Reheating Constraints to Inflationary Models

Liang Dai, Marc Kamionkowski, and Junpu Wang
Department of Physics and Astronomy, Johns Hopkins University, 3400 North Charles Street, Baltimore, Maryland 21218, USA
(Received 28 April 2014; published 23 July 2014)

Evidence from the BICEP2 experiment for a significant gravitational-wave background has focused attention on inflaton potentials $V(\phi) \propto \phi^{\alpha}$ with $\alpha = 2$ ("chaotic" or "$m^2\phi^2$" inflation) or with smaller values of α, as may arise in axion-monodromy models. Here we show that reheating considerations may provide additional constraints to these models. The reheating phase preceding the radiation era is modeled by an effective equation-of-state parameter $w_{\rm re}$. The canonical reheating scenario is then described by $w_{\rm re} = 0$. The simplest $\alpha = 2$ models are consistent with $w_{\rm re} = 0$ for values of n_s well within the current 1σ range. Models with $\alpha = 1$ or $\alpha = 2/3$ require a more exotic reheating phase, with $-1/3 < w_{\rm re} < 0$, unless n_s falls above the current 1σ range. Likewise, models with $\alpha = 4$ require a physically implausible $w_{\rm re} > 1/3$, unless n_s is close to the lower limit of the 2σ range. For $m^2\phi^2$ inflation and canonical reheating as a benchmark, we derive a relation $\log_{10}(T_{\rm re}/10^6\ {\rm GeV}) \simeq 2000(n_s - 0.96)$ between the reheat temperature $T_{\rm re}$ and the scalar spectral index n_s. Thus, if n_s is close to its central value, then $T_{\rm re} \lesssim 10^6$ GeV, just above the electroweak scale. If the reheat temperature is higher, as many theorists may prefer, then the scalar spectral index should be closer to $n_s \simeq 0.965$ (at the pivot scale $k = 0.05\ {\rm Mpc}^{-1}$), near the upper limit of the 1σ error range. Improved precision in the measurement of n_s should allow $m^2\phi^2$, axion monodromy, and ϕ^4 models to be distinguished, even without precise measurement of r, and to test the $m^2\phi^2$ expectation of $n_s \simeq 0.965$.

DOI: 10.1103/PhysRevLett.113.041302 PACS numbers: 98.80.Cq

Introduction.—The imprint of inflationary gravitational waves in the cosmic microwave background polarization [1] reported by the BICEP2 Collaboration [2] implies, if confirmed, that the inflaton field ϕ traversed a distance large compared with the Planck mass during inflation [3,4]. One particularly simple and elegant model for large-field inflation is "$m^2\phi^2$" inflation [5,6] (derived originally as a simple example of chaotic inflation [7]), in which the inflaton potential is simply a quadratic function of ϕ. Reference [8] recently argued that this is perhaps the simplest and most elegant model. They then derived a consistency relation between the scalar spectral index (now constrained to be $n_s - 1 = -0.0397 \pm 0.0073$ [9]) and the tensor-to-scalar ratio (roughly $r \sim 0.2$ according to Ref. [2]) that can be tested with higher-precision measurements of n_s and, in particular, of r. Another promising candidate large-field model, axion monodromy, which suggests a potential $V \propto \phi$ [10] or $V \propto \phi^{2/3}$ [11], has also been receiving considerable attention. We parametrize all of these models by a power-law potential $V \propto \phi^{\alpha}$.

