Ground-State Cooling of a Carbon Nanomechanical Resonator by Spin-Polarized Current

P. Stadler,[1] W. Belzig,[1] and G. Rastelli[1,2]
[1]*Fachbereich Physik, Universität Konstanz, D-78457 Konstanz, Germany*
[2]*Zukunftskolleg, Universität Konstanz, D-78457 Konstanz, Germany*
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We study the nonequilibrium steady state of a mechanical resonator in the quantum regime realized by a suspended carbon nanotube quantum dot in contact with two ferromagnets. Because of the spin-orbit interaction and/or an external magnetic field gradient, the spin on the dot couples directly to the flexural eigenmodes. Accordingly, the nanomechanical motion induces inelastic spin flips of the tunneling electrons. A spin-polarized current at finite bias voltage causes either heating or active cooling of the mechanical modes. We show that maximal cooling is achieved at resonant transport when the energy splitting between two dot levels of opposite spin equals the vibrational frequency. Even for weak electron-resonator coupling and moderate polarizations we can achieve ground-state cooling with a temperature of the leads, for instance, of $T = 10\omega$.

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Beyond proving useful technologically as ultrasensitive detectors of charge [1] and spin [2], nanoelectromechanical systems are also interesting to address fundamental issues as they can enter the quantum regime at low temperature [3,4]. For instance, recent experiments succeeded in approaching the quantum ground state in solid objects formed by a huge number of atoms [5–7]. Particularly interesting nanoelectromechanical systems are suspended carbon nanotube quantum dots (CNTQDs) [8,9]. They emerged as an ideal system for fundamental studies in few electron quantum dots [10] as, for instance, demonstrated by the coherent coupling between the electron spin and its orbital magnetic moment (spin-orbit interaction) [11–13]. In addition, suspended structures also have outstanding mechanical properties as carbon nanoresonators can have frequencies in the range $f \sim$ MHz $-$ GHz and yet large quantum zero-point fluctuations ($\delta u \sim 10$ pm), making them ideal candidates for observing quantum mechanical effects. In these systems, quantized vibrational modes appear in low temperature transport spectroscopy [14–17].

Despite this amazing progress, detecting quantum signatures of flexural modes [Fig. 1(a)] still remains a challenge, hindered by the difficulty of cooling such low-frequency modes to temperatures in the quantum regime, viz., $k_B T < hf$. Although shorter resonators with higher eigenfrequency can in principle overcome the problem [18,19], cooling these modes towards their quantum ground state with phonon occupation number $\bar{n} \ll 1$ remains a demanding achievement. Even at cryogenic temperatures and with suspended nanotubes of length $L \sim 1$ μm, which allow flexible gate-voltage control [20,21], this remains a serious challenge. If proved feasible, such a quantum mechanical mode would be an ideal platform to test decoherence mechanisms and even exotic phenomena such as wave-function collapse theories in quantum states with displaced centers of mass [22,23]. Another possible application is as realization of mechanical qubits in buckled carbon nanotubes [24–27].

In this Letter, we show that the flexural modes can be efficiently cooled towards their quantum limit when a spin-polarized current is injected from ferromagnetic leads and when a vibrational spin-flip interaction is considered (Fig. 1). Considering a flexural mode of frequency ω in a CNTQD with a quality factor $Q = \omega/\gamma_0 \gtrsim 10^4$ (γ_0 is the mechanical damping rate) [8,28], the resonator can be driven towards a nonequilibrium steady state with a phonon occupation $\bar{n} = [\gamma_0 n_B(\omega) + \gamma n]/(\gamma_0 + \gamma)$, in which $n_B(\omega) = 1/[\exp(\omega/T) - 1]$ is the thermal equilibrium occupation ($k_B = \hbar = 1$), and γ and n are, respectively, the damping and the effective phonon occupation induced by the spin-vibration interaction, which we discuss below.

