Three-Body Physics in Strongly Correlated Spinor Condensates

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Spinor condensates have proven to be a rich area for probing many-body phenomena richer than that of an ultracold gas consisting of atoms restricted to a single spin state. In the strongly correlated regime, the physics controlling the possible novel phases of the condensate remains largely unexplored, and few-body aspects can play a central role in the properties and dynamics of the system through manifestations of Efimov physics. The present study solves the three-body problem for bosonic spinors using the hyperspherical adiabatic representation and characterizes the multiple families of Efimov states in spinor systems as well as their signatures in the scattering observables relevant for spinor condensates. These solutions exhibit a rich array of possible phenomena originating in universal few-body physics, which can strongly affect the spin dynamics and three-body mean-field contributions for spinor condensates. The collisional aspects of atom-dimer spinor condensates are also analyzed, and effects are predicted that derive from Efimov physics.

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In recent years, the development of optical traps has stimulated the realization of spinor condensates [1]. The coupling of the degenerate spin degrees of freedom leads to novel quantum phenomena such as the formation of spin domains, spin textures, spin mixing dynamics, and counterintuitive quantum phases. These have been intensively investigated both experimentally [2–13] and theoretically [14–24]. Such phenomena have been shown to be sensitive to the (typically weak) interatomic interactions, characterized by multiple scattering lengths associated with the various atomic hyperfine spins states. Of particular interest is the fact that strongly correlated spinor condensates can enable explorations of spinor physics in exotic dynamical regimes. Although the scattering lengths for most alkali species are modest or even small, one key exception is ^{85}Rb [25]. There have been several proposals to create strongly correlated spinor condensates [26–32] where the scattering lengths substantially exceed the range of interatomic interactions, i.e., the van der Waals length $r_{\rm vdW}$. This enables the spin states to effectively interact even at large distances. In this scenario, one should also consider few-body correlations, notably effects associated with the existence of Efimov states [33–35].

In *single*-spin condensates, when the interactions are enhanced by the presence of a Feshbach resonance [36], an infinity of Efimov states emerges that strongly affects scattering observables at ultracold energies [33–35]. More recently, advances have been made in our understanding of universal Efimov physics in an ultracold quantum gas. Despite the complex nature of the interatomic interactions, recent experimental [37–44] and theoretical [45–49] studies have shown, surprisingly, that the usual three-body parameter is universal, depending only on $r_{\rm vdW}$, which now permits even more quantitative predictions of interesting few-body phenomena to be made.

The present exploration of Efimov physics in a spinor condensate system shows that the additional spin degrees of freedom can fundamentally modify the Efimov trimer's energy spectrum and that scattering processes can strongly affect the condensate spin dynamics. In the context of nuclear physics, where isospin symmetry plays an important role, the work of Bulgac and Efimov [50] demonstrated a much richer structure for Efimov physics when the spin degree of freedom was considered. In this case, multiple families of Efimov states can coexist, depending on the particular spin states and different scattering lengths in the problem [51]. This is in striking contrast to the *standard* Efimov scenario where only a single spin state is available. For the multilevel bosonic systems examined here, with the topologically distinct case of a spinor condensate, the atomic hyperfine states provide the internal atomic structure. Several interesting effects are predicted for the three-body scattering observables controlling the dynamical evolution of spinor condensates. Similarly, our results point to the possibility of exploring spinor physics in an atom-dimer mixture. These results emerge from a calculation of the collisional properties of this system, including a characterization of the signatures of Efimov physics.

