Negative Refraction and Planar Focusing Based on Parity-Time Symmetric Metasurfaces

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We introduce a new mechanism to realize negative refraction and planar focusing using a pair of parity-time symmetric metasurfaces. In contrast to existing solutions that achieve these effects with negative-index metamaterials or phase conjugating surfaces, the proposed parity-time symmetric lens enables loss-free, all-angle negative refraction and planar focusing in free space, without relying on bulk metamaterials or nonlinear effects. This concept may represent a pivotal step towards loss-free negative refraction and highly efficient planar focusing by exploiting the largely uncharted scattering properties of parity-time symmetric systems.

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According to Snell's law of refraction, a consequence of the Huygens-Fermat principle [1], a beam of light hitting the interface between two homogeneous media refracts at an angle related to the ratio between the refractive indices of the two media. Because every known natural material has a positive index, refraction usually occurs in the same direction. Refraction in the negative direction requires one of the two media to have a negative index, a peculiarity that can be observed in artificial electromagnetic materials, or metamaterials, which are engineered to possess simultaneously negative values of the permittivity ε and permeability μ [2–4]. Negative refraction allows us to manipulate electromagnetic waves in new ways, opening exciting venues in a variety of application fields such as antenna technology, electromagnetic absorbers, phase compensation, subwavelength photolithography, and planar focusing lenses. In particular, a negative bending of light is the key to realize a *perfect lens*, a planar device capable of focusing all the spatial Fourier components of a source, realizing a perfect image with, in principle, infinite resolution [5–9]. Figure 1(a) shows a sketch of how light rays emerging from a source can focus on the other side of a negative-index slab after going through negative refraction at the two interfaces, a concept originally introduced in [2] and extended to the evanescent spectrum in [5].

The practical implementation of negative refraction using a bulk double-negative (DNG) metamaterial slab, however, has inherent challenges that severely hinder its applicability. The required electromagnetic properties are in fact typically obtained by exploiting the resonant response of subwavelength inclusions, whose dispersion is fundamentally associated with undesired material losses, a result of Kramers-Kronig relations, which hold for any linear, passive, and causal medium [10]. Loss, finite granularity, and nonideal isotropy of metamaterials severely affect the ultimate resolution of these devices [11,12].

For these reasons, scientists have been looking for alternatives to the use of bulk metamaterials to bend light in the negative direction. In [13] it was demonstrated that the same functionality of a bulk negative-index slab, and focusing of both propagating and evanescent waves, can be achieved by using a pair of identical phase conjugating surfaces. This concept can be implemented at microwaves using parametric or nonlinear wave mixing surfaces [14–17], and in optics with four-wave mixing using two highly nonlinear optical films [18–20]. Phase conjugation on the two surfaces takes the role of the two interfaces of an ideal bulk metamaterial with negative index of refraction, and the ray picture of Fig. 1(a) still holds if one replaces the negative index slab with such a metasurface pair. For this to work, however, each metasurface is required to parametrically amplify the conjugate signals at a level much larger than the impinging signal, with stringent requirements on conversion efficiency that fundamentally limit the overall resolution of this system. Also in this case, inherent loss and imperfections can drastically limit these nonlinear effects in practical scenarios.

In this Letter, we propose a different approach to achieving negative refraction and planar focusing. Rather

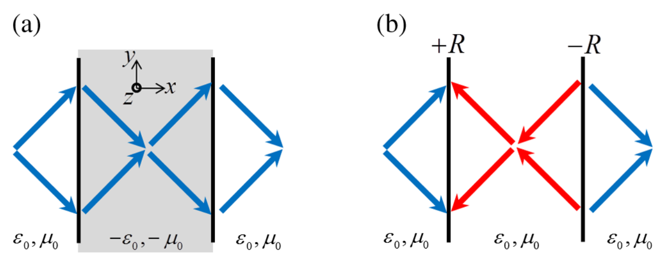

FIG. 1 (color online). (a) Conventional negative refraction in a passive DNG medium, for light rays emitted by a source placed on the left side of the slab. The power flows away from the source, and the phase velocity in the slab is backward. (b) Negative refraction using PT-symmetric metasurfaces with real surface impedances $+R$ and $-R$. In this active scenario, both power flow and the phase velocity are directed from the active surface to the passive one, and negative refraction is obtained without the need for a bulk metamaterial.

