Inflation with Whip-Shaped Suppressed Scalar Power Spectra

Dhiraj Kumar Hazra*
Asia Pacific Center for Theoretical Physics, Pohang, Gyeongbuk 790-784, Korea

Arman Shafieloo†
Asia Pacific Center for Theoretical Physics, Pohang, Gyeongbuk 790-784, Korea and Department of Physics, POSTECH, Pohang, Gyeongbuk 790-784, Korea

George F. Smoot‡
Université Sorbonne Paris Cité, Laboratoire APC-PCCP Université Paris Diderot 10, rue Alice Domon et Leonie Duquet, 75205 Paris Cedex 13, France and Physics Department & LBNL, University of California, Berkeley, California 94720, USA

Alexei A. Starobinsky§
Landau Institute for Theoretical Physics RAS, Moscow 119334, Russia
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Motivated by the idea that inflation occurs at the grand unified theory symmetry breaking scale, in this Letter we construct a new class of large field inflaton potentials where the inflaton starts with a power law potential; after an initial period of relatively fast roll that lasts until after a few *e folds* inside the horizon it transits to the attractor of the slow roll part of the potential with a lower power. Because of the initial fast roll stages of inflation, we find a suppression in scalar primordial power at large scales and at the same time the choice of the potential can provide us a tensor primordial spectrum with a high amplitude. This suppression in scalar power with a large tensor-to-scalar ratio helps us to reconcile the Planck and BICEP2 data in a single framework. We find that a transition from a cubic to quadratic form of inflaton potential generates an appropriate suppression in the power of the scalar primordial spectrum that provides a significant improvement in fit compared to the power law model when compared with Planck and BICEP2 data together. We calculate the extent of non-Gaussianity, specifically, the bispectrum for the best fit potential, and show that it is consistent with Planck bispectrum constraints.

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Introduction.—Detection of B-mode polarization of cosmic microwave background (CMB) photons by BICEP2 [1,2] constrains the primordial tensor amplitude and thereby confirms yet another prediction of inflation. The constraints on the tensor amplitude obtained through BICEP2 data bring in a new question in primordial cosmology since the bounds on the tensors obtained from Planck temperature anisotropy [3,4] data have only limited overlap with the BICEP2 data. This inconsistency between the two data sets can be viewed from another interesting perspective where we can address these two data simultaneously. It has been shown in a number of recent papers [5–9] that a drop in the large scale scalar primordial power spectrum (PPS) allows tensor-to-scalar ratio r to be large and helps to fit the Planck and BICEP2 data simultaneously, and significantly better than power law scalar PPS. Using simple phenomenological models from [10] in a recent paper [5], we have shown that the addition of BICEP2 data rules out the power law form of the primordial spectrum at more than 3σ C.L.

The above results motivate us to build inflationary models that can generate a suppression in scalar power at large scales along with reasonable amplitude of tensor PPS ($r \sim 0.2$). Moreover, keeping the primordial bispectrum constraints from Planck in mind we should ensure that the potential does not generate large non-Gaussianity for a large window of cosmological scales. In this Letter, staying within the canonical Lagrangian for a single scalar field, keeping up to renormalizable terms in the inflaton potential, and using the standard Bunch-Davies vacuum initial conditions for perturbations, we provide an inflationary potential which can address all the issues and fits the data from Planck and BICEP2 significantly better than the power law scalar PPS. Whipped inflation signifies the scalar power spectra generated in these models resembles a long snapped whip.

The Letter is organized as follows. In the first section we shall describe the form of an inflaton potential that we propose and use here. Next, we shall briefly discuss the numerical methods used to solve the potential and to compare the scalar and the tensor PPS to the data. In the results and discussions section we shall present the status of whipped inflation in the light of Planck and BICEP2 data and, finally, we shall conclude with the main features of this analysis.

