Steering Bound Entangled States: A Counterexample to the Stronger Peres Conjecture

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Quantum correlations are at the heart of many applications in quantum information science and, at the same time, they form the basis for discussions about genuine quantum effects and their difference to classical physics. On one hand, entanglement theory provides the tools to quantify correlations in information processing and many results have been obtained to discriminate useful entanglement, which can be distilled to a pure form, from bound entanglement, being of limited use in many applications. On the other hand, for discriminating quantum phenomena from their classical counterparts, Schrödinger and Bell introduced the notions of steering and local hidden variable models. We provide a method to generate systematically bound entangled quantum states which can still be used for steering and, therefore, to rule out local hidden state models. This sheds light on the relations between the various views on quantum correlations and disproves a widespread conjecture known as the stronger Peres conjecture. For practical applications, it implies that even the weakest form of entanglement can be certified in a semidevice independent way.

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Introduction.—Entanglement denotes quantum correlations which cannot be generated in any local way. While the characterization of entanglement for pure two-particle states is straightforward, the task becomes challenging for noisy or mixed quantum states. Here, even the simple question whether or not a given quantum state is entangled is not easy to decide. Apart from that, it is also difficult to characterize the usefulness of entanglement for the mixed state case. Since many quantum information protocols like quantum teleportation or quantum key distribution work with pure maximally entangled states, one may first distill a noisy state to a pure highly entangled state, but characterizing all possible distillation protocols is not straightforward. In fact, it was already shown in 1998 that there are so-called bound entangled quantum states from which no pure state entanglement can be distilled [1]. This shows some irreversibility in entanglement theory, as these states require pure state entanglement for their generation, but then this entanglement can never be recovered again.

In the following years it turned out that bound entangled states are central to many problems in quantum theory. For instance, it has been shown that entangled states with a positive partial transpose (PPT) are bound entangled, but the question whether all bound entangled states are PPT is, despite numerous efforts [2–4], undecided. Using bound entangled states, it has been shown that bound information, an analogue to bound entanglement in classical information theory, exists in the multipartite scenario [5]. Furthermore, bound entangled states are conjectured to have a small dimensionality of entanglement [6]. Finally, it has surprisingly been shown that the correlations of bound entangled states can be used for distilling a secure quantum key [7,8], although no pure state entanglement can be distilled from the state. All these problems and observations clearly justify calling bound entanglement a “mysterious invention of nature” [9].

Besides all the applications in information processing, quantum correlations are also important when contradictions between quantum mechanics and the classical world view should be derived. This was highlighted by Bell, when he showed that no local hidden variable model can reproduce the quantum mechanical correlations [10,11]. Interestingly, a similar question was discussed before by Schrödinger, who asked whether one party (called Alice) can steer the state from the other party (called Bob) by appropriate measurements, a task which is not conceivable in a classical world [12,13]. Mathematically, this problem reduces to finding a local hidden *state* model for the correlations, which is a hidden variable model with the additional constraint that Bob’s measurements are described by quantum mechanics.

Not surprisingly, bound entanglement is also central to several open problems concerning Bell inequalities and steering; see Fig. 1. Most prominently, a conjecture by Peres [14] states that bound (and therefore especially PPT) entangled states always admit a local hidden variable model [15]. It is known that this conjecture is wrong in the multipartite case under various different notions of bound entanglement [16,17], or in the dimension bounded bipartite case [18], but it is still open in the bipartite case. Here it