The study of few-body physics in spinor condensates requires proper inclusion of the multichannel nature of interatomic interactions, originating from the underlying atomic hyperfine structure. Our study begins from the multichannel generalization of the zero-range Fermi pseudopotential [52–54] for *s*-wave interactions (in a.u.), namely:

$$\hat{v}(r) = \frac{4\pi\hat{A}}{m}\delta^3(\vec{r})\frac{\partial}{\partial r}r, \tag{1}$$

where $\delta^3(\vec{r})$ is the usual three-dimensional Dirac-δ function and $\hat{A}$ is the scattering length *matrix* written in the two-body spin basis denoted by $\{|\sigma\rangle\}$. Within this framework, the three-body problem is solved in the adiabatic hyperspherical representation, using the Green's function method developed in Refs. [52,53]. In this representation, the hyperradius R determines the overall size of the system, and the internal motion is described by a set of five hyperangles, collectively denoted by Ω. Briefly, the adiabatic fixed-R eigenvalue equation reads

$$\left[\frac{\hat{\Lambda}^2(\Omega)+\frac{15}{4}}{2\mu R^2}+\hat{V}(R,\Omega)+\hat{E}_\Sigma\right]\Phi(R;\Omega) = U(R)\Phi(R;\Omega), \tag{2}$$

where $\mu = m/\sqrt{3}$ is the three-body reduced mass, $\hat{\Lambda}$ is the grand angular momentum operator [55], $\hat{V}$ is the sum of pairwise interactions, and $\hat{E}_\Sigma$ is the sum of the atomic energy levels, which is diagonal in the three-body spin basis $\{|\Sigma\rangle\}$. The channel functions $\Phi(R;\Omega)$ and the three-body potentials $U(R)$ describe the physical properties of the system and are obtained by solving Eq. (2) for fixed values of R. Application of the zero-range potential model reduces the problem to solving a transcendental equation whose roots $s_\nu(R)$ determine $U_\nu(R)$ through

$$U_\nu(R) = \frac{s_\nu(R)^2 - 1/4}{2\mu R^2}. \tag{3}$$

(See the outline of our formulation in Supplemental Material [56].) For three-identical bosons, for instance, solving Eq. (2) in the limit $R/a \to 0$ yields a *single* imaginary root, independent of R, with numerical value $s_0 \approx 1.0062i$. Insertion of s_0 into Eq. (3) produces the *attractive* $1/R^2$ potential that supports an infinity of three-body bound states characteristic of the Efimov effect. In the present study, the threshold energy levels $\hat{E}_\Sigma$ in Eq. (2) are degenerate and are set equal to zero. In spinor condensates at vanishingly small magnetic fields, the atomic levels are $(2f+1)$-fold degenerate (f is the atomic hyperfine angular momentum and $m_f = -f, \ldots, f$ its azimuthal component). In fact, this degeneracy leads to fundamentally different three-body physics than is obtained for the usual Efimov case with atoms in a single spin state.

The interatomic interaction for spinor condensates [1] is spin dependent, and we assume the scattering length operator in Eq. (1) can be represented as

$$\hat{A} = \sum_{F_{2b}M_{F_{2b}}} a_{F_{2b}}|F_{2b}M_{F_{2b}}\rangle\langle F_{2b}M_{F_{2b}}|, \tag{4}$$

where F_{2b} and $M_{F_{2b}}$ are the two-body total spin and its projection. Because of bosonic symmetry, only the symmetric spin states ($F_{2b} \equiv$ even) are allowed to interact with rotationally invariant scattering lengths $\{a_0, a_2, \ldots, a_{2f}\}$. These scattering lengths set important length scales in the problem, and their relative magnitudes and signs determine many-body properties such as the miscibility of the different spin components. Moreover, the scattering lengths also determine the nature of the three-body interactions and many of the scattering properties of the system, potentially impacting the spin dynamics of condensates.

Figure 1 shows the three-body potentials for $f = 1$ atoms for the allowed values of the total three-body hyperfine spin $|F_{2b} - f| \le F_{3b} \le F_{2b} + f$. These are, of course, independent of $M_{3b} = M_{F_{2b}} + m_f$. [The $F_{3b} = 0$ states are spatially antisymmetric and thus noninteracting in the potential model of Eq. (1).] The results in Fig. 1 were obtained by solving Eq. (2) in the spin basis $\{|\Sigma\rangle\}$ (Supplemental Material [56]) and with $a_0 = 10^2 r_{\rm vdW}$ and

