Additional Strange Hadrons from QCD Thermodynamics and Strangeness Freezeout in Heavy Ion Collisions

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We compare lattice QCD results for appropriate combinations of net strangeness fluctuations and their correlations with net baryon number fluctuations with predictions from two hadron resonance gas (HRG) models having different strange hadron content. The conventionally used HRG model based on experimentally established strange hadrons fails to describe the lattice QCD results in the hadronic phase close to the QCD crossover. Supplementing the conventional HRG with additional, experimentally uncharted strange hadrons predicted by quark model calculations and observed in lattice QCD spectrum calculations leads to good descriptions of strange hadron thermodynamics below the QCD crossover. We show that the thermodynamic presence of these additional states gets imprinted in the yields of the ground-state strange hadrons leading to a systematic 5–8 MeV decrease of the chemical freeze-out temperatures of ground-state strange baryons.

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Introduction.—With increasing temperature, the strong interaction among constituents of ordinary nuclear matter, mesons and baryons, results in the copious production of new hadronic resonances. The newly produced resonances account for the interaction among hadrons to an extent that bulk thermodynamic properties become well described by a gas of uncorrelated hadronic resonances [1]. The hadron resonance gas (HRG) model is extremely successful in describing the hot hadronic matter created in heavy ion experiments [2]. Abundances of various hadron species measured in heavy ion experiments at different beam energies are well described by thermal distributions characterized by a freeze-out temperature and a set of chemical potentials $\vec{\mu} = (\mu_B, \mu_Q, \mu_S)$ for net baryon number (B), electric charge (Q), and strangeness (S) [3]. Nonetheless, details of the freeze-out pattern may provide evidence for a more complex sequential freeze-out pattern [4,5]. In particular, in the case of strange hadrons, arguments have been put forward in favor of a greater freeze-out temperature than that of nonstrange hadrons [6–8].

At the temperature $T_c = (154 \pm 9)$ MeV [9] strong interaction matter undergoes a chiral crossover to a new phase. In the same crossover region, HRG-based descriptions of the fluctuations and correlations of conserved charges for light [10], strange [6,11], as well as charm [12] degrees of freedom break down. Below T_c, bulk thermodynamic properties as well as conserved charge distributions are generally well described by a HRG containing all experimentally observed resonances (PDG-HRG) listed in the particle data tables [13]. However, there are also some notable differences between lattice QCD results and the PDG-HRG predictions. At temperatures below T_c, the trace anomaly, i.e., the difference between energy density and 3 times the pressure, is found to be greater in lattice QCD calculations [14–16] than that of the PDG-HRG. Fluctuations of net strangeness and correlations between net strangeness and baryon number fluctuations are also larger in QCD compared to those of the PDG-HRG [17,18]. It has been argued that the former provides evidence for contributions of additional, experimentally still-unobserved hadron resonances to the thermodynamics of strong interaction matter [19,20]. Indeed, a large number of additional resonances has been identified in lattice QCD [21] and quark model calculations [22,23]. The presence of such additional flavored hadrons in a thermal medium enhances fluctuations of the associated quantum numbers and modifies their correlations with other quantum numbers. In fact, lattice QCD results on net charm fluctuations and their correlations with baryon number, electric charge, and strangeness fluctuations have also been compared with the expectations from a HRG containing additional, experimentally unobserved charmed hadrons predicted by quark model calculations [12]. Such comparisons have provided evidence for the thermodynamic importance of additional charmed hadrons in the vicinity of the QCD crossover [12].

In this Letter, we show that discrepancies between lattice QCD results and PDG-HRG predictions for strangeness fluctuations and correlations below the QCD crossover can be quantitatively explained through the inclusion of additional, experimentally unobserved strange hadrons. The thermodynamic presence of these additional strange hadrons also gets imprinted on the yields of ground-state strange baryons, resulting in observable consequences for the chemical freeze-out of strangeness in heavy ion collision experiments.