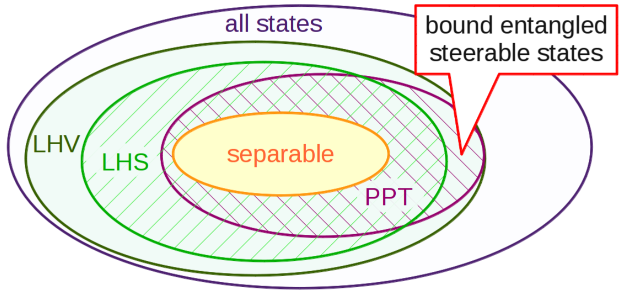

FIG. 1 (color online). A schematic view on the space of all quantum states. The set of all states is convex with the separable states as a subset; states which are not separable are entangled. The PPT states are bound entangled, as no pure state entanglement can be distilled from these weakly entangled states. Some states admit a local hidden state (LHS) model, and if this is not the case, they can be used for steering. A larger set of states admits a local hidden variable (LHV) model, and if this is not the case, the state violates some Bell inequality. In this Letter we present a method to generate PPT states which are steerable. In this figure we have, according to the Peres conjecture, depicted the PPT states as a subset of the LHV states, but the family of states presented in this Letter may also be outside of the LHV states.

is known to hold true for various cases [19–25], but it has also been shown that with the help of additional states and operations, any entangled state shows nonlocal behavior [26,27]. Similarly, it has been conjectured that all bound entangled states do admit even a hidden state model and are thus useless for steering scenarios [28,29]. This conjecture is termed the stronger Peres conjecture and recently strong evidence in favor of it has been claimed [28,29]. In this Letter we disprove it by giving an explicit counterexample.

More precisely, we present a method to generate systematically bound entangled states that violate a steering inequality and thus do not admit a hidden state model. This not only delivers the desired counterexample, it also provides candidates for the other conjectures concerning bound entanglement. For instance, these states are natural candidates for testing the original Peres conjecture or the existence of bipartite bound information [5]. Finally, the resulting states are interesting from a practical point of view as their entanglement may be verified in experiments without any assumptions on the measurements on one party.

Framework and notation.—Steering can be viewed as entanglement verification in a so-called semidevice independent scenario [13]. One of the parties, say, Alice, is totally untrusted and only the number of settings and respective outcomes is specified, while for the other party, Bob, one has a perfect quantum description of the measurements.

We consider the case that Alice can choose between different measurements, each having the same number of possible results. We use $x = 1, \ldots, m$ to label the setting, $a = 1, \ldots, n$ for the result of the measurement, and $a|x$ for the combination. For Bob, we assume that he performs full tomography on his d-dimensional system, so that he can reconstruct the state for each possibility $a|x$ of Alice. Thus, the available data of this scenario are fully specified by the ensemble of conditional states for Bob that we describe by the collection of unnormalized density operators $\mathcal{E} = \{\rho_{a|x}\}_{a,x}$, such that $P(a|x) = \mathrm{tr}(\rho_{a|x})$. Note that nonsignaling means that $\sum_a \rho_{a|x} = \rho$ is independent of the setting x, and if this is fulfilled then the ensemble $\mathcal{E}$ indeed has a quantum representation [12,30].

Note that in a general steering scenario Bob only measures a few characterized observables, e.g., only the Pauli matrices σ_x and σ_z, or, similarly to Alice, he chooses a setting y and obtains a result b by doing a fixed measurement described by the positive operator valued measure $\{M_{b|y}\}_b$. Then the available data are given by the joint conditional probability distributions $P(a,b|x,y)$, which admit a local hidden state model if they can be written as

$$P(a,b|x,y) = \sum_\lambda P(\lambda) P(a|x,\lambda) \mathrm{tr}(M_{b|y}\sigma_\lambda). \quad (1)$$

Here, λ is a hidden variable, occurring with probability $P(\lambda)$ and σ_λ are quantum states. In contrast to this, a local hidden variable model would not have such a constraint for Bob's conditional distribution $P(b|y,\lambda)$. Note that any distribution, as, for instance, also $P(a|x,\lambda)$, can still always be written as an appropriate measurement on a quantum state [31]—via this one sees that Eq. (1) can be obtained by measuring a separable state. But the important point is that Bob's measurement is fixed.