Different ways to achieve cooling of flexural modes have been analyzed [29,30]. The spin valve that we propose has two important advantages. First, the spin is directly coupled to the vibration so that efficient ground-state cooling $\bar{n} \ll 1$ is achieved even for small spin polarization of the contacts. Second, the operating regimes, in which cooling or heating of the resonator is realized, can be controlled not only electrically but even magnetically: The spin valve switches from one to the other regime either by varying the gate or the bias voltage or either by only reversing the magnetic polarization in one or in both ferromagnetic leads. Such a system represents, hence, a promising candidate for the thermal control of nanoresonators in spintronic devices. Previous works also demonstrated that interplay between spin and nanomechanics can lead to interesting effects such as mechanical self-excitations [31].

Spin-vibration interaction.—The system is sketched in Fig. 1(a). For a single flexural mode n with frequency ω_n and oscillating along the x axis, suspended CNTQDs are

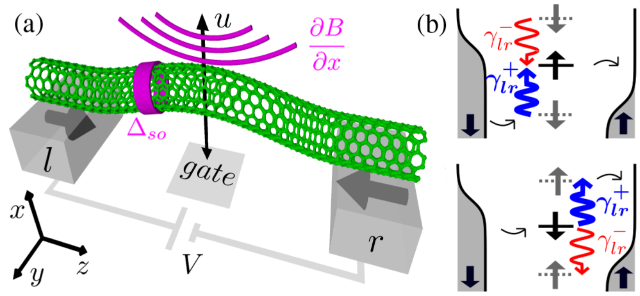

FIG. 1 (color online). (a) Schematic view of a carbon nanotube quantum dot suspended between two ferromagnetic leads. Because of the nanotube spin-orbit interaction and/or a magnetic field gradient, the dot spin's component parallel to the mechanical displacement u is coupled to the flexural mode. (b) Examples of inelastic vibron-assisted tunneling through a single level with spin up or down (upper and lower graph). The spin-vibration interaction allows spin-flip tunneling through emission (red thin arrow) or absorption (blue thick arrow) of an energy quantum $\hbar\omega$ to or from the vibrational mode. These processes are characterized by the rates γ^{+}_{lr} and γ^{-}_{lr}, respectively.

characterized by a spin-vibration interaction of the form $\hat{H}_n = \lambda_n \hat{\sigma}_x(\hat{b}_n + \hat{b}_n^\dagger)$ in which $\hat{\sigma}_x$ is the component of the spin operator (Pauli matrix) parallel to the mechanical motion and $\hat{b}_n$ ($\hat{b}_n^\dagger$) is the bosonic creation (annihilation) operator associated with the harmonic mode. This kind of interaction can be achieved extrinsically or intrinsically.

In the first case, the interaction arises from the relative motion of the suspended nanotube in a magnetic gradient added to the homogeneous magnetic field B in a similar setup as used, e.g., in magnetic resonance force microscopy experiments [2,32,33] or in magnetized microcantilevers coupled to nitrogen vacancy centers in diamond [34,35]. For small harmonic oscillations, one obtains $\lambda_n \simeq \mu_B(\partial B_x/\partial x)X_n$ with μ_B the Bohr magneton, $\partial B_x/\partial x$ the average gradient along the tube's axis, $X_n = u_n\langle f_n(z)\rangle$ the amplitude of the single vibrational mode with $u_n = 1/\sqrt{2m\omega_n}$, $f_n(z)$ the mode waveform, and $\langle\cdots\rangle$ the average over the electronic orbital in the dot. We estimated $\lambda = 0.5$ MHz for the fundamental (even) mode with $\partial B_x/\partial x = 5\times 10^6$ T/m [36,37]

In the second case, the spin-orbit coupling due to the circumferential orbital motion mediates the interaction between the electron spin and the flexural modes [41–43]. In the one-orbital (valley) subspace, the interaction coupling constant reads $\lambda_n \simeq (\Delta_{\mathrm{SO}}/2)dX_n/dz$ with Δ_{SO} the spin-orbit coupling constant and $dX_n/dz = u_n\langle df_n(z)/dz\rangle$ [37]. In this case, one can estimate $\lambda \sim 2.5$ MHz for the first odd mode [37,42]. We notice that for a quantum dot formed in a nanotube with symmetric orbital electronic density, the two interactions discussed here couple vibrational modes of different parity. Other microscopic mechanisms lead also to similar coupling [44].

