Silk Damping at a Redshift of a Billion: New Limit on Small-Scale Adiabatic Perturbations

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We study the dissipation of small-scale adiabatic perturbations at early times when the Universe is hotter than $T \approx 0.5$ keV. When the wavelength falls below the damping scale k_D^{-1} , the acoustic modes diffuse and thermalize, causing entropy production. Before neutrino decoupling, k_D is primarily set by the neutrino shear viscosity, and we study the effect of acoustic damping on the relic neutrino number, primordial nucleosynthesis, dark-matter freeze-out, and baryogenesis. This sets a new limit on the amplitude of primordial fluctuations of $\Delta_R^2 < 0.007$ at 10^4 Mpc⁻¹ $\leq k \leq 10^5$ Mpc⁻¹ and a model-dependent limit of $\Delta_R^2 \leq 0.3$ at $k \leq 10^{20-25}$ Mpc⁻¹.

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Introduction.—A wealth of astronomical observations, especially measurements of the cosmic microwave background (CMB) temperature and polarization anisotropies, have elevated the hot big bang to a detailed and precise model for the early Universe. There is also strong evidence that the Universe underwent a period of inflationary expansion which sets the initial conditions for the growth of large-scale structure from primordial curvature perturbations. Given the initial conditions, a simple model of a flat Universe that is filled with baryons, cold dark matter, neutrinos and a cosmological constant (Λ CDM), already describes the data extraordinarily well [1,2].

The success of ΛCDM on large scales is reflected in the precise determination of cosmological parameters, such as the contributions of baryons, $\Omega_{\rm b}h^2 = 0.02205 \pm 0.00028$, and cold dark matter (DM), $\Omega_c h^2 = 0.1199 \pm 0.0027$, to the universal energy density, or the effective number $N_{\rm eff} =$ 3.30 ± 0.27 of massless neutrinos ν [2]. The latter informs us about the particle content at matter-radiation equality and is currently consistent with that of the Standard Model (SM) of particle physics. It is furthermore common belief that a Universe, endowed with such minimal field content, had an "uneventful" thermal history between the epochs of big-bang nucleosynthesis (BBN)-or possibly even between dark-matter freeze-out (FO)-and hydrogen recombination, so that the number-to-entropy ratios remain constant, $(N_{b,c,\nu} - N_{\bar{b},\bar{c},\bar{\nu}})/S|_{\text{CMB}} = (N_{b,c,\nu} - N_{\bar{b},\bar{c},\bar{\nu}})/S|_{\text{BBN/FO}}$. Importantly, this allows one to perform cosmological concordance tests from BBN light-element yields, to judge the viability of DM models from their expected FO abundance, to infer parameters for successful baryogenesis, or to contemplate or discard extensions of the SM ν sector.

In this *Letter*, we emphasize that the above rationale carries the implicit assumption $\int \Delta_{\mathcal{R}}^2(k) d \ln k \ll 1$, where $\Delta_{\mathcal{R}}^2(k)$ is the variance of the primordial curvature amplitude on wavelengths k. This is because a fraction $\delta \rho / \rho \propto \zeta$ of the total energy density is stored in the primordial curvature perturbations ζ . Once a mode with wave number k enters the horizon, it becomes dynamical (an "acoustic wave") and dissipates its energy by particle diffusion, commonly referred to as Silk damping [3] when it regards the photon-baryon fluid in the post-BBN era. This process leads to entropy production, or more concisely changes in the particle number, and consequently affects the early thermal history.

While the amplitude $\Delta_{\mathcal{R}}^2(k) \equiv \langle |\zeta|^2 \rangle \simeq \mathcal{O}(10^{-9})$ of the primordial power spectrum at scales 10^{-3} Mpc⁻¹ $\lesssim k \lesssim$ 3 Mpc⁻¹ is tightly constrained by the CMB [2,4], galaxy clustering [5], and the Lyman- α forest [6], $\Delta_{\mathcal{R}}^2$ remains essentially unconstrained on smaller scales (larger *k*). Upper limits at $k \gtrsim 3$ Mpc⁻¹ available in the literature are derived from limits on CMB spectral distortions [7–11], the absence of evidence for primordial black holes [12–15], and from indirect constraints of DM annihilation inside ultracompact minihalos [16]. Here, we add an independent constraint for $k \gtrsim 10^4$ Mpc⁻¹ that can be viewed as more robust in that it derives directly from an altered thermal history of the early Universe and is independent of any new physics beyond the SM.

