lattice which resonantly modifies the site-to-site tunneling. For the right choice of parameters a net atomic current is generated. The transport velocity can be oriented oppositely to the bias force, with its amplitude and direction controlled by the detuning between the pump laser and the cavity. The transport can also be enhanced through imbalanced pumping of the two counterpropagating running wave cavity modes. Our results add to the cold atoms quantum simulation toolbox, with implications for quantum sensing and metrology.

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Periodic potentials play a prominent role in condensed matter systems, and highlight some of the fundamental differences between classical and quantum dynamics: a quantum particle undergoes strong scattering when its de Broglie wavelength satisfies the lattice Bragg condition, and can undergo tunneling through classically forbidden regions between sites. Furthermore, if a constant bias force F is applied a quantum particle is not transported in the direction of the force but instead performs Bloch oscillations with no net displacement at a frequency $\omega_B = Fd/\hbar$, where d is the lattice period [1]. Indeed, transport of electrons in lattices with an applied dc electric field only occurs as a result of dephasing processes such as scattering from lattice defects.

Cold atoms present an especially attractive platform for studies of lattice systems because all of the critical parameters governing the dynamics are tunable in real time. In particular, it is possible to control tunneling and transport by modulating the potential in time. Transport in statistical phase space has been demonstrated in pulsed lattices, realizing the quantum delta-kicked rotor [2] and leading to dynamical localization [3] and chaos-assisted tunneling [4]. Directed transport has been observed through ratchet effects in driven dissipative [5] and Hamiltonian lattices [6]. Tunneling control has been achieved through harmonic shaking of lattices without [7–9] and with [10,11] a bias force. It is thus possible to control the superfluid-Mott insulator transition [12,13] and to induce macroscopic delocalization [14] and transport [15] of Bloch oscillating atoms. Recently, photon-assisted tunneling [16] has been studied in strongly correlated quantum gases [17,18], and artificial vector gauge potentials have been generated [19].

What is missing in these schemes is backaction by the atoms upon the electromagnetic fields generating the lattice. Contrast this with the strong backaction effects seen in solids, such as lattice-phonon mediated Cooper pairing and the Meissner effect in superconductors. In principle, an optically trapped atomic gas causes refraction of the lattice light, but extremely low densities render this negligible under normal conditions. An exception is inside a high finesse optical cavity, where the multipass effect can increase the effective optical path length by several orders of magnitude [20], leading to a shift of the cavity resonance that depends on the density distribution of the atoms. If this shift is on the order of the cavity linewidth, the number of cavity photons is modified by the atomic wave function and *vice versa*. This backaction leads to richer dynamics than is otherwise possible [21–23], such as collective atomic recoil lasing [24–26] and single-photon bistability [27,28]. In this context the system can be considered an application of cavity optomechanics [29–31], where collective excitations of the atoms play the role of material oscillators which are dispersively coupled to one or more cavity modes [32].

It has been shown theoretically that such systems allow continuous, nondestructive measurements of atomic Bloch oscillations through detection of intensity and phase modulation of the transmitted light [33,34]. Despite the backaction on the lattice effected by the atoms, Bloch's acceleration theorem remains valid, and this modulation of the lattice potential occurs predominantly at the expected Bloch oscillation frequency (i.e., calculated for an equivalent static tilted lattice) and its harmonics. It is therefore natural to ask whether this dynamical modulation of the lattice can drive the renormalization of atomic tunneling which is now familiar from experiments with free-space lattices. The central result of this Letter is to show that backaction-driven modulation of an intracavity lattice can lead to a directed atomic current with tunable magnitude and direction.