Inflationary potential.—The potential we are proposing in this Letter consists of a rapidly varying and a slowly varying part [Eq. (1)]. Afterwards, in this section we provide the motivation of the construction of this potential,

$$V(\phi) = V_S(\phi) + V_R(\phi). \tag{1}$$

The basic construction of our potential depends on two facts. First, to generate large r we need to work with large field models. Second, since the largest scale modes leave the Hubble radius in the earliest times, an initial brief period of relatively fast roll (fast compared to the next slow roll phase) can help suppress the large scale power while at the same time produce higher tensor mode perturbations.

In this Letter we choose to work with the following rapid and slow part of the potential as in Eq. (2).

$$\begin{aligned} V_S(\phi) &= \gamma\phi^p, \\ V_R(\phi) &= \lambda(\phi-\phi_0)^q\Theta(\phi-\phi_0), \end{aligned} \tag{2}$$

where $\Theta(\phi-\phi_0)$ denotes the Heaviside theta function which cuts off the contribution of the rapid part beyond $\phi \leq \phi_0$. Note that both $V_S(\phi)$ and $V_R(\phi)$ contain power law potentials. We start near the minima of the potential $V_R(\phi)$, i.e., near $\phi = \phi_0$ (but still for $\phi_{\rm initial} > \phi_0$ and $\phi_{\rm initial} - \phi_0 \sim 3$–$4$ $\mathrm{M_{PL}}$) where the field rolls relatively fast (yet $\epsilon_H = -\dot{H}/H^2 < 1$) and then reaches the attractor of $V_S(\phi)$. Since the inflaton ϕ will reach $V_S(\phi)$ after an initial fast roll period, the power p of the slowly varying part will determine the amplitude of the tensor PPS at smaller scales. $V_R(\phi)$ on the other hand defines the suppression at large scales.

A similar type of transition was originally used in [11] for $q = 1$; however, since r was a free parameter for this model, it could not be predicted in advance. (A similar PPS was also studied in [12].) For $q = 2$, Refs. [13,14] discuss the effects in the primordial power spectrum in hybrid inflation scenarios where the potential generates a step in the spectral index, modulated by characteristic oscillations. It was recently generalized to an arbitrary q in [15] based on the hypothesis of the first order phase transition in the inflaton field at the grand unified theory (GUT) scale with formation of Coleman–de Luccia bubbles, following the previous papers on this topic [16,17], and similar to what was originally proposed in [18], but followed by $N \sim 60$ e-folds of more standard slow roll inflation. However, this picture is not the only possibility. Alternatively, as was assumed in [11,13,14], such potential may arise due to a fast phase transition in *another* massive field coupled to the inflaton. This transition is of the first order for $q = 1$ and second order for $q = 2$; see [19]. In this case, slow roll inflation is only temporarily weakly broken around the moment of this transition.

In this Letter, however, keeping in mind that we need to introduce a large scale suppression in scalar power for a wide range of cosmological scales, we need an initial and extended fast roll period with a smooth transition to the slow roll phase more than localized features. Hence, the choice of $V_S(\phi)$ and $V_R(\phi)$ are most important here in order to generate appropriate suppression within the *single canonical scalar field* model with the minimal number of extra parameters. In our analysis, we choose to work with $(p,q) = (2,3)$, $(2,4)$, and (3,4). Note that the higher the q, the higher the tensor amplitude will be. We understand that assuming higher powers in q in the second part of Eq. (2) requires more fine tuning—in other words, a greater amount of hidden symmetry of inflaton interactions with itself and other fields. However, our aim is to investigate phenomenological consequences of such an assumption. These values of p and q are chosen since they work reasonably fine and because they are suitable as one has some guides from previous attempts at Higgs induced inflation and vacuum stability on the Higgs potential.

Since release of the first year WMAP data, there were indications of large scale suppression in scalar power [20], and model independent reconstructions [21] too suggest the possibility of large scale suppression and different features in the scalar spectrum. Note that large scale suppression of scalar PPS due to an inflaton fast roll stage prior to its slow roll stage has been discussed in [22–24]. Through intermediate fast roll during inflation, the suppression in power was addressed in different papers; see Refs. [25]. In [26], keeping the importance of tensor perturbations in mind, the complete scalar and tensor power spectra were confronted with different combinations of CMB data sets in canonical and noncanonical scalar field scenarios.