Our work expands on earlier investigations that primarily discuss the damping of perturbations at low redshift $z \lesssim 2 \times 10^6$ (the spectral-distortion era), where energy injection from dissipation is not fully thermalized but rather leads to a readjustment of the photon spectrum [9,17,18]. At high redshift $z \gtrsim 2 \times 10^6$ (the blackbody era), photon-number-changing interactions quickly restore a blackbody spectrum, so that any direct observable from the CMB is wiped out [19–23]. Therefore the blackbody era has received little attention in the past. As we show below, though, early energy release modifies the thermal history of the Universe at $T \gtrsim \text{keV}$ and thus the standard calculations of neutrino number, BBN, baryon-to-photon ratio η_b , and dark-matter relic density.

Dissipation of acoustic modes.-Let us denote the total energy density of relativistic particles in equilibrium with photons as $\rho = \sum \rho_i$ and the energy density of individual species *i* by ρ_i . Similarly, we write $N = \sum N_i$ for the average number density of particles. For adiabatic initial conditions, the photon density perturbations outside the horizon $\delta^i_{\gamma}(\mathbf{k}) = \delta \rho_{\gamma} / \rho_{\gamma}$ are related to the primordial curvature perturbation $\zeta(\mathbf{k})$ by $\delta^i_{\gamma}(\mathbf{k}) = -(4/3)C\zeta(\mathbf{k})$, where C = 1 and $C = (1 + 4/15R_{\nu})^{-1}$ before and after neutrino decoupling, respectively [24], $R_{\nu} \equiv \rho_{\nu}/(\rho_{\nu} + \rho_{\gamma})$, and we assume neutrino decoupling as instantaneous at temperature $T_{\nu,\text{dec}} = 1.5$ MeV. After entering the horizon, radiation-density perturbations evolve as $\delta_{\gamma}(t, \mathbf{k}) \approx$ $3\delta_{\gamma}^{i}(\mathbf{k})\cos[kr_{s}(t)]\exp[-k^{2}/k_{D}^{2}(t)]$, where r_{s} is the sound horizon at time t [25,26] and $k_D(t)$ is the diffusion scale below which $(k = |\mathbf{k}| > k_D)$ modes are being dissipated.

The presence of primordial perturbations implies a universal average photon energy and number density of $\rho_{\gamma} \simeq a_{\mathcal{B}} \overline{T}^4 (1 + 6\langle \Theta^2 \rangle)$ and $N_{\gamma} \simeq b_{\mathcal{B}} \overline{T}^3 (1 + 3\langle \Theta^2 \rangle)$. Here, $\overline{T} = \langle T \rangle$ is the average temperature of the Universe, and $\Theta(t, \mathbf{x}, \hat{n}) = \Delta T/\overline{T}$ denotes the local temperature perturbation at some fixed time *t* in different directions \hat{n} . The angle brackets $\langle ... \rangle$ denote averages over space at some fixed time. In comparison, a blackbody at temperature *T* has $\rho_{\gamma} = a_{\mathcal{B}}T^4$ and $N_{\gamma} = b_{\mathcal{B}}T^3$. From this, one finds that the presence of perturbations is associated with a momentary lack of photons, $\Delta N_{\gamma}/N_{\gamma} \approx (3/2)\langle \Theta^2 \rangle$ (which will be replenished by the thermalization process) and a corresponding excess energy density, $Q_{\gamma} \simeq 2\rho_{\gamma} \langle \Theta^2 \rangle$. A similar picture holds for any other species contributing to ρ defined above.