In the presence of magnetic fields, the four-level structure of a single quantum dot shell can be tuned. In particular, close to a crossing point, it is possible to have two levels of opposite spin and the same orbital so that their energy separation is smaller than the temperature T or the bias voltage V and yet larger than the energy distance from other levels [12,37,42]. Focusing on the transport on this two-level subspace, we consider the model Hamiltonian

$$\hat{H} = \hat{H}_l + \hat{H}_d + \lambda(d_+^\dagger d_- + d_-^\dagger d_+)(\hat{b}^\dagger + \hat{b}) + \omega\hat{b}^\dagger\hat{b}, \quad (1)$$

in which the dots part reads $\hat{H}_d = \sum_\sigma \varepsilon_\sigma \hat{d}_\sigma^\dagger \hat{d}_\sigma$ and the lead part reads

$$\hat{H}_l = \sum_{\alpha\sigma k}[\varepsilon_{k\sigma}\hat{c}^\dagger_{\alpha\sigma k}\hat{c}_{\alpha\sigma k} + (t_{\alpha\sigma}\hat{c}^\dagger_{\alpha\sigma k}\hat{d}_\sigma + \mathrm{H.c.})]. \quad (2)$$

The operators $\hat{c}^\dagger_{\alpha\sigma k}$ and $\hat{d}^\dagger_\sigma$ are creation operators for the electronic states k in the $\alpha = l, r$ (left, right) leads and the dot states with spin $\sigma = \pm$. The latter have energy $\varepsilon_\sigma = \varepsilon_0 + \sigma\varepsilon_z/2$ with the energy separation ε_z. The ferromagnets are magnetized in the z directions and their effect on the spin-polarized tunneling is captured in spin-dependent tunneling rates $\Gamma^\sigma_\alpha = \pi|t_{\alpha\sigma}|^2\rho_{\alpha\sigma}$. Here, $\rho_{\alpha\sigma}$ denotes the spin-σ density of states at the Fermi level of lead α, and $t_{\alpha\sigma}$ the tunneling amplitude, and we can define a polarization $p_\alpha = (\Gamma^+_\alpha - \Gamma^-_\alpha)/(\Gamma^+_\alpha + \Gamma^-_\alpha)$.

Results.—In the regime of weak spin-vibrational interaction, electrons tunneling from the leads to the dot yield a (small) renormalization of the vibration frequency and a damping of the mechanical motion with friction coefficient γ. In addition, at finite bias voltage, the electron current drives the mechanical oscillator to a steady nonequilibrium regime with a phonon occupation n. To determine these quantities, we employ the Keldysh Green functions technique to calculate the phonon propagator $D(t,t') = -i\langle T_{\mathcal{C}}\hat{u}(t)\hat{u}(t')\rangle$, where $T_{\mathcal{C}}$ denotes the time-ordering operator on the Keldysh contour $\mathcal{C}$ [45]. We have solved the Dyson equation with the self-energy associated with the spin-vibration interaction [Eq. (2)] to the leading order in λ. This approximation is sufficient for $\gamma \ll \omega$ [46]. We find $\gamma = \sum_{\alpha\beta s} s\gamma^s_{\alpha\beta}$ ($s = \pm 1$) and for the occupation

$$n = \frac{1}{\gamma}\sum_{\alpha\beta s} s\gamma^s_{\alpha\beta} n_B[\omega + s(\mu_\alpha - \mu_\beta)]. \quad (3)$$

Here, we introduced the lead chemical potentials μ_α and

$$\gamma^s_{\alpha\beta} = \frac{\lambda^2}{2}\int\frac{d\varepsilon}{2\pi}T^s_{\alpha\beta}(\varepsilon,\omega)f_\alpha(\varepsilon)[1 - f_\beta(\varepsilon + s\omega)], \quad (4)$$

with the Fermi function $f_\alpha(\varepsilon) = \{1 + \exp[(\varepsilon - \mu_\alpha)/T]\}^{-1}$, $T^s_{\alpha\beta}(\varepsilon,\omega) = \sum_\sigma L^\sigma_\alpha(\varepsilon)L^{-\sigma}_\beta(\varepsilon + s\omega)$, and $L^\sigma_\alpha(\varepsilon) = 2\Gamma^\sigma_\alpha/[(\Gamma^\sigma_l + \Gamma^\sigma_r)^2 + (\varepsilon - \varepsilon_\sigma)^2]$.