In this work too, a mild amount of temporal fast roll is used in whipped inflation. We should emphasize that according to the present demand of the Planck and BICEP2 data combination [5] and being within a renormalizable canonical scalar field theory, whipped inflation is the first framework that can provide the overall suppression in scalar power, appropriate tensors, and low non-Gaussianity.

A few comments on the numerical methods.—We solve the background inflationary equations and the scalar and tensor perturbation equations using the publicly available code BI-spectra and Non-Gaussianity Operator, BINGO [27]. We have fixed the initial conditions for inflaton by allowing sufficient e-folds ~ 70 and by using initial slow roll. We have assumed that the pivot scale $k_* = 0.05\ \mathrm{Mpc}^{-1}$ leaves the Hubble radius 50 e-folds before the end of inflation. However, for $(p,q) = (3,4)$ case we have also worked with the assumption that k_* leaves 60 e-folds before the inflation ends. We shall denote this number of e-folds by the usual convention N_*. The tensor part is calculated using a modified version of the same code, yet to be publicly available. We have modified CAMB [28,29] to work with the BINGO outputs directly. We have used the COSMOMC [30,31] to find the best fit using Powell's BOBYQA method of iterative minimization [32]. We have used the COMMANDER and CAMSPEC likelihood to estimate the low-ℓ and high-ℓ likelihood from Planck data [3].

We have used WMAP low-ℓ (2–23) E-mode polarization data [33] (denoted as WP in the results section). The complete BICEP2 likelihood is calculated using band powers for 9 bins for E- and B-mode polarization data. We should also mention here that to make our analysis robust, we have allowed the background cosmological parameters and the 14 Planck foreground nuisance parameters vary along with the inflationary potential parameters. To calculate the non-Gaussianity for this inflationary model, we again use BINGO in the equilateral limit. Note that POLARBEAR B-mode polarization [34] data can also help in order to constrain the cosmology better and for a complete parameter estimation we expect to include POLARBEAR data in a future analysis with whipped inflation. In all our analyses we have assumed a spatially flat Friedmann-Lemaître-Robertson-Walker metric universe. We have defined ${\rm M_{PL}}^2 = 1/(8\pi G)$ and used $\hbar = c = 1$ throughout the Letter.

Results and discussion.—In this section we provide the best fit results obtained for the potential in Eq. (1) for different choices of p and q. Table I contains the best fit values of the inflationary potential parameters and different cosmological parameters. Best fit $\chi^2_{\rm eff}$ $(-2\ln\mathcal{L})$ are provided along with their breakdown in different data sets. The value of $-2\Delta\ln\mathcal{L}$ indicates the difference between the log likelihood obtained in a particular model and the power law scalar and tensor PPS (for power law best fit values, see [5]) when compared with Planck + WP + BICEP2 data combinations. Note that when we work with $p, q = 2, 3$ (cubic to quadratic transition), we get maximum improvement in likelihood (approximately 8). The higher tensor-to-scalar ratio ($r \sim 0.15$–0.25) in all these models helps to fit BICEP2 data as good as (or better than) the power law model. Importantly, the suppression at the large scale scalar PPS, originated from the initial fast roll phase, helps to fit the Planck data significantly better then the power law model. For quartic to quadratic transition ($p = 2$ and $q = 4$) we find that the improvement in fit decreases marginally from $p, q = 2, 3$. This decrease can be attributed to the shape of the transition, which indicates that along with the slow roll part of the potential, the initial *type* of fast roll part is also important in order to address the data better. When we allow the transition to cubic potentials, i.e., $p = 3$, we know that due to even higher tensor-to-scalar ratio ($r \sim 0.2$–0.25 compared to the quadratic potential, we are able to fit the BICEP2 data better than the power law (where the power law optimizes the likelihood between Planck and BICEP2 data) but the fit to the Planck data becomes a bit worse. [The fit to the Planck data gets worse, since (i) corresponding to the high r, the suppression is not enough which fits COMMANDER worse, and (ii) this model produces more red tilt in the scalar PPS ($n_S \sim 0.95$) at small scales than demanded by Planck data]. This model, though, provides an overall 4 improvement in fit compared to the power law model. The result for quartic to cubic transition with $N_* = 60$ is also provided, where we notice further improvement in likelihood compared to the same transition with $N_* = 50$.