Here we point out that consideration of the process by which the Universe reheats may provide additional constraints to these models [12–16]. After inflation ends, there must be a period of reheating (see Ref. [17] for a review) when the energy stored in the inflaton field is converted to a plasma of relativistic particles after which the standard radiation-dominated evolution of the early Universe takes over. Although the physics of reheating is highly uncertain and unconstrained, there is a simple canonical scenario [18] whereby the cold gas of inflaton particles that arise from coherent oscillation of the inflaton field about the minimum of a quadratic potential decay to relativistic particles. This scenario implies a reheating era that lasts for a time $\sim\Gamma^{-1}$, where Γ is the inflaton-decay rate, and in which the effective equation-of-state parameter (in which the energy density scales with scale factor a as $\rho \propto a^{-3(1+w_{\rm re})}$) is $w_{\rm re} = 0$. The radiation-dominated era is then initiated at a temperature $T_{\rm re} \sim (\Gamma M_{\rm pl})^{1/2}$. Still, there are more complicated possibilities. For example, resonant [19,20] or tachyonic [21] instabilities can lead to a short preheating phase of rapid and violent dissipations by exciting inhomogeneous modes. After preheating, inhomogeneous modes of the inflaton or its decay products could become turbulent [22] and eventually evolve to a state of equilibrium. Numerical studies of this thermalization phase suggest a range of variation $0 \lesssim w_{\rm re} \lesssim 0.25$ [23]. The bottom line, though, is that $w_{\rm re} > -1/3$ is needed to end inflation, but $w_{\rm re} > 1/3$ is difficult to conceive since it requires a potential dominated by high-dimension operators (higher than ϕ^6) near its minimum, unnatural from a quantum-field-theoretical point of view.

In this Letter, we show that current measurements of n_s seem to favor $m^2\phi^2$ inflation over axion-monodromy inflation. If n_s is within its current 1σ error range, then axion-monodromy models require an extended phase of reheating involving exotic physics with $w_{\rm re} < 0$. Axion monodromy is consistent with canonical reheating only if n_s is above the current 1σ range. Moreover, if $m^2\phi^2$

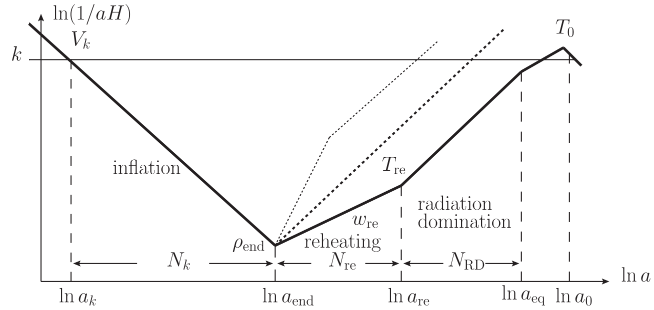

FIG. 1. The evolution of the comoving Hubble scale $1/aH$. The reheating phase connects the inflationary phase and the radiation era. Compared to instantaneous reheating (thick dotted curve), a reheating equation-of-state parameter $w_{\rm re} < 1/3$ implies more postinflationary e-folds of expansion. Fewer postinflationary e-folds requires $w_{\rm re} > 1/3$ (thin dotted curve).

inflation occurred and was followed by canonical reheating, then $n_s = 0.96$ (its central value) implies a reheat temperature just above the electroweak scale. If the reheat temperature was considerably higher, as may be required to accommodate models that explain the baryon asymmetry, then $m^2\phi^2$ inflation (with a high reheat temperature) predicts a value $n_s \simeq 0.965$, at the high end of the currently allowed 1σ range, and a prediction that may be testable with future cosmic microwave background (CMB) data and galaxy surveys. As we will see below, these conclusions are robust to the current order-unity uncertainty in r.

We start by sketching the cosmic expansion history in Fig. 1. At early times, the inflaton field ϕ drives the quasi–de Sitter phase for N_k e-folds of expansion. The comoving horizon scale decreases as $\sim a^{-1}$. The reheating phase begins once the accelerated expansion comes to an end and the comoving horizon starts to increase. After another $N_{\rm re}$ e-folds of expansion, the energy in the inflaton field has been completely dissipated into a hot plasma with a reheating temperature $T_{\rm re}$. Beyond that point, the Universe expands under radiation domination for another $N_{\rm RD}$ e-folds, before it finally makes a transition to matter domination.

It is clear from Fig. 1 that the number of e-folds between the time that the current comoving horizon scale exited the horizon during inflation and the end of inflation must be related to the number of e-folds between the end of inflation and today if the dependence of $(aH)^{-1}$ on a during reheating is known. The expansion history also allows us to trace the dilution of the energy density in the Universe. To match the energy density during inflation, as fixed by r, to the energy density today, a second relation must be satisfied. These two matching conditions, for scale and for energy density, respectively, underly the arguments that follow.