Hadron resonance gas models.—The partial pressure of all open strange hadrons can be separated into mesonic and baryonic components $P_{\text{tot}}^{S,X} = P_M^{S,X} + P_B^{S,X}$,

$$P_{M/B}^{S,X}(T,\vec{\mu}) = \frac{T^4}{2\pi^2}\sum_{i\in X} g_i \left(\frac{m_i}{T}\right)^2 K_2(m_i/T) \times \cosh\left(B_i\hat{\mu}_B + Q_i\hat{\mu}_Q + S_i\hat{\mu}_S\right). \quad (1)$$

Here, M (B) labels the partial pressure of open strange mesons (baryons), g_i is the degeneracy factor for hadrons of mass m_i, and $\hat{\mu}_q \equiv \mu_q/T$, with $q = B, Q, S$. The sum is taken over all open strange mesons or baryons listed in the particle data tables ($X = \text{PDG}$) or a larger set including additional open strange mesons [23] and baryons [22] predicted by quark models ($X = \text{QM}$). Throughout this work, we refer to the HRG model containing these additional, quark model predicted, experimentally undiscovered hadrons as the QM-HRG. In Eq. (1), the classical, Boltzmann approximation has been used, which is known to be appropriate for all strange hadrons at temperatures $T \lesssim T_c$ [11].

The masses and, more importantly, the number of additional states are quite similar in the quark model calculations and the strange hadron spectrum of lattice QCD [21]. HRG models based on either one, thus, give very similar results. As the lattice computations of the strange hadron spectrum have so far been performed with unphysically heavy up and down quark masses, for definiteness we have chosen to compare our finite temperature lattice results with the quark model predictions that generally reproduce the masses of the experimentally known states rather well.

Figure 1 compares partial pressures of open strange mesons and baryons calculated within PDG-HRG and QM-HRG models. The additional strange baryons present in the QM-HRG lead to a large enhancement of the partial baryonic pressure relative to that obtained from the PDG-HRG model. In the mesonic sector, changes are below 5% even at $T = 170$ MeV, i.e., above the applicability range of any HRG [11]. This simply reflects that a large part of the open strange mesons is accounted for in the PDG-HRG model, and the additional strange mesons contributing to the QM-HRG model are too heavy to alter the pressure significantly.

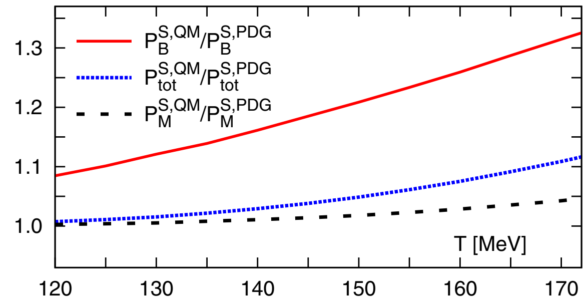

FIG. 1 (color online). Ratios of partial pressures of open strange hadrons ($P_{\text{tot}}^{S,X}$), mesons ($P_M^{S,X}$), and baryons ($P_B^{S,X}$) calculated in HRG with particle spectra from the particle data table, $X = \text{PDG}$, and with additional resonances predicted by the relativistic quark model, $X = \text{QM}$, respectively (see text).

Strangeness fluctuations and correlations.—We calculate cumulants of strangeness fluctuations and their correlations with baryon number and electric charge in (2 + 1)-flavor QCD using the highly improved staggered quark action [24]. In these calculations the strange quark mass (m_s) is tuned to its physical value and the masses of degenerate up and down quarks have been fixed to $m_l = m_s/20$. In the continuum limit, the latter corresponds to a pion mass of about 160 MeV. In the relevant temperature range, 145 MeV $\le T \le$ 170 MeV, we have analyzed $(10\text{–}16) \times 10^3$ configurations, separated by 10 time units in rational hybrid Monte Carlo updates, on lattices of size 6×24^3 and 8×32^3. Some additional calculations on 8×32^3 lattices have been performed with physical light quarks, $m_l = m_s/27$, to confirm that the quark mass dependence of observables of interest is indeed small.

To analyze the composition of the thermal medium in terms of the quantum numbers of the effective degrees of freedom, we consider generalized susceptibilities of the conserved charges,

$$\chi_{klm}^{BQS} = \left.\frac{\partial^{(k+l+m)}[P(T,\hat{\mu}_B,\hat{\mu}_Q,\hat{\mu}_S)/T^4]}{\partial\hat{\mu}_B^k \partial\hat{\mu}_Q^l \partial\hat{\mu}_S^m}\right|_{\vec{\mu}=0}, \quad (2)$$

where P denotes the total pressure of the hot medium. For brevity, we drop the superscript when the corresponding subscript is zero.