TABLE I. Best fit parameters for the whipped inflaton potential 1 and the best fit cosmological parameters when compared with Planck + WP + BICEP2 data combination. The breakdown of best fit likelihood in different data sets are provided along with the difference in log likelihood compared to the best fit power law scalar PPS model [5]. The table contains the best fit parameters for different choices of p and q. Note that for $p = 2, q = 3$ we are able to provide a significant ($-2\Delta\ln\mathcal{L} \sim -8$) better fit to the data compared to the power law scalar PPS (or, equivalently, scalar PPS from a strict slow roll inflation). For other choices of p and q we also get substantial improvement. Note that when we allow the $p = 3$, due to the higher tensor-to-scalar ratio, the model fits the BICEP2 data better, but it does not fit the Planck low-ℓ data equivalently compared to the other cases. However, for $N_* = 60$ we find the model with $p = 3, q = 4$ is fitting the data marginally better than $N_* = 50$.

Inflation potential [Eq. (1)] and cosmological parameters.	$p = 2$, $q = 3$ $N_* = 50$	$p = 2$, $q = 4$ $N_* = 50$	$p = 3$, $q = 4$ $N_* = 50$	$p = 3$, $q = 4$ $N_* = 60$
$\Omega_b h^2$	0.022 06	0.022 08	0.022 08	0.022 06
$\Omega_{\rm CDM} h^2$	0.1189	0.1193	0.1198	0.1191
100θ	1.041	1.041	1.041	1.041
τ	0.098	0.096	0.097	0.085
γ	2.6×10^{-11}	2.6×10^{-11}	1.5×10^{-12}	9.4×10^{-13}
λ	1.1×10^{-10}	5.5×10^{-11}	4.6×10^{-11}	5.2×10^{-11}
$\phi_0({\rm M_{Pl}})$	14.27	14.11	17.17	18.97
Ω_m	0.31	0.31	0.315	0.31
H_0	67.6	67.4	67.25	67.5
$-2\ln\mathcal{L}$				
COMMANDER $\ell = 2$–49	−8.49	−6.83	−4.08	−5.33
CAMSPEC $\ell = 50$–2500	7796.45	7796.67	7797.69	7797.76
WP	2013.52	2013.46	2013.1	2013.4
BICEP2	40.43	40.4	38.9	39
Total	9841.91	9843.7	9845.61	9844.83
$-2\Delta\ln\mathcal{L}$	−7.67	−5.88	−3.97	−4.57

In Fig. 1, we plot the best fit results corresponding to the values of the parameters given in Table I. We plot the best fit potentials and their derivatives and the first slow roll parameter $\epsilon_H = -\dot{H}/H^2$. Note that the potentials and their derivatives are normalized to 1 at the point where inflaton transits to the slow roll phase, i.e., at $\phi = \phi_0$. The plot of slow roll parameter ϵ_H shows the fast roll to slow roll transition during the initial inflationary epoch. For $p = 3$ ($N_* = 50$), we find that the slow roll parameter settles down to a larger value as expected, indicating a higher r than what we obtain from $p = 2$. We also plot the scalar $[P_S(k)]$ and tensor $[P_T(k)]$ PPS from different models and power law PPS. Note that the different kinds of suppression