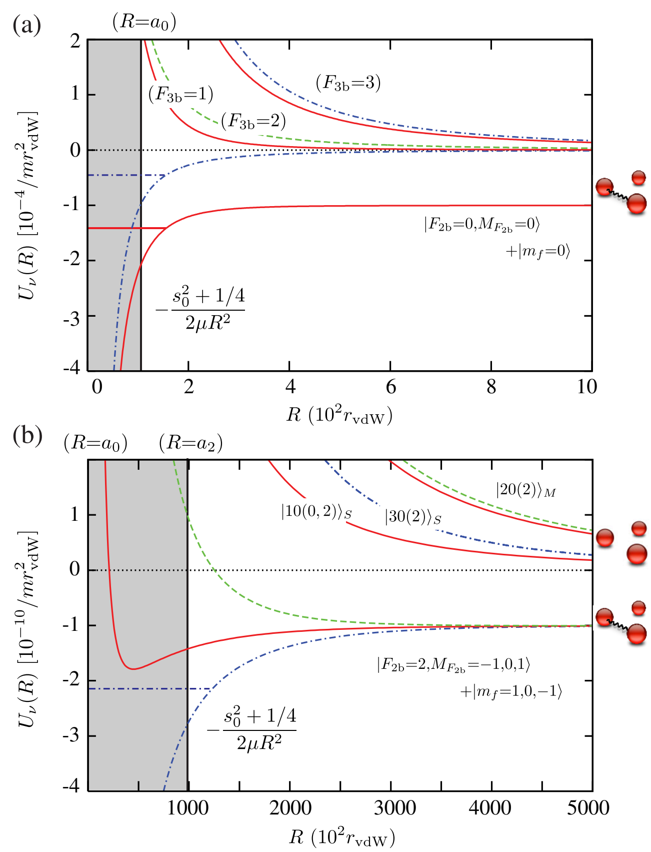

FIG. 1 (color online). $F_{3b} = 1$ (red solid line), 2 (green dashed line), and 3 (blue dash-dotted line) hyperspherical adiabatic potentials for $f = 1$ spinors with $a_0 = 10^2 r_{\rm vdW}$ and $a_2 = 10^5 r_{\rm vdW}$. (a) For $R \le \{a_0, a_2\}$ (shaded region) two attractive potentials exist (both with $s_0 \approx 1.0062i$), allowing for two families of Efimov states, and for $R > a_0$, one of these potentials turns into an atom-dimer channel $|F_{2b} = 0, M_{F_{2b}} = 0\rangle + |m_f = 0\rangle$. (b) For $a_0 \le R \le a_2$ (shaded region), only one family of Efimov states exists ($s_0 \approx 1.0062i$), and for $R \gg a_2$ three (asymptotically degenerate) potentials describe atom-dimer channels, $|F_{2b} = 2, M_{F_{2b}} = -1, 0, 1\rangle + |m_f = 1, 0, -1\rangle$.

$a_2 = 10^5 r_{\rm vdW}$. [Note that Figs. 1(a) and 1(b) illustrate only one of the possible scenarios in which Efimov physics can be manifested in $f = 1$ spinor systems.] Figure 1(a) emphasizes the three-body physics for $R \leq \{a_0, a_2\}$ (shaded region) where *two* attractive potentials exist for $F_{3b} = 1$ (red solid line) and 3 (blue dash-dotted line). Both potentials are associated with $s_0 \approx 1.0062i$ and allow for the coexistence of *two* families of Efimov states (represented in Fig. 1 by the horizontal solid and dash-dotted lines)—a feature absent in systems of single state atoms. For $R > a_0$, the $F_{3b} = 1$ potential turns into an atom-dimer channel describing collisions between an $|F_{2b} = 0, M_{F_{2b}} = 0\rangle$ dimer, with energy $-1/ma_0^2$, and an $|m_f = 0\rangle$ atom. For $a_0 \leq R \leq a_2$ [shaded region in Fig. 1(b)], only the $F_{3b} = 3$ family of Efimov states persists. For $R \gg a_2$, this $F_{3b} = 3$ potential and two other $F_{3b} = 1$ and $F_{3b} = 2$ potentials converge to the dimer energy $-1/ma_2^2$ and describe atom-dimer collisions in states $|F_{2b} = 2, M_{F_{2b}} = -1, 0, 1\rangle + |m_f = 1, 0, -1\rangle$. This offers an interesting scenario in which one can study atom-dimer spinor mixtures, some of whose collisional properties are described in Supplemental Material [56]. In Fig. 1(b), the repulsive potentials for $R \gg a_2$ describe collisions between three free atoms in the symmetric spin states $|F_{3b}, M_{F_{3b}}(F_{2b})\rangle = |30(2)\rangle_S$ and $|10(0,2)\rangle_S$ and the mixed symmetry state $|20(2)\rangle_M$ (see Supplemental Material [56]). Note that only $F_{3b} = 1$ states are sensitive to both a_0 and a_2.