The correlation of net strangeness with net baryon number fluctuations normalized to the second cumulant of net strangeness fluctuations, χ_{11}^{BS}/χ_2^S is a sensitive probe of the strangeness carrying degrees of freedom [25]. Consistent continuum extrapolations for this ratio have been obtained with two different staggered fermion discretization schemes [17,18]. In Fig. 2 (top) we show our present, refined results obtained for lattices with temporal

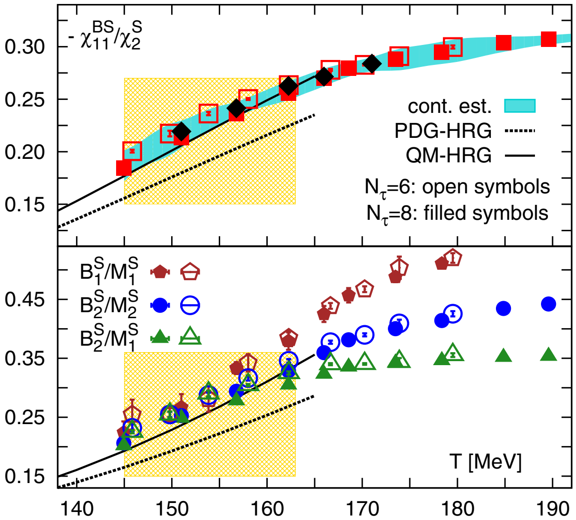

FIG. 2 (color online). Top: BS correlations normalized to the second cumulant of net strangeness fluctuations. Results are from (2 + 1)-flavor lattice QCD calculations performed with a strange to light quark mass ratio $m_s/m_l = 20$ (squares) and $m_s/m_l = 27$ (diamonds). The band depicts the improved estimate for the continuum result facilitated by the high statistics $N_\tau = 6$ and 8 data. Bottom: Ratios of baryonic (B_i^S) to mesonic (M_i^S) pressure observables defined in Eq. (3). The dotted and solid lines show results from PDG-HRG and QM-HRG model calculations, respectively. The shaded region denotes the continuum extrapolated chiral crossover temperature at physical values of quark masses $T_c = (154 \pm 9)$ MeV [9].

extents $N_\tau = 6$ and 8, which are in agreement with the earlier results, together with an improved estimate for the continuum result based on the enlarged statistics for these lattices. In the crossover region and also at lower temperatures in the hadronic regime, the lattice QCD results for $-\chi_{11}^{BS}/\chi_2^S$ are significantly greater than those of the PDG-HRG model predictions.

In the validity range of HRG models, the BS correlation χ_{11}^{BS} is a weighted sum of partial pressures of strange baryons, while the quadratic strangeness fluctuations χ_2^S are dominated by the contribution from strange mesons. The larger value of $-\chi_{11}^{BS}/\chi_2^S$ found in lattice QCD calculations compared to that of PDG-HRG model calculations, thus, reflects the stronger increase of $P_B^{S,\mathrm{QM}}/P_B^{S,\mathrm{PDG}}$ compared to $P_M^{S,\mathrm{QM}}/P_M^{S,\mathrm{PDG}}$ (see Fig. 1). As a consequence, the QM-HRG model provides a better description of QCD thermodynamics in the hadronic phase. This can be more directly verified by considering the ratio of two observables, which in a HRG model give P_M^S and P_B^S, respectively. There is a large set of "pressure observables" that can be constructed for this purpose by using second- and fourth-order cumulants of strangeness fluctuations and correlations with net baryon number [11]. They will all give identical results in a gas of uncorrelated hadrons, but differ otherwise. In particular, they can yield widely different results in a free quark gas. We use two linearly independent pressure observables for the open strange meson (M_1^S, M_2^S) and baryon (B_1^S, B_2^S) partial pressures, respectively,

$$
\begin{aligned}
M_1^S &= \chi_2^S - \chi_{22}^{BS},\\
M_2^S &= \frac{1}{12}(\chi_4^S + 11\chi_2^S) + \frac{1}{2}(\chi_{11}^{BS} + \chi_{13}^{BS}),\\
B_1^S &= -\frac{1}{6}(11\chi_{11}^{BS} + 6\chi_{22}^{BS} + \chi_{13}^{BS}),\\
B_2^S &= \frac{1}{12}(\chi_4^S - \chi_2^S) - \frac{1}{3}(4\chi_{11}^{BS} - \chi_{13}^{BS}).
\end{aligned} \tag{3}
$$