However, since we assume that Bob obtains full tomography, his exact measurement procedure does not matter. If he obtains full information for instance via separate settings and respective outcomes, then the set of all operators $\{M_{b|y}\}_{b,y}$ spans the full operator space, so that the conditions given by Eq. (1) can only be fulfilled if we have already a corresponding equality on the state space level. Thus, an ensemble $\mathcal{E}$ has a local hidden state model if $\rho_{a|x} = \sum_\lambda P(\lambda) P(a|x,\lambda)\sigma_\lambda$ holds for all choices $a|x$. If this is not possible, the ensemble $\mathcal{E}$ is called steerable, referring to the phenomena that Alice can steer the decomposition of Bob's reduced state in a nontrivial way.

Before we proceed, note that the problem can be simplified if one collects all randomness of Alice's measurement into $P(\lambda)$ and Bob's states σ_λ. This results in considering only the finite number of deterministic strategies for Alice that we label by $\lambda_{i_1 i_2 \ldots i_m}$, such that the subscripts i_k encode the triggered outcome for each setting; i.e., $P(a|x,\lambda_{i_1\ldots i_m}) = \delta_{i_x,a}$. Then, the ensemble $\mathcal{E}$ is nonsteerable if and only if there exists a set of positive semidefinite operators $\omega_{i_1 i_2 \ldots i_m} \geq 0$ with $i_k = 1, \ldots, n$ for each $k = 1, \ldots, m$, such that

$$\rho_{a|x} = \sum_{i_1,\ldots,i_m} \delta_{i_x,a}\omega_{i_1 i_2 \ldots i_m} \quad (2)$$

holds for all possible a, x [28].

All the steering inequalities.—Hence, to show that an ensemble $\mathcal{E}$ is steerable one must certify that it is not of the form given by Eq. (2). This certificate is called steering inequality, and is similar in spirit to Bell inequalities [10,15,32] or entanglement witnesses [33]. A linear steering inequality is a linear function of the given ensemble $C(\mathcal{E})$ such that $C(\mathcal{E}) \geq 0$ holds for all nonsteerable ensembles $\mathcal{E}$, so that $C(\mathcal{E}) < 0$ witnesses steering.

In order to derive the form of all such steering inequalities one can proceed as follows: The question given Eq. (2) is a special convex optimization problem called semidefinite programming, i.e., $\min_{x\in\mathbb{R}^n}\{c^T x | F_0 + \sum_i x_i F_i \geq 0\}$ with $c \in \mathbb{R}^n$ and Hermitian matrices F_0 and all F_i. Because of the convex structure of the problem one can solve the alternative problem, called dual $\max_{Z\geq 0}\{-\mathrm{tr}(ZF_0)|\mathrm{tr}(ZF_i) = c_i\}$, which lower bounds the original problem and usually attains the same optimal value. This dual problem is effectively the optimization over all steering inequalities. Thus, to derive all steering inequalities we put Eq. (2) into the form of a semidefinite program and invoke its dual. This approach has been used in a quantification of steering [29].

For our intended goal we consider only a steering inequality for the case $m = 2$ and $n = 3$, since this is the setting of our counterexample. It should be noted, however, that our approach can be applied also for more than two measurements or more outcomes. In the following, we state the form of all such inequalities and verify $C(\mathcal{E}) \geq 0$ for all nonsteerable ensembles.

Steering inequality.—Consider the described steering scenario for $m = 2$ and $n = 3$. Suppose we have a set of operators $\mathcal{Z} = \{Z_{13}, Z_{23}, Z_{31}, Z_{32}, Z_{33}\}$, each positive semidefinite $Z \geq 0$ for all $Z \in \mathcal{Z}$, and further satisfying

$$Z_{11} = Z_{13} + Z_{31} - Z_{33} \geq 0, \tag{3}$$

$$Z_{21} = Z_{23} + Z_{31} - Z_{33} \geq 0, \tag{4}$$

$$Z_{12} = Z_{13} + Z_{32} - Z_{33} \geq 0, \tag{5}$$

$$Z_{22} = Z_{23} + Z_{32} - Z_{33} \geq 0. \tag{6}$$

Then, the linear function

$$C(\mathcal{E}) = \mathrm{tr}(Z_{13}\rho_{1|1}) + \mathrm{tr}(Z_{23}\rho_{2|1}) + \mathrm{tr}(Z_{31}\rho_{1|2}) + \mathrm{tr}(Z_{32}\rho_{2|2}) + \mathrm{tr}[Z_{33}(\rho - \rho_{1|1} - \rho_{2|1} - \rho_{1|2} - \rho_{2|2})] \tag{7}$$

is non-negative for all nonsteerable ensembles of Eq. (2), and thus $C(\mathcal{E}) < 0$ shows steering.