To see how this happens, we consider a Bose-Einstein condensate trapped along one leg of an optical ring cavity, as shown schematically in Fig. 1. Transverse degrees of freedom are assumed to be frozen out by external confinement, effectively reducing the dynamics to a single spatial dimension z. The two running-wave modes of the cavity are pumped through a lossless input-output coupling mirror by a laser with frequency $\omega_0 = ck_r$, where c is the speed of light in vacuum and $\hbar k_r$ is the recoil momentum. The light is detuned far enough from the atomic resonance that the excited state of the atoms can be adiabatically eliminated. For simplicity, we also ignore atomic collisions, which may be negligible in an experiment either because the scattering cross section is naturally small [14], or has been made small through the use of a tunable Feshbach resonance [15]. In a frame rotating at ω_0, and in the dipole and rotating wave approximations, the Hamiltonian is then given by

$$\hat{H} = -\hbar\sum_{k=\pm}[\Delta_c \hat{a}_k^\dagger \hat{a}_k + i(\eta_k^* \hat{a}_k - \eta_k \hat{a}_k^\dagger)] + \int dz \hat{\psi}^\dagger \left(-\frac{\hbar^2}{2m}\partial_z^2 + \hbar U_0 \hat{\mathcal{E}}^\dagger \hat{\mathcal{E}} - Fz\right)\hat{\psi}, \quad (1)$$

where the annihilation operators $\hat{a}_+$ and $\hat{a}_-$ acting on the cavity modes, and $\hat{\psi}(z)$ acting on the atomic field, all obey bosonic commutation relations. $\Delta_c = \omega_0 - \omega_c$ is the detuning of the driving laser from the bare cavity resonance frequency ω_c, and $\eta_k = \sqrt{J_k \kappa/2}$ for an incident flux of J_k photons per unit time and a photon number decay rate of $2\kappa\langle \hat{a}_k^\dagger \hat{a}_k\rangle$ for each mode. The dimensionless positive-frequency component of the electric field is given by $\hat{\mathcal{E}}(z,t) = \hat{a}_+ \exp(ik_r z) + \hat{a}_- \exp(-ik_r z)$. The depth of the lattice is proportional to U_0, which is a function of the atomic dipole moment, the cavity mode volume, and the detuning from atomic resonance, and $F < 0$ is the uniform and constant bias force.

We can alternatively express the cavity modes in a standing wave basis, described by the annihilation operators $\hat{a}_c = (\hat{a}_+ + \hat{a}_-)/\sqrt{2}$ and $\hat{a}_s = i(\hat{a}_+ - \hat{a}_-)/\sqrt{2}$, where the c (s) mode has cosine (sine) spatial symmetry. In this basis the interaction term in the second line of (1) takes the form

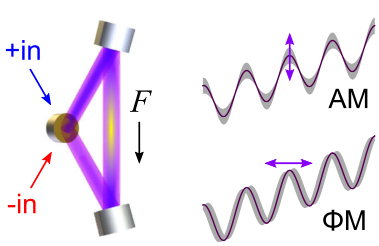

FIG. 1 (color online). An optical lattice is created by pumping the two running wave modes of a ring cavity. A trapped Bose-Einstein condensate (yellow ellipse) undergoes Bloch oscillations due to the bias force F. Atomic backaction leads to lattice amplitude and phase modulation (AM and ΦM, respectively), which in turn induces coherent directed transport of the condensate.

$$\hat{H}_i = \hbar U_0[\hat{n}\hat{N} + (\hat{n}_c - \hat{n}_s)\mathcal{C} + (\hat{a}_c^\dagger \hat{a}_s + \hat{a}_s^\dagger \hat{a}_c)\mathcal{S}], \quad (2)$$