Three independent ratios B_i^S/M_j^S are shown in Fig. 2 (bottom). They start to coincide in the crossover region giving identical results only below $T \lesssim 155$ MeV. This reconfirms that a description of QCD thermodynamics in terms of an uncorrelated gas of hadrons is valid till the chiral crossover temperature T_c. Below T_c, the information one extracts from B_i^S/M_j^S agrees with that of $-\chi_{11}^{BS}/\chi_2^S$. In the hadronic regime, the ratios calculated in lattice QCD are significantly greater than those calculated in the PDG-HRG model. QM-HRG model calculations are in good agreement with lattice QCD. These results provide evidence for the existence of additional strange baryons and their thermodynamic importance below the QCD crossover.

Implications for strangeness freeze-out.—Since the initial nuclei in a heavy ion collision are net strangeness free, the HRG at the chemical freeze-out must also be strangeness neutral. Obviously, for such a strangeness neutral medium, all three thermal parameters T, μ_B, and μ_S are not independent; the strangeness chemical potential can be expressed as a function of T and μ_B. While $\mu_S(T, \mu_B)$ is unique in QCD, for a HRG it clearly depends on the relative abundances of the open strange baryons and mesons. For fixed T and μ_B, a strangeness neutral HRG having a larger relative abundance of strange baryons over open strange mesons naturally leads to a larger value of μ_S.

Calculations of $\mu_S(T, \mu_B)$ in a strangeness neutral HRG are straightforward. For QCD, this can be obtained from lattice QCD computations of μ_S/μ_B using next-to-leading-order Taylor expansion of the net strangeness density [3,26]. The ratio $\mu_S/\mu_B = s_1(T) + s_3(T)(\mu_B/T)^2 + \mathcal{O}(\mu_B^4)$ is closely related to the ratio χ_{11}^{BS}/χ_2^S shown in Fig. 2. At leading order, it only receives a small correction from nonzero electric charge chemical potential μ_Q/μ_B,

$$
\left(\frac{\mu_S}{\mu_B}\right)_{\mathrm{LO}} \equiv s_1(T) = -\frac{\chi_{11}^{BS}}{\chi_2^S} - \frac{\chi_{11}^{QS}}{\chi_2^S}\frac{\mu_Q}{\mu_B}. \tag{4}
$$

The next-to-leading-order correction $s_3(T)(\mu_B/T)^2$ [3] is small for $\mu_B \lesssim 200$ MeV. We show the leading order result in Fig. 3. At a given temperature, the strangeness neutrality constraint gives rise to a larger value, consistent with lattice QCD results, of μ_S/μ_B for the QM-HRG compared to the PDG-HRG. In other words, for a given μ_B/T, the required

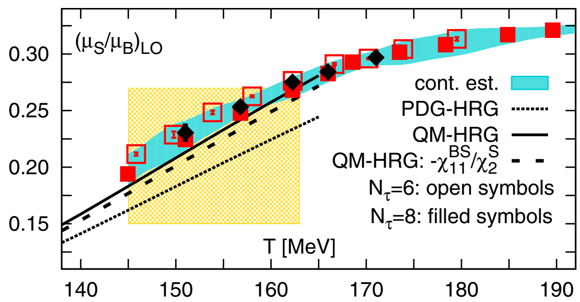

FIG. 3 (color online). The leading-order result for the ratio of strangeness and baryon chemical potentials versus temperature. Data and HRG model results are for a strangeness neutral thermal system having a ratio of net electric charge to net baryon number density $N_Q/N_B = 0.4$. The dashed line shows the QM-HRG result for vanishing electric charge chemical potential. All other curves and labels are as in Fig. 2.

value of μ_S/μ_B necessary to guarantee strangeness neutrality is achieved at a lower temperature in the QCD and QM-HRG models than in the PDG-HRG model.