In the CMB rest frame and at subhorizon scales, we have $\langle \Theta^2 \rangle \simeq \langle \Theta_0^2 \rangle + 3 \langle \Theta_1^2 \rangle \simeq \langle |\Theta_0|^2 \rangle$, where we used that in the tight-coupling regime the amplitude of the photon dipole is $|\Theta_1| \simeq |\Theta_0|/\sqrt{3}$ and $\pi/2$ out of phase with the monopole [25]; in Fourier space, $|\Theta_0|_k \approx (3/4)\delta_\gamma^i \exp(-k^2/k_D^2) = C\zeta \exp(-k^2/k_D^2)$. Assuming adiabatic perturbations, $\delta \rho_i / \rho_i \simeq \delta \rho_\gamma / \rho_\gamma$ for all *i*, the average fractional energy release (from the acoustic waves to the average plasma) between time t_1 and t_2 is then given by [9,11]

$$\frac{\Delta Q}{\rho} \approx 2 \left[\langle \Theta^2 \rangle_{t_1} - \langle \Theta^2 \rangle_{t_2} \right] \approx 2C^2 \int_{k_D(t_2)}^{k_D(t_1)} \frac{\mathrm{d}k}{k} \Delta_{\mathcal{R}}^2(k). \quad (1)$$

Here, $\Delta_{\mathcal{R}}^2(k)$ is related to the primordial curvature power spectrum by $\Delta_{\mathcal{R}}^2(k) \equiv k^3 P_{\zeta}(k)/(2\pi^2)$.

For $z \gtrsim 2 \times 10^6$, the energy release above yields the entropy production, or the change in comoving number density of relativistic particles, as $d \ln a^3 N/dt \approx$ $-(3/2)\partial_t \langle \Theta^2 \rangle$, from which we calculate the photon number density as

$$N_{\gamma}(z) \approx N_{\gamma}^{*}(z) \exp\left[-\frac{3C^{2}}{2} \int_{0}^{z} \Delta_{\mathcal{R}}^{2}(k_{D}) \frac{d\ln k_{D}}{d\ln z} d\ln z\right].$$
(2)

Note that similar relations hold for all relativistic particles thermally coupled to photons. Here, $N_{\star}^{*}(z)$ is the average photon number without thermalization but taking into the account the smoothing of perturbations by particle diffusion. That is, $N_{\nu}^{*}(z)$ is the photon number density at redshift z extrapolated from the CMB temperature today $T_0 =$ 2.726 K and the standard thermal history including the entropy transfer from e^{\pm} annihilation, etc. Equation (2) defines an effective photon temperature, $T \equiv (N_{\gamma}/b_{\beta})^{1/3}$, specific to the average number of photons in the Universe, and $T^* \equiv (N_{\gamma}^*/b_{\mathcal{B}})^{1/3}$, the temperature that appears in the usual thermal history calculation. At this point, it is worth stressing that the total radiation energy density in the Universe stays practically unchanged throughout the diffusion and thermalization process, because it simply redistributes the energy stored inside of the perturbations to the median and the thermalization only changes particle numbers, but not the energy density. As a result, the expansion history at early times is the same as the usual calculation and controlled by $T^*(z)$.

Diffusion scale.—We calculate k_D from the damping rate $\Gamma(k, t)$, $k_D^{-2} = k^{-2} \int_0^t dt' \Gamma(k, t')$. At early times, heat conduction and bulk viscosity of the plasma are negligible [27], and Γ is dominated by shear viscosity η , i.e., $\Gamma(k, t) \approx \frac{2}{3} [k^2/a^2(\rho + p)]\eta(t)$. Here, $p \simeq \rho/3$ is the pressure of the primordial fluid and η is roughly given by [27,28]

$$\eta = \frac{16}{45} \rho_{\gamma} t_{\gamma} + \frac{4}{15} \rho_{\nu} t_{\nu} \Theta(T - T_{\nu,\text{dec}}).$$
(3)