The essential point of our proposal is that z (or y) spin polarized electrons injected in the dot are perpendicular to the spin component coupled to the nanotube oscillations (x axis) so that spin-flip transitions are needed to exchange energy with the vibrational mode. These inelastic processes are characterized by the rates $\gamma^s_{\alpha\beta}$ [Eq. (4)] describing a spin flip of an electron tunneling from lead α to lead β accompanied by the absorption ($s = +$) or emission ($s = -$) of an energy quantum of the vibron. The weighted sum gives the total damping coefficient γ.

On the one side, for the parallel configuration of the ferromagnets $p_l p_r > 0$, we always found heating of the oscillator at finite bias voltage and we will not further consider this case. On the other side, for the antiparallel configuration, we obtain heating and efficient cooling also for different polarizations $|p_l| \neq |p_r|$. We found similar results even in the limit of one unpolarized lead (see discussion below). Hereafter, we restrict our discussion to the antiparallel configuration with the same polarization $p_r = p$ and $p_l = -p$ with $\mathrm{sgn}(p) = \mathrm{sgn}(\varepsilon_z)$. We note that the inverted polarizations with $\mathrm{sgn}(p) = -\mathrm{sgn}(\varepsilon_z)$ is equivalent to a reversed voltage. Depending on the sign of the voltage, we also found a strong overheating of the mechanical resonator $\bar{n} \gg 1$ for which the system approaches an instability region with a negative damping $\gamma < 0$. This configuration corresponds to the operating regime in which phonon lasing has been discussed recently [47]. Electromechanical instability was also obtained in a different microscopic model based on the magnetomotive interaction between current and vibration in Ref. [31] in which it was shown that the feedback action of the vibration on the current can lead to mechanical self-excitations in a suspended CNTQD in contact with a single ferromagnet. In the remainder of the Letter we consider antiparallel magnetizations with $p > 0$, $\varepsilon_z > 0$, and $V > 0$.

For $T \gg \Gamma^\sigma_\alpha$, one can use an analytic approximation for the rates $\gamma^s_{\alpha\beta}$, which is in excellent agreement with the full results [Eq. (4)]. The analysis of such an incoherent regime can also be addressed by using a Pauli master equation [48]. The Lorentzian functions appearing in Eq. (4) can be treated separately as δ functions in the integral and we can cast each rate as the sum of two rates $\gamma^s_{\alpha\beta} \simeq \sum_\sigma \gamma^{s\sigma}_{\alpha\beta}$, for tunneling through the dot level σ, respectively. They read

$$\gamma^{s\sigma}_{\alpha\beta} = \frac{\lambda^2}{\Gamma^\sigma_l + \Gamma^\sigma_r}\{\Gamma^\sigma_\alpha \Gamma^{-\sigma}_\beta T^{s\sigma}_+ f_\alpha(\varepsilon_\sigma)[1 - f_\beta(\varepsilon_\sigma + s\omega)] + \Gamma^{-\sigma}_\alpha \Gamma^\sigma_\beta T^{s\sigma}_- f_\alpha(\varepsilon_\sigma - s\omega)[1 - f_\beta(\varepsilon_\sigma)]\} \quad (5)$$

with $T^{s\sigma}_\pm = 1/[(\Gamma^{-\sigma}_l + \Gamma^{-\sigma}_r)^2 + (\sigma\varepsilon_z \pm s\omega)^2]$.