Quantitative analysis.—We consider power-law potentials

$$V(\phi) = \frac{1}{2} m^{4-\alpha}\phi^{\alpha}, \tag{1}$$

for the inflaton, with power-law index α and mass parameter m. From the attractor evolution of the inflaton field $3H\dot{\phi} + V_{,\phi} \simeq 0$, one can determine the number

$$N = \int_{\phi}^{\phi_{\rm end}} \frac{H d\phi}{\dot{\phi}} \simeq \frac{\phi^2 - \phi_{\rm end}^2}{2\alpha M_{\rm pl}^2} \simeq \frac{\phi^2}{2\alpha M_{\rm pl}^2} \tag{2}$$

of e-folds from the time that the field value is ϕ until the end of inflation. Note that the field value at the end of inflation $\phi_{\rm end}$ is small compared to that during slow roll. The conventional slow-roll parameters are then given by

$$\epsilon = \alpha/(4N) \quad \text{and} \quad \eta = (\alpha - 1)/(2N). \tag{3}$$

For power-law potentials, the scalar spectral tilt $n_s - 1$ and the tensor-to-scalar ratio r are inversely proportional to the number of e-folds,

$$n_s - 1 = -(2+\alpha)/(2N), \qquad r = 4\alpha/N. \tag{4}$$

Simultaneous measurements of $n_s - 1$ and r with high precision, in principle, pin down both N and α. However, given the current uncertainty in r, we treat α as a model input and use $n_s - 1$ to infer *both* N and r. As we shall see, the precise value of r does not affect our results.

In cosmology we observe perturbation modes on scales that are comparable to that of the horizon. For example, the pivot scale at which Planck determines n_s lies at $k = 0.05\ \mathrm{Mpc}^{-1}$. The comoving Hubble scale $a_k H_k = k$ when this mode exited the horizon can be related to that of the present time:

$$\frac{k}{a_0 H_0} = \frac{a_k}{a_{\rm end}} \frac{a_{\rm end}}{a_{\rm re}} \frac{a_{\rm re}}{a_{\rm eq}} \frac{a_{\rm eq} H_{\rm eq}}{a_0 H_0} \frac{H_k}{H_{\rm eq}}. \tag{5}$$

Here quantities with subscript k are evaluated at the time of horizon exit. Similar subscripts refer to other epochs, including the end of inflation (end), reheating (re), radiaton-matter equality (eq), and the present time (0). Using $e^{N_k} = a_{\rm end}/a_k$, $e^{N_{\rm re}} = a_{\rm re}/a_{\rm end}$, and $e^{N_{\rm RD}} = a_{\rm eq}/a_{\rm re}$, we obtain a constraint on the total amount of expansion [24]:

$$\ln \frac{k}{a_0 H_0} = -N_k - N_{\rm re} - N_{\rm RD} + \ln \frac{a_{\rm eq} H_{\rm eq}}{a_0 H_0} + \ln \frac{H_k}{H_{\rm eq}}. \tag{6}$$

The Hubble parameter during inflation is given by $H_k = \pi M_{\rm pl}(rA_s)^{1/2}/\sqrt{2}$, with the primordial scalar amplitude $\ln(10^{10}A_s) = 3.089^{+0.024}_{-0.027}$ from Planck [9]. For a given power-law index α, N_k and r are determined from $n_s - 1$, and hence $\ln H_k$ is known.