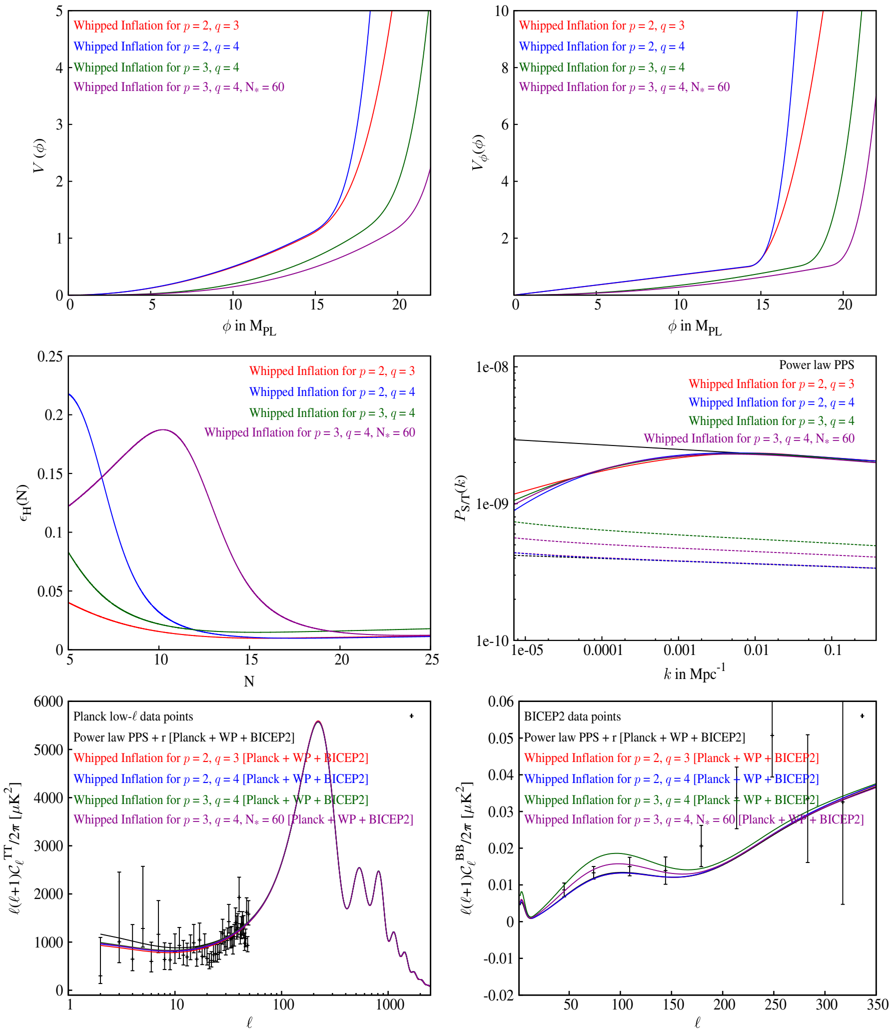

FIG. 1 (color online). Top : Best fit inflaton potentials (left) and their derivatives (right) are plotted corresponding to the best fit values in Table I obtained for whipped inflationary potential [Eq. (1)]. Middle left : First slow roll parameter, $\epsilon_H = -\dot{H}/H^2$, for different cases are plotted. Middle right : Best fit primordial scalar (solid) and tensor (dashed) power spectra are plotted along with power law best fit PPS (in black). Bottom : The best fit angular power spectra $\mathcal{C}_\ell^{\mathrm{TT}}$ (left) and $\mathcal{C}_\ell^{\mathrm{BB}}$ (right) are plotted along with corresponding data points from Planck and BICEP2 data. The power law best fit spectra are plotted in black. Note that in all the cases, our whipped inflation model fits Planck data significantly better than the power law through the suppression in large scale $\mathcal{C}_\ell^{\mathrm{TT}}$.

in scalar power for different models are evident from the plot. Finally, in the same figure we plot the best fit $\mathcal{C}_\ell^{\mathrm{TT}}$ and $\mathcal{C}_\ell^{\mathrm{BB}}$ along with the corresponding data points from Planck (low-ℓ) and BICEP2 data. Note that our models, for all given values of p and q, are able to address the data. Note