The relative yields of strange anti-baryons ($\bar{H}_S$) to baryons (H_S) at freeze-out are controlled by the freeze-out parameters (T^f, μ_B^f, μ_S^f),

$$R_H \equiv \frac{\bar{H}_S}{H_S} = e^{-2(\mu_B^f/T^f)(1-(\mu_S^f/\mu_B^f)|S|)}. \qquad (5)$$

(For simplicity of the argument we will ignore here the influence of a nonvanishing electric charge chemical potential. As shown in Fig. 3 a nonzero μ_Q/T has only a small influence on μ_S/μ_B.) While this relation does not explicitly depend on the content and spectra of hadrons in a HRG, the presence of additional strange hadrons implicitly enters through the strangeness neutrality constraint. As discussed before, for different HRG models at fixed T and μ_B strangeness neutrality leads to different values of μ_S/μ_B. Once μ_B^f/T^f and μ_S^f/μ_B^f are fixed through experimental yields of strange hadrons, it is obvious from Fig. 3 that a given value of μ_S^f/μ_B^f is realized at a larger temperature in the PDG-HRG model than in the QM-HRG model.

Ratios of the freeze-out parameters μ_B^f/T^f and μ_S^f/μ_B^f can be obtained by fitting the experimentally measured values of the strange baryon ratios $R_\Lambda = \bar{\Lambda}/\Lambda$, $R_\Xi = \Xi^+/\Xi^-$ and $R_\Omega = \Omega^+/\Omega^-$ to Eq. (5). Fits of these strange anti-baryon to baryon yields to Eq. (5) result in $(\mu_S^f/\mu_B^f, \mu_B^f/T^f) = (0.213(10), 1.213(30))$ for the NA57 results at $\sqrt{s} = 17.3$ GeV [27] and (0.254(7),0.697(20)) for the STAR preliminary results at $\sqrt{s} = 39$ GeV [28]. In Fig. 4 we show comparisons of these $(\mu_S^f/\mu_B^f, \mu_B^f/T^f)$ values with the lattice QCD, QM-HRG, and PDG-HRG predictions for μ_S/μ_B at $\mu_B/T = \mu_B^f/T^f$. By varying the temperature ranges, one can match the values of μ_S/μ_B to μ_S^f/μ_B^f and, thus, determine the freeze-out temperatures T^f. As

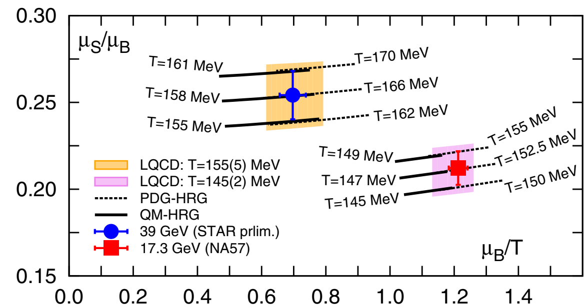

FIG. 4 (color online). Values of $(\mu_S^f/\mu_B^f, \mu_B^f/T^f)$ extracted from fits to multiple strange hadrons yields (see text) are compared to μ_S/μ_B predictions, obtained by imposing strangeness neutrality, from lattice QCD calculations (shaded bands) as well as from QM-HRG (solid lines) and PDG-HRG (dotted lines) models. The predictions are shown for $\mu_B/T = \mu_B^f/T^f$. For each case, the temperature ranges are chosen such that the predicted values reproduce μ_S^f/μ_B^f.

expected from Fig. 3, the QM-HRG predictions are in good agreement with lattice QCD results and lead to almost identical values for T^f. The PDG-HRG-based analysis, however, results in freeze-out temperatures for strange baryons that are larger by about 8 (5) MeV for the smaller (larger) value of T^f.

Conclusions.—By comparing lattice QCD results for various observables of strangeness fluctuations and correlations with predictions from PDG-HRG and QM-HRG models, we have provided evidence that additional, experimentally unobserved strange hadrons become thermodynamically relevant in the vicinity of the QCD crossover. We have also shown that the thermodynamic relevance of these additional strange hadrons modifies the yields of the ground-state strange hadrons in heavy ion collisions. This leads to significant reductions in the chemical freeze-out temperature of strange hadrons. Compared to the PDG-HRG, the QM-HRG model provides a more complete description of the lattice QCD results on thermodynamics of strange hadrons at moderate values of the baryon chemical potential. This suggests that the QM-HRG model is probably the preferable choice for the determination of the freeze-out parameters also at greater values of the baryon chemical potential beyond the validity of the present lattice QCD calculations.