In addition to Eq. (6), a second relation between the various e-folds of expansion can be derived by tracking the postinflationary evolution of the energy density and temperature. The inflaton field at the end of inflation has a value $\phi_{\rm end} = (\alpha^2 M_{\rm pl}^2/2\epsilon_0)^{1/2}$ under the estimate that inflation terminates at $\epsilon = \epsilon_0 \simeq 1$, while its value during inflation satisfies $N_k = \phi_k^2/(2\alpha M_{\rm pl}^2)$. Therefore, the final stage of the inflation phase has potential energy $V_{\rm end} = V_k(\phi_{\rm end}/\phi_k)^\alpha$, where $V_k = 3M_{\rm pl}^2 H_k^2 = (3\pi^2/2)M_{\rm pl}^4 r A_s$. The energy density is $\rho_{\rm end} = (1+\lambda)V_{\rm end}$, with the ratio $\lambda = \epsilon_0/(3-\epsilon_0)$ of kinetic energy to potential energy.

The duration,

$$N_{\rm re} = [3(1+w_{\rm re})]^{-1} \ln(\rho_{\rm end}/\rho_{\rm re}), \tag{7}$$

of reheating determines the dilution of the energy density. Here for simplicity we assume $w_{\rm re}$ is a constant. The final energy density determines the reheating temperature through $\rho_{\rm re} = (\pi^2/30) g_{\rm re} T_{\rm re}^4$, with $g_{\rm re}$ being the effective number of relativistic species upon thermalization. The subsequent expansion is mainly driven by hot radiation, except for very recently nonrelativistic matter and dark energy. Although it remains a possiblity before big bang nucleosynthesis at $z > 10^9$, for simplicity we assume that no immense entropy production takes place after $T_{\rm re}$. Under this assumption, the reheating entropy is preserved in the CMB and neutrino background today, which leads to the relation

$$g_{s,{\rm re}} T_{\rm re}^3 = \left(\frac{a_0}{a_{\rm re}}\right)^3 \left(2T_0^3 + 6 \times \frac{7}{8} T_{\nu 0}^3\right), \tag{8}$$

with the present CMB temperature $T_0 = 2.725$ K, the neutrino temperature $T_{\nu 0} = (4/11)^{1/3} T_0$, and the effective number of light species for entropy $g_{s,{\rm re}}$ at reheating. We therefore relate the reheating temperature to the present CMB temperature through

$$\frac{T_{\rm re}}{T_0} = \left(\frac{43}{11 g_{s,{\rm re}}}\right)^{1/3} \frac{a_0}{a_{\rm eq}} \frac{a_{\rm eq}}{a_{\rm re}}. \tag{9}$$

Combining Eq. (7), Eq. (9), and other relations lead to a second equation relating the various e-folds,

$$\frac{3(1+w_{\rm re})}{4} N_{\rm re} = \frac{1}{4}\ln\frac{30}{g_{\rm re}\pi^2} + \frac{1}{4}\ln\frac{\rho_{\rm end}}{T_0^4} + \frac{1}{3}\ln\frac{11 g_{s,{\rm re}}}{43} + \ln\frac{a_{\rm eq}}{a_0} - N_{\rm RD}. \tag{10}$$

We now combine Eq. (6) and Eq. (10) and

$$N_{\rm re} = \frac{4}{1-3w_{\rm re}}\left[-N_k - \ln\frac{k}{a_0 T_0} - \frac{1}{4}\ln\frac{30}{g_{\rm re}\pi^2} - \frac{1}{3}\ln\frac{11 g_{s,{\rm re}}}{43} + \frac{1}{4}\ln\frac{\pi^2 r A_s}{6} - \frac{\alpha}{8}\ln\frac{r}{16\epsilon_0} - \frac{\ln(1+\lambda)}{4}\right]. \tag{11}$$

The required duration $N_{\rm RD}$ of radiation domination and the reheating temperature $T_{\rm re}$ can then be obtained. We clarify that in Eq. (11) we compute the required value of $r = -8\alpha(n_s - 1)/(2+\alpha)$ for given α. However, the results are essentially unchanged if we simply set $r \simeq 0.2$.

It is worth noting that Eq. (11) has only logarithmic dependence on ϵ_0, $g_{\rm re}$, and $g_{s,{\rm re}}$, so it suffices to take fiducial values $\epsilon_0 = 1$ and $g_{\rm re} = g_{s,{\rm re}} = 100$. The expression is not affected by the precise values of r and A_s, as the dependence on these quantities is only logarithmic. Nevertheless, the expression depends linearly on $n_s - 1$ through N_k, and is sensitive to $w_{\rm re}$.