Table I summarizes the values of s_ν relevant for $f = 1$ and 2 spinor condensates, covering all possible regions of R and for different magnitudes of the relevant scattering lengths. For $f = 1$ the values of s_ν are listed according to the value of F_{3b} while for $f = 2$ they are not assigned in detail (see the complete assignment in Supplemental Material [56]). Notably, for $f = 1$ ensembles, the imaginary values of s_ν agree exactly with the ones for single-level atoms, except with the important distinction that such roots can be degenerate in the spinor case. It is well known that the existence of overlapping series of states can lead to formation of ultra-long-lived states [58]. In our present case, the $F_{3b} = 1$ and 3 Efimov states can interact for finite (but small) magnetic fields and such controllability can not only produce long-lived states but also, due to their weakly bound character, affect the spin dynamics of the condensate. The occurrence of such effects, however, will depend on the three-body short-range physics [47]. Further analysis of this parameter space is beyond the scope of the present study. This feature opens up exciting possibilities for the study of Efimov trimers in spinor condensates in a well-controlled manner. More interestingly, $f = 2$ ensembles exhibit several values of s_ν that differ from those for single-level atoms. The physics controlling the appearance of these new roots is due to the fact that the atoms in the two-body states $|F_{2b} M_{F_{2b}}\rangle$ are not in a pure quantum state. Instead, they are in a mixture of states, with the amount of mixing controlled by the angular momentum algebra.

TABLE I. Values of s_ν relevant for $f = 1$ and 2 spinor condensates covering all possible regions of R for the different ranges of the relevant scattering lengths. For $f = 1$, we list the lowest few values of s_ν for each F_{3b} while for $f = 2$ we only list the values of s_ν and their multiplicity (superscript), instead of the specific value of F_{3b} where they occur.

$(f = 1)$	$F_{3b} = 1$	$F_{3b} = 2$	$F_{3b} = 3$
$R \ll \lvert a_{\{0,2\}}\rvert$	$1.0062i$, 2.1662	2.1662	$1.0062i$, 4.4653
$\lvert a_0\rvert \ll R \ll \lvert a_2\rvert$	0.7429	2.1662	$1.0062i$, 4.4653
$\lvert a_2\rvert \ll R \ll \lvert a_0\rvert$	0.4097	4	2
$R \gg \lvert a_{\{0,2\}}\rvert$	2	4	2
$(f = 2)$	$F_{3b} = 0, 1, \ldots, 6$		
$R \ll \lvert a_{\{0,2,4\}}\rvert$	$1.0062i^{(5)}$, $2.1662^{(5)}$		
$\lvert a_0\rvert \ll R \ll \lvert a_{\{2,4\}}\rvert$	$1.0062i^{(4)}$, $0.4905^{(1)}$		
$\lvert a_2\rvert \ll R \ll \lvert a_{\{0,4\}}\rvert$	$1.0062i^{(1)}$, $0.7473i^{(1)}$, $0.6608^{(1)}$		
$\lvert a_4\rvert \ll R \ll \lvert a_{\{0,2\}}\rvert$	$1.0062i^{(1)}$, $0.5528i^{(1)}$, $0.3788i^{(1)}$, $0.5219^{(1)}$		
$\lvert a_{\{0,2\}}\rvert \ll R \ll \lvert a_4\rvert$	$1.0062i^{(1)}$, $0.6608^{(1)}$		
$\lvert a_{\{0,4\}}\rvert \ll R \ll \lvert a_2\rvert$	$1.0062i^{(1)}$, $0.5528i^{(1)}$, $0.5219^{(1)}$		
$\lvert a_{\{2,4\}}\rvert \ll R \ll \lvert a_0\rvert$	$0.6861^{(1)}$		
$R \gg \lvert a_{\{0,2,4\}}\rvert$	$2^{(5)}$, $4^{(2)}$		