To show this, note that a given ensemble $\mathcal{E}$ with $m = 2$ and $n = 3$ is nonsteerable if and only if there exists $\omega_{ij} \geq 0$ with $i, j = 1, 2, 3$, such that

$$\rho_{1|1} = \omega_{11} + \omega_{12} + \omega_{13}, \quad \rho_{1|2} = \omega_{11} + \omega_{21} + \omega_{31}, \tag{8}$$

$$\rho_{2|1} = \omega_{21} + \omega_{22} + \omega_{23}, \quad \rho_{2|2} = \omega_{12} + \omega_{22} + \omega_{32}, \tag{9}$$

and

$$\rho = \rho_{1|1} + \rho_{2|1} + \rho_{3|1} = \rho_{1|2} + \rho_{2|2} + \rho_{3|2} = \sum_{ij} \omega_{ij} \tag{10}$$

hold. Using these relations in Eq. (7) one can verify that this expression equals to $C(\mathcal{E}) = \sum_{ij}\mathrm{tr}(Z_{ij}\omega_{ij})$ and, hence, is non-negative since all occurring operators are positive semidefinite.

Strategy for generating counterexamples.—Now we can present our method of generating a counterexample. Let us assume that we have fixed a linear steering inequality, i.e., a set of valid operators $\mathcal{Z}$ satisfying the conditions from the previous section. From this one can obtain an entanglement witness [33] by employing any choice of measurements for Alice in C. For the case of $n = 2$, $m = 3$ this means that

$$W = A_{1|1} \otimes Z_{13} + A_{2|1} \otimes Z_{23} + A_{1|2} \otimes Z_{31} + A_{2|2} \otimes Z_{32} + (\mathbb{1} - A_{1|1} - A_{2|1} - A_{1|2} - A_{2|2}) \otimes Z_{33} \tag{11}$$

is non-negative on separable states for any set of operators $A_{a|x}$ satisfying $A_{a|x} \geq 0$ and $\sum_a A_{a|x} = \mathbb{1}$ for all combinations a, x, and one readily gets $C = \mathrm{tr}(W\rho_{AB})$.

The method is then as follows: We assume that Alice and Bob both have qutrits and that Alice makes a projective measurement in two mutually unbiased bases. After that we look for a "good" steering inequality, i.e., a good set $\mathcal{Z}$. To do so we randomly choose a pure state, compute its ensemble $\mathcal{E}$ using the fixed measurements of Alice, and determine the best steering inequality $\mathcal{Z}$. Afterwards, we build up the given witness W and minimize its expectation value with respect to all PPT states. If this optimum is negative, then we have already a counterexample. If this fails then we start over. However, once we found a PPT state violating the randomly chosen steering inequality, we can use this state, compute its ensemble, and look for an even better inequality. And, similarly, once we have a better steering inequality we can look for an even better state. This further amplifies the violation of the PPT entangled state and we repeat this until the violation saturates.

Note that the occurring optimizations are semidefinite programs and thus can be done efficiently [34,35]. Furthermore, we should add that we normalize the steering inequality such that each $Z \in \mathcal{Z}$ satisfies $\mathrm{tr}(Z) = 1$.

Counterexample.—Running the explained procedure quickly results in bound entangled states which serve as counterexamples to the stronger Peres conjecture. Interestingly, if one amplifies the violation, we always end up with a violation of $C = -0.0029$. From the numerical solution one can infer the following analytical solution.