 $t_{\gamma} = (n_{e^{\pm}}\sigma_{\rm KN})^{-1}$ denotes the mean free scattering time of a photon with $n_{e^{\pm}}$ being the electron-positron number density and $\sigma_{\rm KN}(x)$ the Klein-Nishina cross section for which we use the expression in [29] with $x = 2.7(T/m_e) \{3(T/m_e) +$ $[K_1(m_e/T)/K_2(m_e/T)]$ as a thermal averaged quantity; and K_{ν} is a Bessel function of the second kind. Before neutrinos stream freely, their mean free time t_{ν} is determined by weak interactions with $\sigma_{\nu} \approx (G_F T)^2 = 5.3 \times 10^{-44} T_{\text{MeV}}^2 \text{ cm}^2$. We also estimate k_D at temperatures above the electroweak phase transition $T_W = O(100 \text{ GeV})$ from electroweak interactions of the SM content. For $T \gg T_W = 100 \text{ GeV}$ the temperature dependence of the scattering cross section becomes one of a gauge interaction $\sigma_{\nu} \propto T^{-2}$ and, for simplicity, we assume an instant (second order) phase transition and take Z and W bosons as massless for $T > T_W$. In our estimation of k_D we have included all Standard Model states that interact weakly; a more detailed numerical study will neither change the qualitative picture nor the quantitative analysis by much. At low redshift, $z \leq 10^8$, and with the inclusion of heat conduction χ in Γ , the damping rate reduces to the expression familiar in the CMB literature [26].

As a general rule, the particle that is most weakly interacting, yet still kinetically coupled and as abundant as radiation, controls k_D . This is because it will have the largest product $t_i\rho_i$ in a generalization of Eq. (3) for η . For

the purpose of this work we assume a SM field content and a massive DM particle with an electroweak-strength interaction. It is then the massless SM degrees of freedom which suffice to be taken into account for calculating k_D .

Importantly, from Eq. (1) we see that $k_D(z)$ informs us about the scales k that dissipate at a given redshift z. The redshift evolution of the diffusion scale is shown as a red, solid line in Fig. 1. The diffusion scale at $T > T_W$ (where anisotropic shear from γ , W^{\pm} , Z bosons are all important), $k_D \approx 4.5 \times 10^{14} (T/\text{MeV})^{0.51} \text{ Mpc}^{-1}$. After electroweak symmetry breaking and before neutrino decoupling, $T_{\nu,\text{dec}} < T < T_W$, k_D is dominated by neutrino shear viscosity, $k_D \approx 5 \times 10^4 (T/\text{MeV})^{2.7} \text{ Mpc}^{-1}$. For $T < T_{\nu,\text{dec}}$, $k_D \approx 10^5 \text{ Mpc}^{-1}$ remains constant until $T \approx 2 \text{ keV}$. This is because neutrinos previously erased near-horizon sized modes (dot-dashed line) so that the uptake of photon diffusion is delayed until a later epoch at $T \approx 2 \text{ keV}$ since $t_{\gamma} \ll t_{\nu}$. This makes additional photon production almost negligible during the epoch of BBN.

Revised thermal history.—In the spectral-distortion era, the limits on $\Delta_{\mathcal{R}}^2(k)$ from μ distortions of the CMB are already quite stringent [e.g., [10]] and photon heating is not relevant. Therefore, we focus on dissipation in the thermalization era ($z > 2 \times 10^6$, $k \gtrsim 10^4$ Mpc⁻¹). For simplicity, let us assume that the amplitude of primordial curvature fluctuations is scale-invariant on small scales with amplitude $\Delta_{\mathcal{R}0}^2$. Thus, the average photon temperature becomes



FIG. 1 (color online). Redshift and temperature dependence of the diffusion scale k_D (top) and temperature parameter Θ_p (bottom) defined in Eq. (4). The blue dashed line captures photon diffusion (without neutrino shear viscosity); the red solid line includes neutrino diffusion and represents the full result. Neutrino shear viscosity dominates dissipation before neutrino decoupling, with a diffusion scale that is close to the comoving horizon $k_{\rm H} = aH$ (black dot-dashed line). Lines at T > 200 GeV are thin, as that part of the graph may be modified if there are particles or interactions beyond the SM.