The above results illustrate the rich structure of Efimov states in spinor systems. Such richness also appears in the three-body scattering observables. Here, we use a WKB model [51] to determine the scattering length and energy dependence of collision rates for all relevant scattering processes for $f = 1$ spinor condensates. The results for $F_{3b} = 1$, 2, and 3 are summarized in Table II. Notice that scattering observables can display log-periodic interference and resonant effects due to the multiple families of Efimov states and collision pathways available in spinor ensembles. Interference and resonance effects are parametrized according to, respectively,

$$M^{\eta}_{s_0}(a) = \alpha e^{-2\eta}\left[\sin^2\left(|s_0| \ln\frac{a}{r_\phi}\right) + \sinh^2\eta\right], \quad (5)$$

$$P^{\eta}_{s_0}(a) = \beta \frac{\sinh 2\eta}{\sin^2(|s_0| \ln\frac{a}{r_\phi}) + \sinh^2\eta}, \quad (6)$$

where $r_\phi = r_{\rm vdW} e^{-\phi/|s_0|}$ is the three-body parameter, incorporating the short-range physics through the phase ϕ [47], and η is the three-body inelasticity parameter [34,35], which encapsulates the probability for decay into deeply bound molecular states. In the above equations, α and β are universal constants that can be evaluated for each F_{3b}. In Table II, $K_3^{(0)}$ and $K_3^{(2)}$ denote the collision rate for three-body recombination into weakly bound $F_{2b} = 0$ and 2 dimers, respectively. Such dimers can still remain trapped and further dissociate into free atoms via collision with other atoms with dissociation rates $D_3^{(0)} \propto K_3^{(0)}(k^4 a_0)$ and

TABLE II. Scattering length dependence for $F_{3\mathrm{b}} = 1$, 2, and 3 three-body scattering observables relevant for $f = 1$ spinor condensates. Here, K_3 is the three-body recombination rate [superscripts (0) and (2) indicating recombination into weakly bound $F_{2\mathrm{b}} = 0$ and 2 dimers and (d) recombination rate into deeply bound states] and $a_{3\mathrm{b}}$ the three-body scattering length matrix element. $M(a)$, $P(a)$, and $T(a)$ are given in Eqs. (5)–(8). Here, γ is a universal constant that can be calculated for each observable, $s_1 \approx 0.7429$ ($|a_2| \gg |a_0|$) and $s_1 \approx 0.4097$ ($|a_0| \gg |a_2|$) for $F_{3\mathrm{b}} = 1$ and for $F_{3\mathrm{b}} = 2$, $s_1 \approx 2.1662$, and $k^2 = 2\mu E$.