where $\hat{n} = \hat{n}_c + \hat{n}_s = \hat{a}_c^\dagger \hat{a}_c + \hat{a}_s^\dagger \hat{a}_s$ is the total number of photons, $\hat{N}$ is the number of atoms in the condensate, and $\mathcal{C}[\hat{\psi}] = \langle\cos(2k_r z)\rangle$ and $\mathcal{S}[\hat{\psi}] = \langle\sin(2k_r z)\rangle$ depend implicitly on the atomic state. We will be most interested in cases where $\langle\hat{n}_c\rangle \gg \langle\hat{n}_s\rangle$, so that the intracavity lattice has predominantly cosine symmetry. Then the quantity $\mathcal{C}$ characterizes the degree of spatial ordering of the atoms and $\mathcal{S}$ is related to the coherence between the lowest and first excited Bloch bands [33]. Viewed in the optomechanical picture, $\mathcal{C}$ and $\mathcal{S}$ represent the occupations of the lowest momentum modes of the condensate, with $\mathcal{S} = 0$ in the absence of Bloch oscillations or a symmetry-breaking optical bistability [35]. However, even if we choose the η_k to initially give $\mathcal{S} = 0$, during Bloch oscillations $\mathcal{S}$ becomes nonzero and the intensity and spatial phase of the lattice vary dynamically.

To solve the full nonlinear dynamics we write the Heisenberg-Langevin equations, $i\hbar\partial_t \hat{a}_\mu = [\hat{a}_\mu, \hat{H}] - i\hbar\kappa\hat{a}_\mu$ for $\mu = c, s$ and $i\hbar\partial_t\hat{\psi} = [\hat{\psi}, \hat{H}]$, ignoring all input noise operators, whose means are zero, and neglecting atom losses over the time scales of interest so that $N = \langle\hat{N}\rangle$ is constant. Letting $\alpha_\mu = \langle\hat{a}_\mu\rangle$ and $\psi = \langle\hat{\psi}\rangle/\sqrt{N}$, and factoring the expectation values of operator products, we obtain the mean-field equations,

$$\partial_t\alpha_c = -(\kappa - i\Delta_+)\alpha_c - iU_0 N\mathcal{S}\alpha_s + \eta_c, \quad (3)$$

$$\partial_t\alpha_s = -(\kappa - i\Delta_-)\alpha_s - iU_0 N\mathcal{S}\alpha_c + \eta_s, \quad (4)$$

$$i\hbar\partial_t\psi = \left(-\frac{\hbar^2}{2m}\partial_z^2 + \hbar U_0|\mathcal{E}(z)|^2 - Fz\right)\psi. \quad (5)$$

Here, $\Delta_\pm = (\Delta_c - U_0 N) \mp U_0 N\mathcal{C}$ are the effective detunings, and $\mathcal{E}(z,t) = \langle\hat{\mathcal{E}}(z,t)\rangle$ is the dimensionless electric field. The standing-wave modes are pumped at rates $\eta_c = (\eta_+ + \eta_-)/\sqrt{2}$ and $\eta_s = i(\eta_+ - \eta_-)/\sqrt{2}$. Again we see that the lattice modulation is driven by changes in $\mathcal{C}$ and $\mathcal{S}$ during Bloch oscillations—$\mathcal{C}$ drives amplitude modulation through changes of the Stark detuning Δ_+ of the dominant cosine mode, and $\mathcal{S}$ induces shaking of the lattice (i.e., phase modulation) through coherent coupling of the sine and cosine modes.

In Fig. 2, we show the dynamics for ^{88}Sr atoms accelerating under gravity in a 689 nm ring cavity lattice (i.e., lattice spacing $d = \pi/k_r = 344.5$ nm). In this case we have $\omega_B = 2\pi \times 745$ Hz, and the recoil frequency $\omega_r \equiv \hbar k_r^2/(2m) = 2\pi \times 4.78$ kHz. Here we choose $U_0 N = -\kappa$ corresponding to the onset of collective strong coupling, and compare the dynamics with $\kappa = 2\pi \times 1$ kHz and 1 MHz. Bloch oscillations induce lattice amplitude modulation of ~10% peak to peak in both cases, with negligible shaking. The fast oscillations on top of the main modulation observed for larger κ were identified previously, and are predominantly higher harmonics of ω_B associated with the nonlinearity of the coupled atom-light