Fully polarized contacts.—To gain insight into the problem, we describe in detail the case of fully polarized ferromagnets although efficient cooling is achieved even for small polarizations. For $p = 1$, the diagonal rates vanish $\gamma^s_{ll} = \gamma^s_{rr} = 0$, as the electron cannot come back to its original lead after a spin flip. Moreover, in the high-voltage limit $eV \gg T$ ($e > 0$), we can safely neglect the processes $\gamma^s_{rl} \simeq 0$ being $V > 0$ as electrons tunneling from the right lead are Pauli blocked. Accordingly, the total damping reduces to the sum of only two processes $\gamma \simeq \gamma^+_{lr} - \gamma^-_{lr}$ and the expression of n simplifies to the average distribution resulting from these two competing processes

$$n \simeq \frac{\gamma^+_{lr} n_B(\omega + eV) - \gamma^-_{lr} n_B(\omega - eV)}{\gamma^+_{lr} - \gamma^-_{lr}} \simeq \frac{\gamma^-_{lr}}{\gamma^+_{lr} - \gamma^-_{lr}}. \quad (6)$$

The second step in Eq. (6) holds for $eV \gg \omega$, when the nonequilibrium phonon occupation is completely ruled by the ratio $\gamma^+_{lr}/\gamma^-_{lr}$. Although in the region of stability defined by $\gamma^+_{lr} > \gamma^-_{lr}$ the total damping is always positive, n can show heating or cooling: for $\gamma^+_{lr} \gtrsim \gamma^-_{lr}$ the mechanical oscillator is almost undamped and it is actively heated to $n \gtrsim n_B(\omega)$ whereas for $\gamma^+_{lr} \gg \gamma^-_{lr}$ the dominant emission processes yield an efficient cooling of the oscillator, viz., $n \ll n_B(\omega)$. This is the main mechanism of cooling underlying our proposal.

We now discuss the result for the fully polarized case in Fig. 2. Since $\Gamma^+_l = \Gamma^-_r = 0$ one of two terms appearing in Eq. (5) vanishes for each spin channel. For symmetric contacts $\Gamma^-_l = \Gamma^+_r = \Gamma$ and setting $T^s = \lambda^2\Gamma/[\Gamma^2 + (\omega - s\varepsilon_z)^2]$, the single spin-channel rates read

$$\gamma^{s\sigma}_{lr} = T^s f_l(\varepsilon_\sigma - s\omega\delta_{\sigma+})[1 - f_r(\varepsilon_\sigma + s\omega\delta_{\sigma-})]. \quad (7)$$

In Fig. 2(a) we show an example of the case $\varepsilon_z \gg \omega$ for an asymmetric voltage bias for which only the spin-down level is involved in transport. In this limit $T^+ \simeq T^-$ and the difference between the absorption rate γ^+_{lr} and emission rate γ^-_{lr} is mainly given by the product of the electronic occupations in Eq. (7). The system is expected to switch from cooling to heating when we move from the regime $\gamma^+_{lr} \gg \gamma^-_{lr}$ to the regime $\gamma^+_{lr} \gtrsim \gamma^-_{lr}$. In a simple picture, the switch is expected close to the line $\mu_r = \varepsilon_-$. For $\mu_r > \varepsilon_-$ (cooling region) the emission processes are suppressed due to the occupation of the low energy level in the right lead [left inset of Fig. 2(a)]. For $\mu_r < \varepsilon_-$ (heating region) emission processes are relevant and they compete with the absorption ones [right inset of Fig. 2(a)]. At finite temperature, the thermal broadening of the Fermi functions causes a smooth transition between the two regimes so that the crossing line corresponding to $n = n_B$ occurs at $\varepsilon_- = \mu_r + 8T^2/\varepsilon_z$ to leading order in T/ε_z and for $T \gg \omega$. Note that, in this discussion, the left lead plays only the role of a source for injecting one electron with spin up in the dot level. Hence cooling is achieved even for a normal left contact ($p_l = 0$).

The minimum of the phonon occupation as a function of voltage decreases with the ratio ε_z/ω. The optimal cooling is achieved at $\varepsilon_z = \omega$. At this point and in the limit $eV \gg (T, \omega, \varepsilon_0)$, $f_l \simeq 1$ and $f_r \simeq 0$ and the phonon occupation of Eq. (3) becomes $n \simeq (\Gamma/\omega)^2$. Further decreasing the ratio ε_z/ω does not improve the cooling.