$(F_{3\mathrm{b}} = 1)$	$a_2 \gg a_0$	$a_2 \gg \lvert a_0\rvert$	$\lvert a_2\rvert \gg a_0$	$\lvert a_2\rvert \gg \lvert a_0\rvert$	$(F_{3\mathrm{b}} = 2)$	$a_2 \gg r_{\mathrm{vdW}}$	$\lvert a_2\rvert \gg r_{\mathrm{vdW}}$
$K_3^{(0)}/a_2^4$	$M^{\eta}_{s_0}(a_0)(a_0/a_2)^{2s_1}$	$\cdots$	$M^{\eta}_{s_0}(a_0)(a_0/a_2)^{2s_1}$	$\cdots$	$K_3^{(2)}/a_2^8$	γk^4	$\cdots$
$K_3^{(2)}/a_2^4$	$\gamma + M^{\eta}_{s_0}(a_0)(a_0/a_2)^{4s_1}$	$\gamma + P^{\eta}_{s_0}(a_0)(a_0/a_2)^{4s_1}$	$\cdots$	$\cdots$	$K_3^{(d)}/a_2^8$	γk^4	$\gamma k^4 (r_{\mathrm{vdW}}/a_2)^{2s_1}$
$K_3^{(d)}/a_2^4$	$\gamma(a_0/a_2)^{2s_1}$	$P^{\eta}_{s_0}(a_0)(a_0/a_2)^{2s_1}$	$\gamma(a_0/a_2)^{2s_1}$	$P^{\eta}_{s_0}(a_0)(a_0/a_2)^{2s_1}$	$a_{3\mathrm{b}}^{(2)}/a_2^4$	$\cdots$	$\cdots$
$a_{3\mathrm{b}}^{(1)}/a_2^4$	$\gamma + O^{\eta}_{s_0}(a_0)(a_0/a_2)^{4s_1}$	$\gamma + T^{\eta}_{s_0}(a_0)(a_0/a_2)^{4s_1}$	$\gamma + O^{\eta}_{s_0}(a_0)(a_0/a_2)^{4s_1}$	$\gamma + T^{\eta}_{s_0}(a_0)(a_0/a_2)^{4s_1}$			
$(F_{3\mathrm{b}} = 1)$	$a_0 \gg a_2$	$a_0 \gg \lvert a_2\rvert$	$\lvert a_0\rvert \gg a_2$	$\lvert a_0\rvert \gg \lvert a_2\rvert$	$(F_{3\mathrm{b}} = 3)$	$a_2 \gg r_{\mathrm{vdW}}$	$\lvert a_2\rvert \gg r_{\mathrm{vdW}}$
$K_3^{(0)}/a_0^4$	$\gamma + M^{\eta}_{s_0}(a_2)(a_2/a_0)^{4s_1}$	$\gamma + P^{\eta}_{s_0}(a_2)(a_2/a_0)^{4s_1}$	$\cdots$	$\cdots$	$K_3^{(2)}/a_2^4$	$M^{\eta}_{s_0}(a_2)$	$\cdots$
$K_3^{(2)}/a_0^4$	$M^{\eta}_{s_0}(a_2)(a_2/a_0)^{2s_1}$	$\cdots$	$M^{\eta}_{s_0}(a_2)(a_2/a_0)^{2s_1}$	$\cdots$	$K_3^{(d)}/a_2^4$	γ	$P^{\eta}_{s_0}(a_2)$
$K_3^{(d)}/a_0^4$	$\gamma(a_2/a_0)^{2s_1}$	$P^{\eta}_{s_0}(a_2)(a_2/a_0)^{2s_1}$	$\gamma(a_2/a_0)^{2s_1}$	$P^{\eta}_{s_0}(a_2)(a_2/a_0)^{2s_1}$	$a_{3\mathrm{b}}^{(3)}/a_2^4$	$O^{\eta}_{s_0}(a_2)$	$T^{\eta}_{s_0}(a_2)$
$a_{3\mathrm{b}}^{(1)}/a_0^4$	$\gamma + O^{\eta}_{s_0}(a_2)(a_2/a_0)^{4s_1}$	$\gamma + T^{\eta}_{s_0}(a_2)(a_2/a_0)^{4s_1}$	$\gamma + O^{\eta}_{s_0}(a_2)(a_2/a_0)^{4s_1}$	$\gamma + T^{\eta}_{s_0}(a_2)(a_2/a_0)^{4s_1}$			