Numerical results.—In Fig. 2, we apply the results above to compute $N_{\rm re}$ and $T_{\rm re}$ as functions of $n_s - 1$. We study potentials with power-law indexes $\alpha = 2/3, 1, 2, 4$. Moreover, we focus on effective reheating equation-of-state parameters $w_{\rm re} \geq -1/3$ (as required if inflation has ended). As discussed above, a matterlike $w_{\rm re} = 0$ is favored for canonical reheating, but $w_{\rm re} > 1/3$ is disfavored from model building. Still, for illustration, we will show results even for $w > 1/3$.

Our results indicate that the quadratic model $\alpha = 2$ implies a prolonged reheating epoch for the central value $n_s \simeq 0.96$ and canonical reheating ($w_{\rm re} = 0$). A number $N_{\rm re} \simeq 30$ of e-folds is required in this case, and $T_{\rm re} \simeq 10^6$ GeV. A scalar tilt bluer than that, though, requires smaller $N_{\rm re}$ and allows for higher reheating temperature. For $m^2\phi^2$ inflation and canonical reheating, we approximate the numerical results by a relation $\log_{10}(T_{\rm re}/10^6\ {\rm GeV}) \simeq 2000(n_s - 0.96)$ between the reheat temperature $T_{\rm re}$ and the scalar spectral index n_s. If a reheat temperature considerably above the electroweak scale is desired, then n_s will have to be larger than its central value. For example, if reheating was nearly instantaneous and set $T_{\rm re} \simeq 10^{16}$ GeV, as may be required by grand-unification-scale baryogenesis models, then $m^2\phi^2$ inflation with canonical reheating requires $n_s \simeq 0.965$. (Note here that this n_s corresponds to the pivot scale $k = 0.05\ {\rm Mpc}^{-1}$ used by Planck. The value inferred for n_s increases to roughly $n_s \simeq 0.967$ for the WMAP pivot scale $k = 0.002\ {\rm Mpc}^{-1}$.)

For models with smaller power-law indexes (e.g., $\alpha = 2/3, 1$), canonical reheating is too efficient in diluting the energy density if n_s falls within its 1σ error range. A reheat temperature above even the BBN temperature requires $w_{\rm re} < 0$. Thus, unless n_s turns out to be above the current 1σ upper limit, axion-monodromy models require some exotic mechanism of reheating, beyond that in the canonical scenario. On the other hand, models with larger power-law indexes (e.g., $\alpha = 3, 4$) require $w_{\rm re} > 1/3$ (dilution of energy density faster than that that occurs with the radiation-dominated phase) and thus also pose a challenge for reheating models, unless n_s is near the lower limit of the current 2σ range. Our results also indicate that instantaneous reheating is disfavored by current measurements except for $\alpha = 2$–3. Together, these arguments (and the results shown in Fig. 2) tend to favor the simplest $m^2\phi^2$ models over other power-law models.