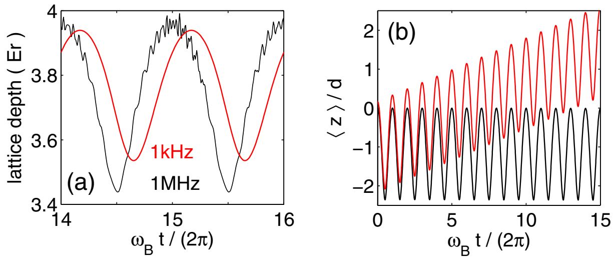

FIG. 2 (color online). Backaction-induced lattice modulation and atomic transport. Only the cosine cavity mode was pumped, with $U_0 = -2\pi \times 1$ Hz and $\Delta_c = U_0 N = -\kappa$. (a) Lattice depth in units of the recoil energy $E_r = \hbar\omega_r$ as a function of time during intracavity Bloch oscillations. The black curve is for $\kappa = 2\pi \times 1$ MHz, and the red curve for 1 kHz. (b) Condensate centroid position as a function of time, showing *uphill* transport for $\kappa \sim \omega_B$. Colors are the same as in (a).

system [34]. In the case of small κ these features are outside the cavity bandwidth and therefore suppressed.

Despite similar modulation depths for small and large κ in Fig. 2, transport is only observed when κ is on the order of ω_B. This is because κ is the relaxation rate for the light field, which in turn sets the relative lag between the backaction-induced lattice modulation and the atomic Bloch oscillation. When $\kappa \gg \omega_B$ the light adapts almost instantaneously to the atomic motion at ω_B and there is consequently no delay between the two; when $\kappa \sim \omega_B$ the light is not able to adiabatically follow the atoms' motion and a phase lag develops, as evident in Fig. 2(a). The effect of such a delay on transport in externally driven free-space lattices is well known [15,36], but in such cases the phase difference is a controlled parameter. In a cavity, on the other hand, the phase difference comes about self-consistently according to Eqs. (3)–(5).

Neglecting for the moment the origin of the modulation in Fig. 2, the resulting transport is similar to that in a free-space optical lattice under modulation of the lattice amplitude or phase, or of the bias force F [37–39]. Physically, transport occurs because the phase lag ensures that the duration for which the lattice is shallower, and therefore tunneling more effective, overlaps more with the motion in one direction than in the other. By breaking this symmetry, the band center is effectively shifted from zero quasimomentum and a net motion occurs [40]. The transport velocity is given by the group velocity $v_g = \hbar^{-1}\partial E(q)/\partial q$ averaged over a full Bloch oscillation, where $E(q)$ is the dispersion relation of the atoms in the untilted lattice, and q is the quasimomentum [36]. If the phase lag is zero, v_g averages to zero by the symmetry of $E(q)$ about $q = 0$, but for a finite lag the lattice modulation falls out of sync with the atoms, leading to transport at velocity

$$v_t = -dT_1 \sin\phi, \tag{6}$$

where T_1 is the tunneling rate between neighboring sites, which is proportional to the modulation depth, and ϕ is the lag between the modulation and the Bloch oscillation. For initial site-to-site coherence, such as we consider here, the result is directed transport superposed on the underlying Bloch oscillation [15]. Given initially random site-to-site phases, one instead observes spatial spreading of the atoms [10,14].

We have extended the above theory for free-space lattices to include the self-consistent optomechanical effects of a cavity. Under the approximation of nearest-neighbor tunneling, which is valid for modulation at ω_B, we find analytic expressions for T_1 and ϕ, which are given in the Supplemental Material [40]. The analytic theory is in excellent agreement with the numerical simulations. The magnitude and direction of transport depend on the cavity pumping parameters Δ_c and $\eta_\pm$, as well as the Stark shift $U_0 N$. We find that $|v_t|$ increases quadratically with $U_0 N$ for small values, and is maximized for intermediate lattice depths ($\sim 3\hbar\omega_r$ for the parameters of Fig. 2). In very shallow lattices, tunneling is strong but the modulation depth is reduced in proportion to the trap depth; in deep lattices tunneling is suppressed and the modulation depth is reduced due to flattening of the Bloch bands. The dependence on detuning Δ_c is shown in Fig. 3. For balanced pumping, with $\eta_+ = \eta_-$ ($\eta_s = 0$), the transport exhibits a dispersive shape around the Stark-shifted cavity resonance ($\Delta_+ = 0$). This is because modulation of $\mathcal{C}$ during the Bloch oscillation effectively dithers the detuning of the cosine cavity mode, as described by Eq. (3). The result is a modulation amplitude which approximately follows the derivative of the Lorentzian cavity response. The modulation phase also changes across the resonance, adding to the detailed shape we observe. Again, these effects are well captured by the analytic tight-binding theory. We note in passing that for larger blue detunings, Raman transitions of the type studied in [41] can become resonant, leading to Rabi oscillations between condensate momentum states and tunneling into higher bands.