$$T(z) = T^*(z) \mathrm{e}^{-\Delta_{\mathcal{R}_0}^2 \Theta_p(z)}.$$
(4)

The definition of $\Theta_p(z)$ can be deduced from Eqs. (1) and (2) [see the bottom panel of Fig. 1]. The plateau value for 2 keV < $T < T_{\nu,\text{dec}}$ is $\Theta_p \simeq 1.6$. We find that $\Theta_p(T^*) \simeq$ 1.3 ln(T^*/MeV) + 1.3 $\simeq 1.3 \ln z - 30$ and $\Theta_p(T^*) \simeq$ 0.25 ln(T^*/MeV) + 13 $\simeq 0.25 \ln z + 7.2$ are good approximations for, respectively, $T_{\nu,\text{dec}} < T < T_W$ and $T > T_W$. Equipped with the modification of the T-z relation, we shall now discuss its consequence for cosmological observables.

Neutrino number density and N_{eff}.--After neutrino decoupling, the comoving neutrino number remains constant. However, photon production from dissipation of acoustic waves continues and N_{ν}/N_{γ} changes with time. At the same time, it is important to note that $N_{\rm eff}$, which measures the energy density of relativistic particles, remains fixed at its standard value. This is because both the neutrino and photon fluid initially shared the same perturbations and energy conservation implies that their relative energy densities are not affected by the presence and dissipation of small-scale modes. Albeit strictly beyond the scope of present experimental capabilities, we note in passing that any direct observation of N_{ν} in mismatch with a value inferred from $N_{\rm eff}$ can in principle allow us to probe the the small-scale power spectrum at $k \lesssim 10^5 \text{ Mpc}^{-1}$. Further information may then be extracted from neutrino spectral distortions, which are caused by mixing of Fermi distributions of slightly different temperature in the neutrino free-streaming phase.

Light-element yields.—As alluded to before, no entropy is produced after neutrino decoupling until $T \approx 2$ keV. This frames the period of nucleosynthesis and the modifications to BBN come from an elevated baryon asymmetry as initial condition (because of post-BBN dissipation of small scale power) and a modification of the average energy per particle, ρ/N .

Impressive progress has been made in the determination of the primordial deuterium abundance from high-z QSO absorption systems, with the most recent mean reported as $(D/H)_p = (2.53 \pm 0.04) \times 10^{-5}$ [30]. A precision measurement of the true primordial He abundance must await future CMB probes; inference of the primordial mass fraction Y_p from extragalactic H-II regions are plagued by systematic uncertainties [31,32] and a conservative range may be taken as $0.24 \le Y_p \le 0.26$. The constraint applies to those modes that dissipate their energy *after* BBN but before the spectral-distortion era, $k \approx 10^4 - 10^5$ Mpc⁻¹. We find

$$Y_p: \Delta^2_{\mathcal{R}0} < 0.007, \qquad (D/H)_p: \Delta^2_{\mathcal{R}0} < 0.2, \quad (5)$$

from the overproduction of He; for D/H we adopted a nominal 2σ lower limit from the quoted mean. Since higher values of primordial D/H are in principle conceivable (e.g., by systematic D absorption on dust grains), we refrain from deriving a limit on the overproduction of D/H. However, no



FIG. 2 (color online). Light-element yields from BBN as a function of $\Delta_{\mathcal{R}0}^2$. The dominant constraint is derived from an overproduction of helium.

known astrophysical sources of D exist, and underproducing D yields a robust constraint, Eq. (5). We note that Li/H increases with larger $\Delta_{\mathcal{R}0}^2$, worsening the cosmological lithium problem (see Fig. 2).

Baryon asymmetry and DM relic abundance.—Entropy production causes a dilution of net particle number once the latter is frozen out. We have already seen the reduction of η_b in the post-BBN era. Here, we will consider the entropy conversion from k_D evolution for temperatures well above 1 MeV. Our results are derived under the premise that k_D is governed by SM fields only.