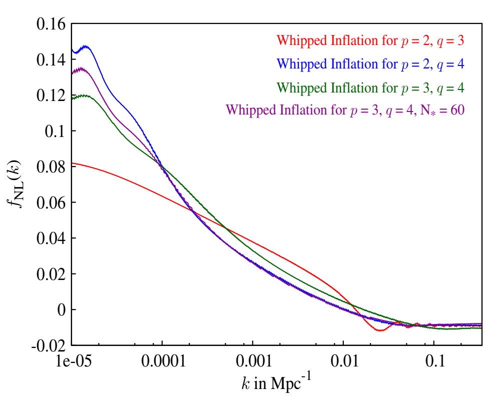

FIG. 2 (color online). The f_{NL} in equilateral triangular configurations ($k_1 = k_2 = k_3 = k$) are plotted for the best fit model are plotted. In all the cases, within cosmological scales, the bispectrum contribution is small and consistent with Planck limits.

that the suppression in low-ℓ $\mathcal{C}_\ell^{\mathrm{TT}}$ is the main factor that improves the likelihood significantly compared to the power law model, as has been tabulated in Table I.

In Fig. 2, we plot the non-Gaussianity, specifically the f_{NL}, in equilateral triangular configurations, corresponding to the best fit values quoted in Table I. The f_{NL} is calculated using the publicly available code BINGO using the methods described in [27,35–37]. Because of the initial fast roll phase we find *relatively* high f_{NL} [$\mathcal{O}(0.2)$] at large scales and at small scales the f_{NL} settles to a value closer to 0. In all the cases, the generated f_{NL} in these models are completely consistent with Planck constraints [38].

Conclusions.—In this Letter we provide an inflaton potential for the canonical scalar field model that can address the CMB temperature and polarization data from Planck and BICEP2 significantly better than the power law model. The potential offers a rapid and a slow roll part and we show that the inflaton, after having fast roll for around 10–15 e-folds, eventually falls on the attractor of the slow roll part of the potential. The initial fast roll period introduces a large scale suppression in scalar PPS. Since we have shown in our recent paper [5] that using Planck temperature anisotropy data and BICEP2 polarization data (mainly B-mode data), a large scale scalar power suppression rules out power law scalar PPS at more than 3σ C.L., the models described in this Letter serve as suitable representative models for consistently addressing Planck and BICEP2 data together.

The results in this Letter indicate the following facts: (i) Staying within the canonical scalar field model, working with only renormalizable terms, and using the simple Bunch-Davies vacuum initial condition, it is possible to have a set of inflationary potential that can resolve the inconsistencies between Planck and BICEP2 data. (ii) The significant improvement in fit ($-2\Delta \ln \mathcal{L} \sim -8$) compared to the power law model (or, equivalently, a strict slow roll model) certainly keeps this class of potentials in a higher ground. (iii) In our recent paper [5], we have argued that we need more flexibilities than a power law power spectrum in order to explain CMB data better. The same statement can be translated in inflationary theory to that we need more flexibilities than what we have in a strict slow roll inflation. Our analysis justifies the addition of three extra parameters, namely, the form of the rapid potential (described by q and λ) and the scale of the transition from fast to slow roll, i.e., ϕ_0. (iv) The type of power suppression at large scale scalar PPS is important. Hence, a proper transition from potential power q to p is necessary. (v) We do not need to construct theories to get a blue tensor PPS tilt to address the data. (vi) Since the feature (suppression) is not localized in a small window of cosmological scales, the suppressions that the potentials offer do not fit statistical uncertainties in the data. (vii) The non-Gaussianity in these models are small and, hence, certainly favored by Planck.

We would like to close the Letter by commenting that, if the value of tensor-to-scalar ratio r, obtained from B-mode polarization data from BICEP2 persists, the potentials discussed in this Letter offer simple and, at the same time, important mechanisms to generate scalar and tensor PPS that can consistently address temperature and the polarization anisotropy data in single framework. The whipped inflation models will then be worthy of an effort to relate them to the GUT symmetry breaking phase transition.

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Note added.—Just before we submitted this Letter, Ref. [42] suggested that the detected B-mode polarization data by BICEP2 might be contaminated by the radiation from galactic radio loops and this could largely affect any cosmological conclusion. We should clarify that our results are based on the BICEP2 and Planck data assuming a good control of the systematics, taking in to account the effects of all foregrounds.

*dhiraj@apctp.org
†arman@apctp.org
‡gfsmoot@lbl.gov
§alstar@landau.ac.ru

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