Because the effects we have described so far are dominated by lattice amplitude modulation, they are

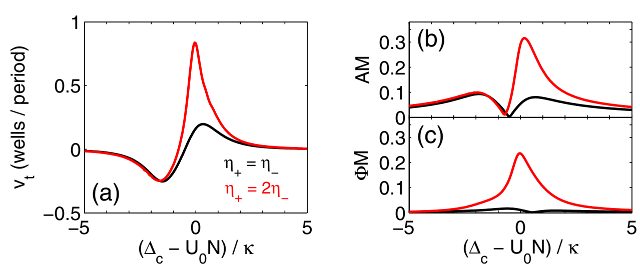

FIG. 3 (color online). Control of transport with detuning. (a) Transport velocity as a function of Δ_c, for the same parameters as Fig. 2 and $\kappa = 2\pi \times 1$ kHz. The black curve is for balanced pumping, and the red curve for a strong imbalance. (b) Relative depth of self-induced lattice amplitude modulation (AM), defined as half the peak-to-peak lattice depth modulation, normalized to the mean lattice depth. (c) Phase modulation (ΦM) defined as half the peak-to-peak spatial extent of the lattice shaking, in units of the lattice period.

qualitatively present in standing wave cavities as well. However, only in ring cavities is it possible to pump the counterpropagating running-wave modes with independent amplitudes, corresponding to direct pumping of the sine mode (not to be confused with a translation of the lattice, such as occurs during lattice shaking). In Fig. 3(a) we observe a strong enhancement of uphill transport around $\Delta_c = U_0 N$, when $\eta_+ = 2\eta_-$, even though the initial trap depth is the same as before. As seen in Figs. 3(b) and 3(c), both the amplitude and phase modulations are increased. Imbalanced pumping increases population of the cavity sine mode, thereby enhancing the backaction-induced lattice shaking. At first it may be surprising that this effect is only pronounced around $\Delta_c = U_0 N$, but the effective detuning of the cosine mode Δ_+ becomes positive here, corresponding to the regime of cavity heating [42], where linear stability analysis for $F = 0$ predicts unstable dynamics [43]. In contrast, near the positive transport peak one finds that both the sine and cosine modes operate in the cavity cooling regime with $\Delta_\pm$ negative, where the $F = 0$ dynamics are damped.

The effect of imbalanced pumping is investigated further in Fig. 4, where we vary the ratio η_+/η_-, for $\Delta_c = U_0 N$ and fixed initial trap depth. Phase modulation is more sensitive than amplitude modulation to small pump asymmetries. We observe that the transport minima are slightly offset from the condition of balanced pumping; the actual minima occur where the time-averaged value of $\mathcal{S}$ over a full Bloch oscillation period vanishes. This is due to weak pumping of the sine mode balancing the atomic dynamics. Simulations at strong imbalances reveal the existence of numerous types of instability. These include Landau-Zener tunneling, symmetry-breaking bistability [35], and collective excitations of the condensate [43], and will be the subject of a future work [44]. Near the onset of instability, the depths of amplitude and phase modulation as defined here become comparable; recall Figs. 3(b) and 3(c). However, we note that AM is more efficient at driving transport when each type of applied modulation is considered in isolation for free-space lattices. Transport appears to be dominated by amplitude modulation in all of the parameter regimes we have studied, with a fourfold increase in $|v_t|$ possible through imbalanced pumping of the cavity.