The photon production from dissipation of acoustic modes dilutes the baryon-to-photon ratio,

$$\ln\left(\frac{N_B}{N_{\gamma}}\right) \approx \ln\left(\frac{N_B}{N_{\gamma}}\right|_*\right) + 3\Delta_{\mathcal{R}0}^2\Theta_p(T^*).$$
(6)

Therefore, in the presence of small-scale power, the initial baryon asymmetry from some baryon-number-generating process has to be larger than in the standard case. Above the QCD phase transition, baryon number is carried by quarks so that $N_B \simeq N_{\gamma}$, implying a principal limit $(N_B - N_{\bar{B}})/N_{\gamma} \lesssim \mathcal{O}(1)$. The latter condition implies

$$\Delta_{\mathcal{R}0}^2 \lesssim 21 \left[39 + 28 \ln\left(\frac{T}{10^{19} \text{ GeV}}\right) \right]^{-1}.$$
 (7)

For baryogenesis scenarios operative at $T \sim \text{TeV}$ to 10^{19} GeV , this gives a rather weak bound $\Delta_{\mathcal{R}0}^2 \leq 0.3$. However, it must be said that the constraint applies to remarkably small scales, $k_D \simeq 10^{20-25} \text{ Mpc}^{-1}$.

DM relic abundance.—The calculation of the darkmatter relic abundance is also affected by a revised temperature-redshift relation. If DM is a weakly interacting massive thermal relic, its abundance freezes out when the annihilation rate equals the expansion rate: $H \simeq n_{\rm DM} \langle \sigma v \rangle$. For given *H*, the DM equilibrium number density is reduced relative to the standard case, leading to an earlier



FIG. 3 (color online). Estimate of the DM annihilation cross section $\langle \sigma v \rangle$ in the presence of small-scale density perturbations as a function of DM mass m_{χ} . The bands indicate the region $\Omega_{\rm c} = 0.2594 \pm 0.0074$ (95% C.L. from [2]).

FO. Conversely, the expansion factor from FO to the present increases by a factor of $e^{3\Delta_{R0}^2\Theta_p}$, and that reduces the relic DM number density. The values $\langle \sigma v \rangle$, required for matching onto the CMB observation of Ω_c , are shown in a sample calculation in Fig. 3.

Conclusion.—We study the dissipation of primordial acoustic waves from adiabatic perturbations, and its impact on the thermal history of the early Universe at redshift $z \gtrsim 2 \times 10^6$. Because of dissipation, the redshift-temperature relation is modified and entropy production leads to a revision of N_{ν}/N_{γ} , $(D/H)_p$, Y_p . From those observables we establish a constraint $\Delta_{\mathcal{R}0}^2 < 0.007$ at comoving scales 10^4 Mpc⁻¹ $\lesssim k \lesssim 10^5$ Mpc⁻¹. Such small scales were previously believed to be inaccessible by direct early Universe observables.

One can take this work into various directions that remain to be explored. For example, we restricted ourselves to a SM particle content. New radiation degrees of freedom that are populated for T > 1 MeV and that have interaction strengths such that their mean free path exceeds the one of neutrinos are likely to dominate the plasma's viscosity. This can lead to more drastic modifications of the thermal history prior to BBN, with consequences for baryogenesis and the DM problem. Even within SM with massive neutrinos, the diffusion scale at high T will be model dependent. For example, say, neutrinos are Dirac particles, and their right-handed counterparts are fully excited for temperatures well above their mass. They may then dominate the diffusion process when the only link to the thermal bath comes from minute Yukawa interactions.

We have also only considered Gaussian primordial fluctuations and wave modes that are statistically independent. However, if long- and short-wavelength fluctuations are correlated (e.g., through local-model non-Gaussianity), the dissipation on small scales will give rise on large scales to an isocurvature fluctuation correlated to the adiabatic perturbation. We leave the study of the effects of these modes to future work.

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