The strong cooling obtained for the resonant regime can be explained as follows. The absorption processes for each spin channel are now the same and we have $\gamma^{++}_{lr} = \gamma^{+-}_{lr}$ as the virtual levels $\varepsilon_- + \omega$ and $\varepsilon_+ - \omega$, which are involved in the spin-flip tunneling for cooling, coincide, respectively, to the real dot spin levels ε_+ and ε_-. This yields a strong enhancement of the (transmission) function T^+ (phonon

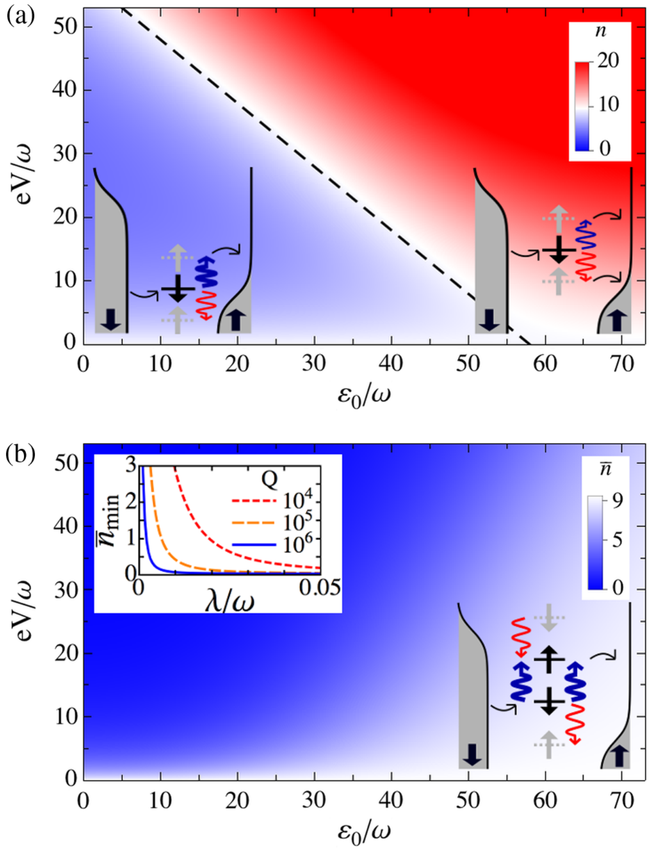

FIG. 2 (color online). Phonon occupation as function of the bias voltage V and ε_0. The parameters are $p = 1$ (fully polarized), $\Gamma_l^- = \Gamma_r^+ = 0.2\omega$, and $T = 10\omega$. White corresponds to $n_B(\omega)$. (a) Vanishing external damping $\gamma_0 = 0$, $\varepsilon_z = 10T$, $\mu_r = \varepsilon_0 - eV$, and $\mu_l = \varepsilon_0$. The black dashed line indicates the transition from cooling to heating (see text). Inset: schematic behavior of the relevant spin-flip processes in the region of cooling (left) and heating (right). (b) Resonant regime $\varepsilon_z = \omega$ with $\gamma_0 = 10^{-5}\omega$, $\lambda/\omega = 0.01$, and $\mu_{l,r} = \varepsilon_0 \pm eV/2$. Inset, left: the minimum occupation $\bar{n}_{\min}$ as a function of the spin-vibration coupling constant λ for different quality factors. Inset, right: schematic behavior of the energy levels and of the inelastic resonant slip-flip tunneling.

absorption) as compared to T^- (phonon emission) in Eq. (7), namely $T^+ \gg T^-$, which explains the strong cooling effect. As a consequence, n has a weak dependence on the alignment of the average level position ε_0 and the lead chemical potential μ_α. In Fig. 2(b) we show the resonant case with a finite intrinsic damping $\gamma_0/\omega = 10^{-5}$ to illustrate the behavior of $\bar{n}$.