than relying on conjugating the electromagnetic fields at the two planar interfaces, as in Fig. 1(a), we exploit the anomalous scattering properties of parity-time (PT) symmetric systems [21–29]. Scattering from PT-symmetric optical structures has been mainly studied for the possibility of inducing unidirectional invisibility [25–29]. Here, we demonstrate their potential to achieve loss-immune, metamaterial-free, and fully linear, negative refraction. In conventional planar focusing using a DNG slab [Fig. 1(a)] negative refraction requires a flip of the longitudinal component of the wave vector in the slab, essentially resulting in a phase velocity distribution consistent with PT symmetry. We heuristically conjecture, therefore, that negative refraction may occur in a PT-symmetric metasurface configuration, as represented in Fig. 1(b). Here, two metasurfaces separated by a distance d in vacuum are characterized by a PT-symmetric impedance distribution $Z_{\mathrm{left}} = -Z^*_{\mathrm{right}}$, where $*$ indicates conjugation. In this configuration an outside field distribution similar to Fig. 1(a) may be induced with similar backward phase flow between the surfaces, but also with a backward power distribution, flowing from the second surface to the first one, as represented by the red arrows. If the second surface is active, it may indeed sustain an outward Poynting vector distribution around it, while the first surface acts as a power sink. The simplest possible PT-symmetric metasurface pair that may support this functionality is a couple of metasurfaces with opposite resistivity, $+R$ on the source side, and $-R$ on the image side, as shown in the figure and assumed in the following analysis.

The scattering matrix elements S_{ij} of the system in Fig. 1(b) can be calculated using the two-port transmission-line network model shown in Fig. 2(a). Two parallel lumped resistors are separated by a portion of transmission line with characteristic impedance Z_0 and length $x = \beta d$, β being the line propagation constant. The resistors have opposite values $\pm R$, and we introduce the dimensionless non-Hermiticity parameter $r = R/Z_0$. The outside medium is also assumed to have characteristic impedance Z_0. If we assume TE incident polarization at an arbitrary angle θ, the wave impedance is $Z_0 = \eta_0/\cos\theta$, and the propagation constant is $\beta = k_0\cos\theta$, where η_0 is the characteristic impedance of free space and $k_0 = \omega\sqrt{\mu_0\varepsilon_0}$. Similar considerations apply to TM incidence, using $Z_0 = \eta_0\cos\theta$. Using the transmission matrix formalism [30], and assuming time harmonic fields $e^{-i\omega t}$, we obtain the scattering parameters

$$S_{11} = \frac{(2r-1)\sin(x)}{\sin x - 2ir^2 e^{-ix}}, \tag{1}$$

$$S_{22} = -\frac{(2r+1)\sin(x)}{\sin x - 2ir^2 e^{-ix}}, \tag{2}$$

$$S_{12} = S_{21} = -\frac{2ir^2}{\sin x - 2ir^2 e^{-ix}}. \tag{3}$$

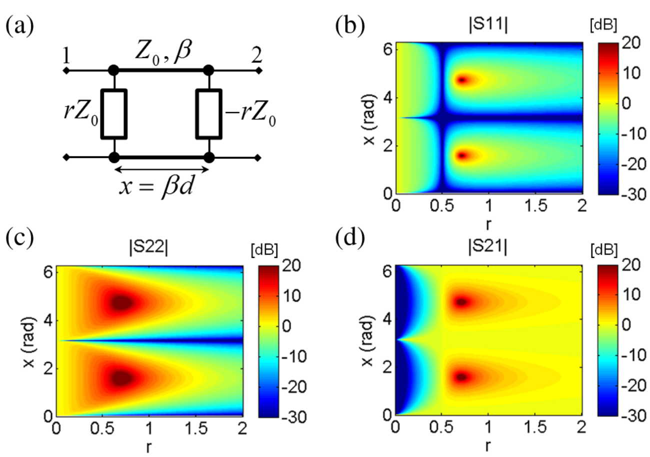

FIG. 2 (color online). (a) Equivalent circuit for the geometry of Fig. 1(b). We study the two-port PT-symmetric system composed of two parallel lumped resistors $+R = rZ_0$ and $-R = -rZ_0$ separated by a portion of transmission line of impedance Z_0, and electrical length x. (b)–(d) Magnitude of S_{11} (b), S_{22} (c), and $S_{21} = S_{12}$ (d) as a function of x and r.

The magnitude of the scattering parameters (in dB) is shown in Figs. 2(b)–2(d), as a function of the variables x and r. Figure 2(b) shows the magnitude of the reflection coefficient from port 1 (the $+R$ side). For particular values of x and r, we obtain spectral singularities for which the magnitude of the reflection coefficient is infinite, for instance, around $r = 0.7$ and $x = \pi/2$, as typical for non-Hermitian systems [31,32]. Remarkably, if one picks $r = 0.5$, corresponding to a resistance being half the background line impedance, the reflection coefficient identically vanishes, regardless of the separation distance x. Under the same condition, Fig. 2(d) shows that the transmittance to port 2 $|S_{21}|$ is unity. The system is essentially always impedance matched from port 1. When excited from port 2, however, the reflection is nonzero, as seen in Fig. 2(c), while the transmission is again unitary, due to reciprocity. Such a unidirectional reflectionless response, with strong reflection asymmetry, is typical of PT-symmetric structures [25]. However, the proposed system, under the condition $r = 0.5$, is not strictly speaking unidirectional invisible, but instead displays another exotic property that is particularly appealing for the purpose of negative refraction. To see this, we evaluate the scattering matrix elements in Eqs. (1)–(3) for $r = 0.5$ and arbitrary x:

$$S = \begin{pmatrix} 0 & e^{-ix} \\ e^{-ix} & 2(e^{-2ix} - 1) \end{pmatrix}. \tag{4}$$

This unidirectional reflectionless system possesses the fascinating property that the transmitted wave undergoes a *phase advance* $-x$ that is exactly opposite to the one that it would have without the PT-symmetric metasurface pair. This property implies a negative phase velocity between surfaces, in complete analogy to the case of the Veselago lens [2,5], confirming the potential of this structure for negative refraction and planar focusing.