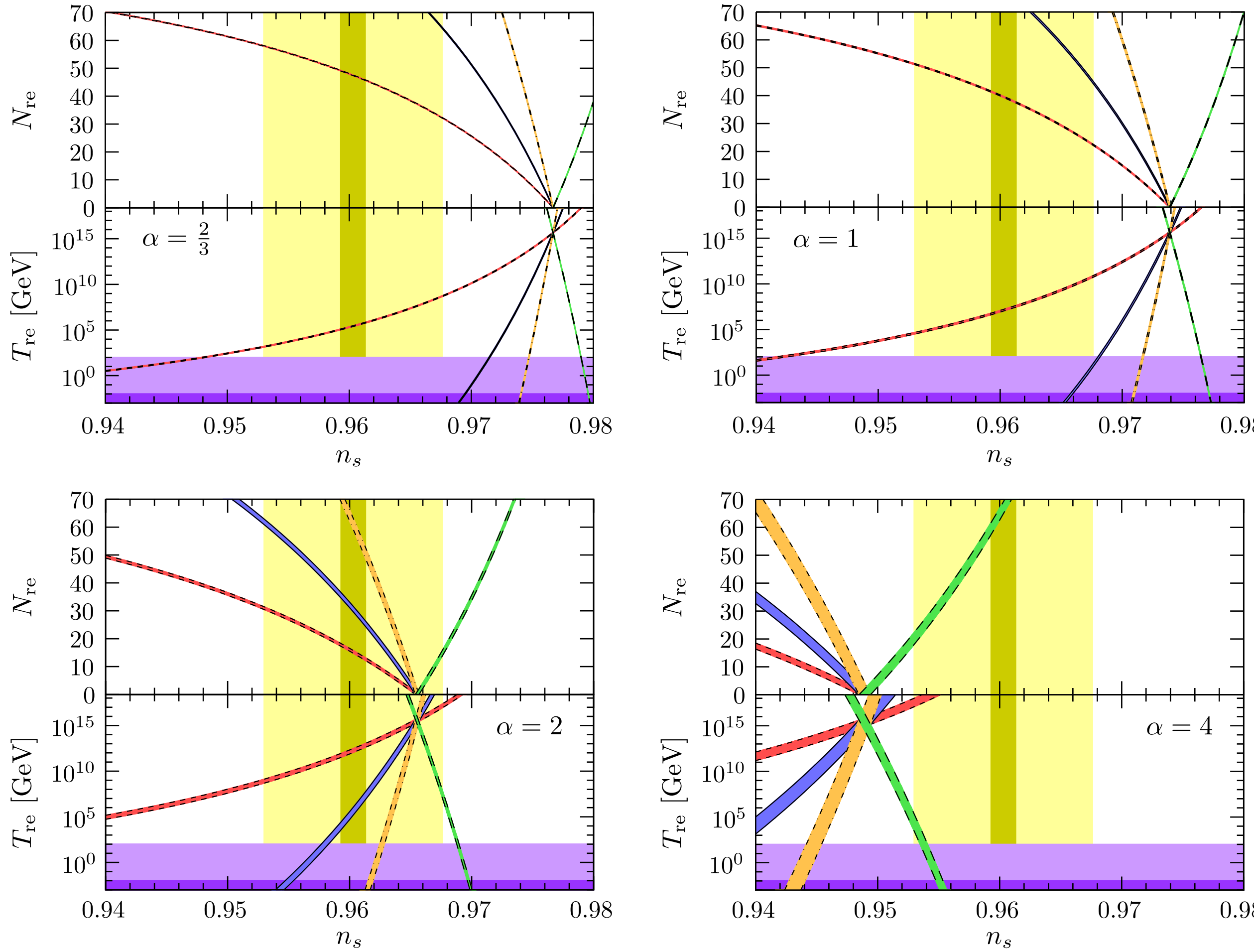

FIG. 2 (color online). We plot N_{re} (upper panels) and T_{re} (lower panels) as determined from Eq. (11) and Eq. (7), respectively. Results for power-law indexes $\alpha = 2/3, 1, 2, 4$ are each shown separately. Different effective equation-of-state parameters for reheating are considered in each case: $w_{re} = -1/3$ (red dashed curve), $w_{re} = 0$ (blue solid curve), $w_{re} = 1/6$ (orange dash-dotted curve), and $w_{re} = 2/3$ (green long-dashed curve). All curves intersect at the point where reheating occurs instantaneously. The width of each curve corresponds to a variation of the termination condition $0.1 \lesssim \epsilon_0 \lesssim 1$ and also roughly the uncertainty in r. The light purple regions are below the electroweak scale $T_{EW} \sim 100$ GeV. The dark purple regions, below 10 MeV, would ruin the predictions of big bang nucleosynthesis. Temperatures above the intersection point are unphysical as they correspond to $N_{re} < 0$. The light yellow band indicates the 1σ range $n_s - 1 = -0.0397 \pm 0.0073$ from Planck [9], and the dark yellow band assumes a projected uncertainty of 10^{-3} [8] for $n_s - 1$, as expected from future experiments (assuming the central value remains unchanged).