Effect of finite polarization.—We discuss now the effect of finite polarization. For symmetrically applied voltage, the results for the minimal value $\bar{n}_{\min}$ as a function of the energy separation for different polarizations are shown in Fig. 3(a). Even in this case, at arbitrary fixed polarization, optimal cooling is again achieved for the resonant regime $\omega = \varepsilon_z$. A finite polarization always reduces the minimum occupation as $\bar{n}_{\min}$ decreases as a function of p independent of the ratio ε_z/ω [Fig. 3(b)]. To discuss this behavior we

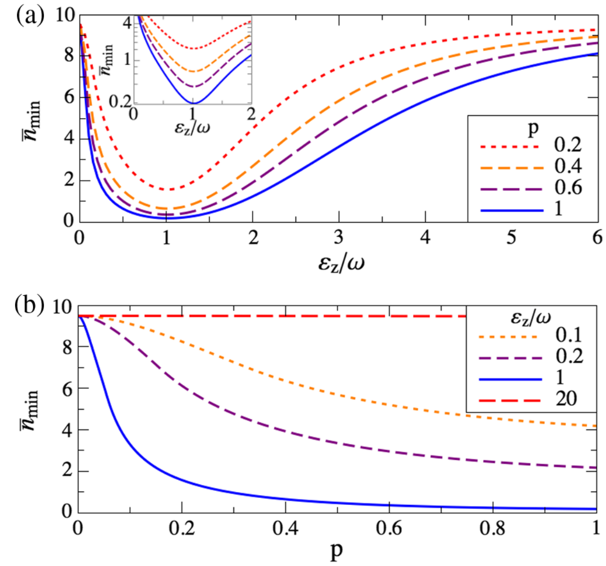

FIG. 3 (color online). Minimum of the phonon occupation $\bar{n}_{\min}$ for an intrinsic damping of $Q = 10^4$ and $\lambda/\omega = 0.05$. The temperature is $T = 10\omega$ and $\Gamma_l = \Gamma_r = 0.2\omega$. (a) Minimal occupation as function of ε_z/ω for different polarizations. The inset (log scale for y axis) shows that maximal cooling is achieved at $\varepsilon_z = \omega$. (b) Minimal phonon occupation as a function of polarization for different energy separations.

consider the analytic high-voltage approximation for the phonon occupation given by

$$n \simeq \frac{\gamma_{lr}^- + n_B(\omega)(\gamma_{ll} + \gamma_{rr})}{\gamma_{lr}^+ - \gamma_{lr}^- + \gamma_{ll} + \gamma_{rr}}, \tag{8}$$

where we set the short notation $\gamma_{\alpha\alpha} = \gamma_{\alpha\alpha}^+ - \gamma_{\alpha\alpha}^-$. From Eq. (8) we observe that the diagonal lead processes $\gamma_{\alpha\alpha}$, which are not present for $p = 1$, have the effect of thermalizing the oscillator. As an example, assuming a strong asymmetry of the leads, as for instance $\Gamma_l \simeq 0$ ($\Gamma_r \simeq 0$), we have $\gamma_{lr}^{\pm} = 0$: the dot is in contact only with one left (right) lead and the oscillator is always at the thermal equilibrium. Such processes compete with the cooling processes γ_{lr}^+ leading to an increase of the minimum phonon occupation.

Clearly, taking into account the intrinsic damping of the mechanical oscillator also increases the minimum phonon occupation. Remarkably, a phonon occupation of $\bar{n}_{\min} \simeq 0.5$ is still achieved for $Q \simeq 10^4$, $\lambda/\omega = 0.05$, and polarizations $p > 0.48$ [Fig. 3(b)]. The minimal phonon occupation reduces to $\bar{n}_{\min} = 0.2$ at $p = 1$. An occupation of $\bar{n}_{\min} \simeq 0.5$ is also obtained for $Q \simeq 10^5$ and $p > 0.3$ ($\bar{n}_{\min} = 0.05$ at $p = 1$). Motivated by a recent experiment that reported large spin-orbit interaction coupling Δ_{SO} [12], one can also consider coupling constants of order $\lambda/\omega = 0.2$, which implies a strong reduction of the polarization required for cooling. As an example, $\bar{n}_{\min} \simeq 0.5$ for $Q \simeq 10^4$ and $p > 0.3$. Therefore, we conclude that even for modest polarizations, which appears feasible in promising experiments with CNTQDs [20,49,50], quantum ground-state cooling is achievable.

Conclusions.—In summary, we discussed a suspended CNTQD forming a nanomechanical spin valve with a direct coupling between the dot spin and the flexural modes showing that ground-state cooling is achievable with moderated spin-current polarization.

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