Finally, we note that the observation regarding the thermodynamic relevance of additional strange hadrons hints that an improved HRG model including further, unobserved light quark hadrons may resolve the current discrepancy between lattice QCD results for the trace anomaly and the results obtained within the PDG-HRG model.

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[1] R. Hagedorn, Suppl. Nuovo Cimento **3**, 147 (1965).
[2] P. Braun-Munzinger, K. Redlich, and J. Stachel, in *Quark-Gluon Plasma*, edited by R. C. Hwa and X.-N. Wang (World Scientific, Singapore, 2004), Vol. 3, p. 491.
[3] A. Bazavov *et al.*, Phys. Rev. Lett. **109**, 192302 (2012).
[4] J. Cleymans, K. Redlich, H. Satz., and E. Suhonen, Z. Phys. C **58**, 347 (1993).
[5] J. Sollfrank and U. W. Heinz, in *Quark-Gluon Plasma*, edited by R. C. Hwa (World Scientific, Singapore, 1995), Vol. 2, p. 555.
[6] R. Bellwied, S. Borsanyi, Z. Fodor, S. D. Katz, and C. Ratti, Phys. Rev. Lett. **111**, 202302 (2013).
[7] S. Chatterjee, R. M. Godbole, and S. Gupta, Phys. Lett. B **727**, 554 (2013).
[8] K. A. Bugaev, D. R. Oliinychenko, J. Cleymans, A. I. Ivanytskyi, I. N. Mishustin, E. G. Nikonov, and V. V. Sagun, Europhys. Lett. **104**, 22002 (2013).
[9] A. Bazavov *et al.* (HotQCD Collaboration), Phys. Rev. D **85**, 054503 (2012).
[10] S. Ejiri, F. Karsch, and K. Redlich, Phys. Lett. B **633**, 275 (2006).
[11] A. Bazavov *et al.*, Phys. Rev. Lett. **111**, 082301 (2013).
[12] A. Bazavov *et al.*, arXiv:1404.4043.
[13] J. Beringer *et al.* (Particle Data Group), Phys. Rev. D **86**, 010001 (2012).
[14] S. Borsanyi, G. Endrődi, Z. Fodor, A. Jakovác, S. D. Katz, S. Krieg, C. Ratti, and K. K. Szabó, J. High Energy Phys. 11 (2010) 077.
[15] S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg, and K. K. Szabó, Phys. Lett. B **730**, 99 (2014).
[16] A. Bazavov *et al.* (HotQCD Collaboration), .
[17] S. Borsanyi, Z. Fodor, S. D. Katz, S. Krieg, C. Ratti, and K. Szabó, J. High Energy Phys. 01 (2012) 138.
[18] A. Bazavov *et al.* (HotQCD Collaboration), Phys. Rev. D **86**, 034509 (2012).
[19] A. Majumder and B. Müller, Phys. Rev. Lett. **105**, 252002 (2010).
[20] J. Noronha-Hostler, M. Beitel, C. Greiner, and I. Shovkovy, Phys. Rev. C **81**, 054909 (2010).
[21] R. G. Edwards, N. Mathur, D. G. Richards, and S. J. Wallace, Phys. Rev. D **87**, 054506 (2013).
[22] S. Capstick and N. Isgur, Phys. Rev. D **34**, 2809 (1986).
[23] D. Ebert, R. N. Faustov, and V. O. Galkin, Phys. Rev. D **79**, 114029 (2009).
[24] E. Follana, Q. Mason, C. Davies, K. Hornbostel, G. Lepage, J. Shigemitsu, H. Trottier, and K. Wong (HPQCD collaboration and UKQCD collaboration), Phys. Rev. D **75**, 054502 (2007).
[25] V. Koch, A. Majumder, and J. Randrup, Phys. Rev. Lett. **95**, 182301 (2005).
[26] S. Borsanyi, Z. Fodor, S. D. Katz, S. Krieg, C. Ratti, and K. K. Szabó, Phys. Rev. Lett. **111**, 062005 (2013).
[27] F. Antinori *et al.* (NA57 Collaboration), Phys. Lett. B **595**, 68 (2004).
[28] F. Zhao (for the STAR Collaboration), *Proc. Sci.*, CPOD2013 (2013) 036.