Recently, Ref. [8] proposed that future measurements of $n_s - 1$ and r with high precision will serve as a nontrivial consistency check of the potential shape. Their method of determining the power-law index α does not rely on good knowledge of the inflationary e-folds N_k, and is independent of the reheating physics. Here our test of the potential shape is complementary to theirs in the sense that it only requires precise determination of $n_s - 1$, and not of r.

Conclusions.—The recent BICEP2 measurement of a large tensor-to-scalar ratio r hints, if confirmed, at large-field power-law inflaton potentials. By matching the end of the inflationary epoch to the beginning of the radiation-dominated phase we can, with improving measurement of the scalar tilt, begin to make quantitative inferences about the physics of reheating. Our analysis suggests that of the power-law inflationary models, those with $\alpha \sim 2$, which includes the $m^2\phi^2$ model, are most compatible with the simplest canonical reheating scenario. Axion-monodromy models (with power-law indexes $\alpha = 1$ or $\alpha = 2/3$) require something more exotic in the way of reheating physics, unless n_s falls above its current 1σ range. Models with $\alpha = 4$, on the other hand, are also disfavored for the 1σ range for n_s. While the statistical significance is not yet conclusive, it is intriguing that the current data do seem to favor a simple quadratic inflaton potential if a simple reheating scenario is assumed. Future more precise measurements of n_s should help make these arguments sharper.

Although we have focused on power-law potentials, the test we propose can, in principle, be applied to other potentials, provided that $r \simeq 0.2$ already fixes the energy density during slow roll.

We have presented a definitive relation between $T_{\rm re}$ and n_s, if inflation does indeed occur via a quadratic potential and is then followed by canonical reheating. Similar relations for $w_{\rm re} \neq 0$ can be readoff from Fig. 2. If, moreover, the reheat temperature is considerably above the electroweak scale, then the central value of n_s should, with more precise measurements, veer upward in value, close to $n_s = 0.965$ as the reheat temperature approaches the grand-unification scale. Fortunately, a precision of $\sim 10^{-3}$ in the value of n_s should eventually be achieved with future experiments such as EUCLID [25] and PRISM [26], and with cosmic 21-cm surveys [27,28]. In case high precision in n_s cannot be achieved soon, one can instead use an r measured to a similar level of precision for the same test.

Finally, laser interferometry experiments [29] are proposed to detect the inflationary gravitational-wave spectrum on solar-system scales, some 40 e-folds below the CMB scales [30]. These gravitational waves reenter the horizon during reheating if $T_{\rm re} < 10^4$ GeV and will thus also probe the physics of reheating [31].

The authors thank Nima Arkani-Hamed, Arthur Kosowsky, David Kaplan, and Jared Kaplan for useful discussions, and Donghui Jeong, Marko Simonovic, and Julian Munoz for useful comments on an earlier draft. This work was supported by the John Templeton Foundation and NSF Grants No. PHY-1316665 and No. PHY-1214000.