$D_3^{(2)} \propto K_3^{(2)}(k^4 a_2)$, where $k^2 = 2\mu E$ with $E > 0$ being the three-body collision energy. This interplay between dimer formation and dissociation can provide an interesting dynamical regime in spinor condensates that is absent when the scattering lengths are small. Three-body recombination into deeply bound molecular states $K_3^{(d)}$ can only lead to losses and can display resonant enhancements due to the formation of Efimov states or can be suppressed due to repulsive three-body interactions. Note that the total rate is obtained by multiplying K_3 in Table II by the appropriate factors that account for the various degeneracies in the problem. From Tables I and II, one can explore the various ways in which the existence of multiple families could be observed. In the case where $a_0 \approx a_2, \ldots, \approx a_{2f} < 0$, multiple nearby resonant features in atom losses should be observed instead of the usual single feature [59]. For $f = 1$ spinors, one should be able to observe two nearby resonances due to the $F_{3\mathrm{b}} = 1$ and 3 Efimov states (see Tables I and II). For $f = 2$, however, a total of five nearby resonances should be observable (Supplemental Material [56]).

While inelastic collisions determine the stability of condensates, three-body *elastic* processes determine how different spins interact and can impact the many-body behavior of the system via the Efimov resonances described above. Within the mean-field description of spinor condensates [1], such effects can be incorporated through the *three-body* scattering length operator

$$\hat{A}_{3\mathrm{b}} = \sum_{\substack{F_{3\mathrm{b}} M_{F_{3\mathrm{b}}} \\ F_{2\mathrm{b}}}} a_{3\mathrm{b}}^{(F_{3\mathrm{b}})} |F_{3\mathrm{b}} M_{F_{3\mathrm{b}}}(F_{2\mathrm{b}})\rangle\langle F_{3\mathrm{b}} M_{F_{3\mathrm{b}}}(F_{2\mathrm{b}})|. \quad (7)$$

This is a natural extension of the three-body scattering length (units of length^4) as defined in Refs. [33,60–63]. The scattering length dependence of $a_{3\mathrm{b}}$ is listed in Table II for $f = 1$ atoms, showing the different ways it is influenced by Efimov physics. (Note that for $F_{3\mathrm{b}} = 2$, $a_{3\mathrm{b}}$ is not defined since for this case a higher centrifugal barrier suppresses collisions at ultracold energies; see Supplemental Material [56].) One interesting case emerges, for instance, when $a_0 < 0$ or $a_2 < 0$ where $a_{3\mathrm{b}}$ can display resonant effects if a_0 or a_2 are tuned near a Efimov resonance—parametrized in Table II by the tangentlike, log-periodic function

$$T^{\eta}_{s_0}(a) = \alpha + \beta \frac{2\sin(|s_0| \ln \frac{a}{r_\phi}) \cos(|s_0| \ln \frac{a}{r_\phi})}{\sin^2(|s_0| \ln \frac{a}{r_\phi}) + \sinh^2 \eta}, \quad (8)$$

where α and β are, again, universal constants. [Note that oscillations in $a_{3\mathrm{b}}$, parametrized by $O^{\eta}_{s_0}(a)$ in the Supplemental Material [56], are also allowed.] In this case $|a_{3\mathrm{b}}|^{1/4} \gg \{|a_0|, |a_2|\}$, and three-body correlations can dominate mean-field interactions, allowing for both attractive ($a_{3\mathrm{b}} < 0$) and repulsive ($a_{3\mathrm{b}} > 0$) three-body interactions.

From the mean-field perspective, in order to understand three-body contributions for spinor condensates, it is convenient to write $\hat{A}_{3b}$ in a way that makes explicit the importance of spin-exchange interactions [14,15]. For $f=1$ atoms, we can rewrite Eq. (7) as (Supplemental Material [56])

$$\hat{A}_{3b} = \alpha_{3b} + \alpha_{3b}^{ex} \sum_{i<j} \vec{f}_i \cdot \vec{f}_j, \tag{9}$$

where $\vec{f}_i$ ($i = 1$, 2, and 3) is the atomic hyperfine angular momentum for the atom i, and the three-body direct and exchange interactions are given, respectively, by