At first, let us describe the steering inequality: The set of operators $Z_{13} = |q_+\rangle\langle q_+|$, $Z_{23} = |q_-\rangle\langle q_-|$, $Z_{32} = Z_{33} = |s\rangle\langle s|$, and $Z_{31} = (1-x)|t\rangle\langle t| + x|2\rangle\langle 2|$ with real, normalized vectors

$$|q_{\pm}\rangle = [a, \sqrt{1-a^2-b^2}, \mp b], \tag{12}$$

$$|s\rangle = [a, -\sqrt{1-a^2}, 0], \qquad |t\rangle = [c, -\sqrt{1-c^2}, 0], \tag{13}$$

and abbreviations

$$a = \sqrt{\frac{2+2x}{3}}, \quad b = \sqrt{\frac{1-2x}{4}}, \quad c = \sqrt{\frac{2-4x}{3-3x}} \tag{14}$$

define a one-parameter family of steering inequalities for $0 \le x \le 1/2$.

This can be seen as follows: With this ansatz we already fulfil the positivity requirements of each individual $Z \in \mathcal{Z}$. Moreover, the additional constraints given by Eqs. (5) and (6) are satisfied automatically since $Z_{32} = Z_{33}$, while from Eqs. (3) and (4) we only need to check one condition, since the unitary matrix $V = \mathrm{diag}(1, 1, -1)$ interchanges Z_{13} with Z_{23}, (i.e., $Z_{23} = V Z_{13} V^{\dagger}$), but leaves Z_{31} and Z_{33} invariant. Thus, we only need to show that $Z_{11} \ge 0$, for which the particular choices of a, b, c become important. These are determined by the identity $Z_{13} + Z_{23} + Z_{31} = \mathrm{diag}(2, 1/2, 1/2)$ that we observed from the numerical solution. Via this choice, the operator Z_{11} then has the same eigenvalues as Z_{31}, i.e., eigenvalues $\{x, 1-x, 0\}$.

Second, before discussing the state, let us fix the two mutually unbiased bases, since we employ some rotated form, which makes the final bound entangled state look simpler. The respective vectors are denoted by $|v_{x|a}\rangle$ and are given by $|v_{1/2|1}\rangle = [1/\sqrt{3}, -1/\sqrt{6}, \mp 1/\sqrt{2}]$, $|v_{3|1}\rangle = [1/\sqrt{3}, \sqrt{2/3}, 0]$, for $a = 1$ and $|v_{1|2}\rangle = [1, 0, 0]$, $|v_{2|2}\rangle = [0, q/\sqrt{2}, \mathbb{i} q/\sqrt{2}]$, $|v_{3|2}\rangle = [0, q^*/\sqrt{2}, -\mathbb{i} q^*/\sqrt{2}]$, with $q = (-1)^{2/3}$ for setting $a = 2$.

Finally, let us turn to the state. Consider the following class of states:

$$\rho_{AB} = \lambda_1 |\psi_1\rangle\langle\psi_1| + \lambda_2 |\psi_2\rangle\langle\psi_2| + \lambda_3 (|\psi_3\rangle\langle\psi_3| + |\tilde{\psi}_3\rangle\langle\tilde{\psi}_3|), \tag{15}$$

using the normalized states

$$|\psi_1\rangle = (|12\rangle + |21\rangle)/\sqrt{2}, \tag{16}$$

$$|\psi_2\rangle = (|00\rangle + |11\rangle - |22\rangle)/\sqrt{3}, \tag{17}$$

$$|\psi_3\rangle = m_1|01\rangle + m_2|10\rangle + m_3(|11\rangle + |22\rangle), \tag{18}$$

$$|\tilde{\psi}_3\rangle = m_1|02\rangle - m_2|20\rangle + m_3(|21\rangle - |12\rangle), \tag{19}$$

with $m_i \ge 0$.