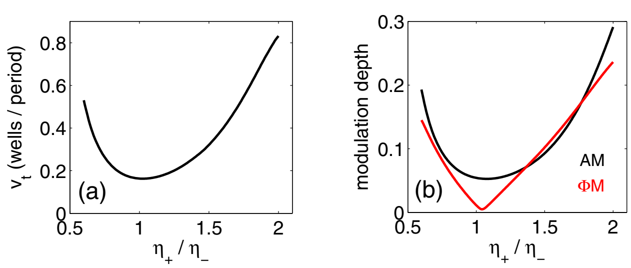

FIG. 4 (color online). Effects of imbalanced pumping of the running-wave cavity modes. (a) Transport velocity as a function of η_+/η_-, with $\Delta_c = U_0 N$ and the same parameters as Fig. 3. (b) Amplitude and phase modulation depths as defined above.

We can now compare our results to experiments in driven free-space lattices. Combined Bloch oscillations and transport have been observed by imaging atoms both *in situ* and in time of flight [14,15]. For intracavity lattices, one could detect the Bloch oscillations nondestructively in the transmitted light [33,34]. In fact, transport has its own unique signature: for the case shown in Fig. 2 we find a factor of two imbalance between the spectral power of the cavity fields in the sidebands at $\pm\omega_B$. Indeed, transport up (down) in our system is analogous to sideband-resolved cavity optomechanical heating (cooling) [45,46]. For a pump detuning $\Delta_+ \sim +\omega_B$, the $-\omega_B$ sideband is near resonance with the cavity and therefore dominates over the further detuned $+\omega_B$ sideband; the atoms undergo energy conserving Raman transitions up the Wannier-Stark ladder [39] of states separated in energy by steps of $\hbar\omega_B$. Conversely, a red-detuned pump with $\Delta_+ \sim -\omega_B$ leads to a dominant $+\omega_B$ sideband, with the atoms descending the Wannier-Stark ladder.

An important feature of the backaction-driven dynamics is that ω_B appears to remain unchanged from its value in a static lattice. Because the lattice in a ring cavity can accelerate, this does not have to be the case [47]. That the system oscillates at $Fd/\hbar$ is critical for applications in metrology and sensing, where the modulation frequency is taken as a measure of the force [33]. It also implies that backaction will not induce so-called super Bloch oscillations [15] because these occur when the lattice modulation is detuned from ω_B. It is worth noting that although the backaction-driven ΦM is much smaller than what has been applied in free-space experiments [48], the values of AM we observe are similar to what was applied in Refs. [38,39]. Finally, we note that our tight-binding theory predicts that in the absence of initial site-to-site coherence, the backaction-induced modulation will decay. This is because $\mathcal{C}$ and $\mathcal{S}$ become constant, cutting off the modulation of the lattice according to Eq. (2).

In conclusion, we have shown that optomechanical effects lead to qualitatively new dynamics for a Bose-Einstein condensate undergoing Bloch oscillations in a high finesse optical cavity. As the condensate quasimomentum samples the first Brillouin zone, the optical lattice depth and position are dynamically modulated, even as the Bloch frequency itself is unchanged. When the cavity damping rate is on the order of the Bloch oscillation frequency, coherent directed transport of the condensate can be observed. Asymmetric pumping of the running wave cavity modes enhances the transport, and in extreme cases makes the system dynamically unstable. Our results extend the study of coherent control of tunneling to include optomechanical lattice excitations. They are relevant to measurements of Bloch oscillations in cavities and other nondestructive atomic probes, and also more generally to attempts to realize neutral atom quantum simulators, where backaction upon electromagnetic fields is significant.

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Note added.—Since the preparation of this manuscript, we have become aware of the experimental observation of transport of a Bloch oscillating Bose-Einstein condensate in a high-finesse standing wave cavity [49].

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