$$\alpha_{3b} = (2a_{3b}^{(3)} + 3a_{3b}^{(1)})/5, \tag{10}$$

$$\alpha_{3b}^{ex} = (a_{3b}^{(3)} - a_{3b}^{(1)})/5. \tag{11}$$

This form for $\hat{A}_{3b}$ is in close analogy to the two-body spinor case, in which $\hat{A}_{2b} = \alpha_{2b} + \alpha_{2b}^{ex}\vec{f}_1 \cdot \vec{f}_2$, where $\alpha_{2b} = (a_0 + 3a_2)/3$ and $\alpha_{2b}^{ex} = (a_2 - a_0)/3$ [14,15]. Mean-field contributions, however, are introduced through the corresponding two- and three-body coupling constants $g_{2b} = (4\pi/m)\alpha_{2b}$, $g_{2b}^{ex} = (4\pi/m)\alpha_{2b}^{ex}$, $g_{3b} = 3^{1/2}(12\pi/m)\alpha_{3b}$, and $g_{3b}^{ex} = 3^{1/2}(12\pi/m)\alpha_{3b}^{ex}$, respectively [63].

Observe that the a_0^4 (or a_2^4) dependence of g_{3b} and g_{3b}^{ex} can quickly make the three-body direct and spin-exchange mean-field energies $g_{3b}n^2$ and $g_{3b}^{ex}n^2$ comparable to their two-body counterparts $g_{2b}n$ and $g_{2b}^{ex}n$. In fact, resonant effects in α_{3b}^{ex} due to Efimov states can strongly affect both ferromagnetic ($g_{2b}^{ex} < 0$) and antiferromagnetic ($g_{2b}^{ex} > 0$) phases in spinor condensates [1,14,15] whenever $|g_{3b}^{ex}|n^2 > |g_{2b}^{ex}|n$ with g_{3b}^{ex} and g_{2b}^{ex} having *opposite* signs. In particular, in the ferromagnetic phase, we speculate that attractive two-body interactions $g_{2b} < 0$ and $g_{2b}^{ex} < 0$ can be stabilized by a repulsive three-body interaction ($g_{3b} > 0$ and $g_{3b}^{ex} > 0$) to form local, self-bound, quantum droplets of spinor characters in a spirit similar to that of Ref. [63]. The study of mean-field three-body contributions and their possible effects in spinor condensates, however, will be a subject of future investigations.

Finally, our present study has also shown the possibility of creating atom-dimer spinor condensates where $F_{2b} \neq 0$ dimers can exchange $M_{F_{2b}}$ by colliding with other atoms in $|m_f\rangle$ states. For instance, for $f=1$ atoms, Fig. 1(b) shows $F_{2b} = 2$ dimers can collide with atoms in states $|-1\rangle$, $|0\rangle$, and $|1\rangle$, and their collisional properties are listed in Table SI of the Supplemental Material [56]. Similar to atomic spinor condensates, in the mean-field approximation the relevant parameter for this atom-dimer mixture is the elastic atom-dimer scattering length matrix $\hat{A}_{ad}$ whose elements a_{ad} are listed in Table SI (Supplemental Material [56]). As one can see, Efimov resonances can also strongly affect the mean-field energy. Moreover, if both $a_0 > 0$ and $a_2 > 0$ are large, both $F_{2b} = 0$ and 2 dimers can remain trapped, leading to an interesting regime where reactive scattering can affect the dynamics of the system (Supplemental Material [56]).

In summary, we have explored universal aspects of Efimov physics in spinor systems and found a rich variety of scattering phenomena that strongly affect the spin dynamics in strongly correlated spinor condensates. The multiple, coexisting families of Efimov states characteristic of spinor systems can lead to nontrivial spin dynamics, dominated by three-body correlations, as well as allowing for the existence of ultra-long-lived Efimov states. We also study few-body aspects of atom-dimer spinor condensates and show that these can offer novel regimes for studying spinlike physics.

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