By construction, this represents a valid quantum state. In order to assure that this state has a positive partial transpose, we make it PPT invariant, i.e., $\rho_{AB} = \rho_{AB}^{T_A}$, for which one must make the off-diagonal blocks Hermitian. These constraints will fix the eigenvalues to

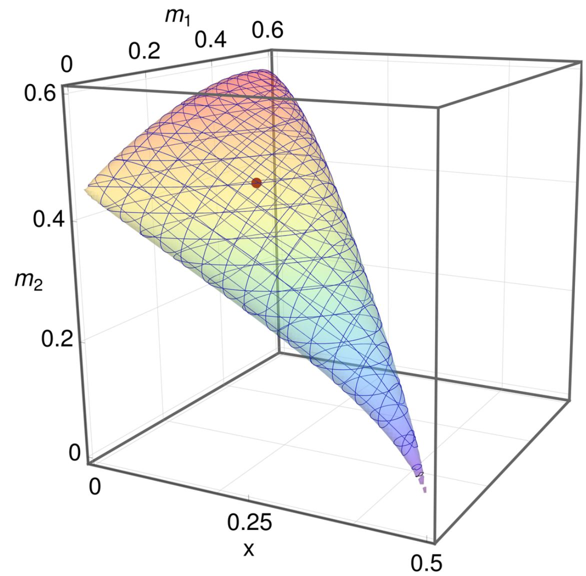

FIG. 2 (color online). The family of states which are counterexamples to the stronger Peres conjecture. The parameters m_1 and m_2 characterize the state, while x characterizes the steering inequality. The red dot corresponds to the values $x = 0.1578$, $m_1 = 0.2162$, $m_2 = 0.4363$ which leads to the highest violation of the steering inequality.

$$\lambda_1 = 1 - \frac{2 + 3m_1 m_2}{4 - 2m_1^2 + m_1 m_2 - 2m_2^2}, \tag{20}$$

$$\lambda_3 = \frac{1}{4 - 2m_1^2 + m_1 m_2 - 2m_2^2}. \tag{21}$$

The parameter $\lambda_2 = 1 - \lambda_1 - 2\lambda_3$ is given by normalization. The λ_i are therefore parametrized by m_1, m_2 and this is only giving non-negative eigenvalues if $m_1^2 + m_2^2 + m_1 m_2 \le 1$.

Summarizing, we have deduced a class of steering inequalities $\mathcal{Z}$ parametrized by $0 \le x \le 1/2$, a set of measurements for Alice given by the two mutually unbiased bases, and a class of PPT states that depend on two non-negative, constrained parameters m_1, m_2. For these choices one can now compute expectation values of the steering inequality, and deduce combinations which verify steering; see Fig. 2. A heuristic optimization over the steering violation gives $C = -0.0029$ for the parameters $x = 0.1578$, $m_1 = 0.2162$, $m_2 = 0.4363$, which coincides with the numerically found solution.

Conclusion.—We provided a way to generate bound entangled states that do not possess a local hidden state model and thus violate a steering inequality. This disproves the stronger Peres conjecture and shows that the original Peres conjecture cannot be proven by considering the stronger steering case. It also means that even the weakest form of entanglement can be verified in a semidevice independent way.

Naturally, the generated bound entangled quantum states are interesting candidates for some of the conjectures concerning bound entanglement. A first question is whether with a few further modifications of our states and measurements one could even find a violation of a Bell inequality and thus disprove also the original Peres conjecture. A second question is whether this bound entangled state could even allow the generation of a secret key in a semidevice independent quantum key distribution protocol. Third, these bound entangled states even provide prominent candidates to investigate whether they could be useful for teleportation or in entanglement swapping in quantum repeaters [36]. Finally, it would be interesting to use our method to generate bound entangled states in higher dimensions, such as a $4 \otimes 4$ system, which can be viewed as a four-qubit system. Given the recent advances in quantum control, such states could probably be observed with entangled photons or ions.

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Note added.—After the appearance of our results on the arXiv it was noted that if one takes the state from our family with $m_1 = 1/60$ and $m_2 = 3/10$, the MUB measurements of Alice and the three dichotomic measurements of Bob characterized by the steering inequality, more precisely the Z_{13}, Z_{23}, Z_{33} with $x = 0.26$, then the corresponding data do not admit a local hidden variable model [37]. This shows that our method can indeed be used to find counterexamples to the original Peres conjecture. Note, however, that this approach is not working for the optimal steering parameters.

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