[1] M. Kamionkowski, A. Kosowsky, and A. Stebbins, Phys. Rev. Lett. **78**, 2058 (1997); U. Seljak and M. Zaldarriaga, Phys. Rev. Lett. **78**, 2054 (1997).
[2] P. A. R. Ade *et al.* (BICEP2 Collaboration), Phys. Rev. Lett. **112**, 241101 (2014).
[3] M. S. Turner, Phys. Rev. D **48**, 5539 (1993).
[4] D. H. Lyth, Phys. Rev. Lett. **78**, 1861 (1997).
[5] T. Piran and R. M. Williams, Phys. Lett. **163B**, 331 (1985).
[6] V. A. Belinsky, I. M. Khalatnikov, L. P. Grishchuk, and Y. B. Zeldovich, Phys. Lett. **155B**, 232 (1985).
[7] A. D. Linde, Phys. Lett. **129B**, 177 (1983).
[8] P. Creminelli, D. Lp. Nacir, M. Simonovi, G. Trevisan, and M. Zaldarriaga, arXiv:1404.1065.
[9] P. A. R. Ade *et al.* (Planck Collaboration), arXiv:1303.5082.
[10] L. McAllister, E. Silverstein, and A. Westphal, Phys. Rev. D **82**, 046003 (2010).
[11] E. Silverstein and A. Westphal, Phys. Rev. D **78**, 106003 (2008).
[12] J. Martin and C. Ringeval, Phys. Rev. D **82**, 023511 (2010).
[13] P. Adshead, R. Easther, J. Pritchard, and A. Loeb, J. Cosmol. Astropart. Phys. 02 (2011) 021.
[14] J. Mielczarek, Phys. Rev. D **83**, 023502 (2011).
[15] S. Dodelson and L. Hui, Phys. Rev. Lett. **91**, 131301 (2003).
[16] R. Easther and H. V. Peiris, Phys. Rev. D **85**, 103533 (2012).
[17] R. Allahverdi, R. Brandenberger, F.-Y. Cyr-Racine, and A. Mazumdar, Annu. Rev. Nucl. Part. Sci. **60**, 27 (2010).
[18] L. F. Abbott, E. Farhi, and M. B. Wise, Phys. Lett. **117B**, 29 (1982); A. D. Dolgov and A. D. Linde, Phys. Lett. **116B**, 329 (1982); A. J. Albrecht, P. J. Steinhardt, M. S. Turner, and F. Wilczek, Phys. Rev. Lett. **48**, 1437 (1982).
[19] L. Kofman, A. D. Linde, and A. A. Starobinsky, Phys. Rev. Lett. **73**, 3195 (1994).
[20] L. Kofman, A. D. Linde, and A. A. Starobinsky, Phys. Rev. D **56**, 3258 (1997).
[21] B. R. Greene, T. Prokopec, and T. G. Roos, Phys. Rev. D **56**, 6484 (1997).
[22] R. Micha and I. I. Tkachev, Phys. Rev. Lett. **90**, 121301 (2003); Phys. Rev. D **70**, 043538 (2004).
[23] D. I. Podolsky, G. N. Felder, L. Kofman, and M. Peloso, Phys. Rev. D **73**, 023501 (2006).
[24] A. R. Liddle and S. M. Leach, Phys. Rev. D **68**, 103503 (2003).
[25] L. Amendola *et al.* (Euclid Theory Working Group Collaboration), Living Rev. Relativity **16**, 6 (2013).
[26] P. Andre *et al.* (PRISM Collaboration), arXiv:1306.2259.
[27] Y. Mao, M. Tegmark, M. McQuinn, M. Zaldarriaga, and O. Zahn, Phys. Rev. D **78**, 023529 (2008).
[28] A. Kosowsky, Physics **3**, 103 (2010).
[29] E. S. Phinney *et al.*, "The Big Bang Observer: Direct detection of gravitational waves from the birth of the Universe to the present," NASA Mission Concept Study, 2004; S. Kawamura, T. Nakamura, M. Ando, N. Seto, K. Tsubono, K. Numata, R. Takahashi, S. Nagano *et al.*, Classical Quantum Gravity **23**, S125 (2006).
[30] A. R. Liddle, Phys. Rev. D **49**, 3805 (1994); **51**, 4603(E) (1995); R. Bar-Kana, Phys. Rev. D **50**, 1157 (1994); M. S. Turner, Phys. Rev. D **55**, R435 (1997); T. L. Smith, M. Kamionkowski, and A. Cooray, Phys. Rev. D **73**, 023504 (2006); S. Chongchitnan and G. Efstathiou, Phys. Rev. D **73**, 083511 (2006); T. L. Smith, M. Kamionkowski, and A. Cooray, Phys. Rev. D **78**, 083525 (2008).
[31] R. Easther and E. A. Lim, J. Cosmol. Astropart. Phys. 04 (2006) 010; J. Garcia-Bellido, D. G. Figueroa, and A. Sastre, Phys. Rev. D **77**, 043517 (2008); J. Garcia-Bellido and D. G. Figueroa, Phys. Rev. Lett. **98**, 061302 (2007).