To gain physical insight into this exotic phenomenon, we performed full-wave simulations for an incident electromagnetic field on this PT-symmetric metasurface pair. Figure 3(a) shows a snapshot in time of the transverse component of the electric field E_z in the case of a plane wave normally incident from the left on a pair of metasurfaces with $R = \pm 0.5\eta_0$. The black arrows in the figure represent the average Poynting vector. The planar wave fronts and parallel Poynting vector contours are fully restored at port 2, indicating that the pair of metasurfaces is indeed transparent to electromagnetic waves. As predicted by our theoretical considerations, the incident plane wave is not reflected at all, and it is totally transmitted through the structure despite the presence of resistive losses at the first interface, which are fully compensated in this PT-symmetric scenario. Remarkably, the wave between surfaces is propagating in the direction opposite to that of the incident wave, with a power flow sustained by the active $-R$ element and feeding the resistive one. The phase velocity is also reversed in the space between metasurfaces, obeying a PT-symmetric distribution, and providing a phase advance to the transmitted wave. This is visually evident looking at animation S1 in the Supplemental Material [33], which shows the time-harmonic evolution for this example.

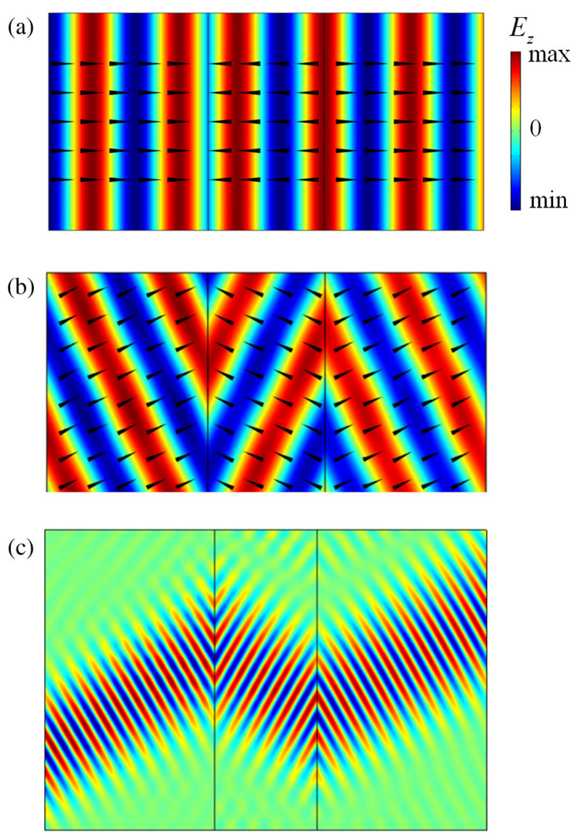

FIG. 3 (color online). PT-based negative refraction for a TE polarized field for (a) a normally incident plane wave, (b) an obliquely incident plane wave, and (c) an obliquely incident Gaussian beam. The black arrows represent the average Poynting vector field, demonstrating the behavior described in Fig. 1(b). See also the corresponding time-harmonic animations in [33].

For oblique incidence, shown in Fig. 3(b), the surface impedances are set to the constant value $R = \pm 0.5\eta_0/\cos\theta$, for $\theta = 25°$. Here, negative refraction is clearly observed between the interfaces, with reversed longitudinal propagation, and power transfer from the active surface to the passive one. Note that this phenomenon is significantly different from previous approaches to negative refraction, as the power in the middle region also flows backward, parallel to the phase velocity, fed by the active surface. Animation S2 [33] shows the time-harmonic evolution for this scenario. It is remarkable that, although the gain interface by itself would produce an infinite reflection coefficient (being the parallel combination of $R = -Z_0/2$ and Z_0, equal to $-Z_0$), the combined gain-loss system of Fig. 3 is inherently stable. This can be verified by calculating the input impedances at both sides of the system using transmission-line theory, which are different from $-Z_0$ for any x.

The stability of the system and the infinite reflection coefficient for waves propagating towards the gain surface enforce the existence of only a backward wave between surfaces when the system is illuminated from the left, as in Fig. 3. This wave is coherently synchronized to have the same amplitude and phase as the incident signal as it reaches the passive (left) surface, because this is the only condition under which the left surface can produce zero reflection and total absorption, ensuring no forward wave between the surfaces. At the same time, since the gain (right) surface radiates symmetrically on both sides, the incident wave is perfectly restored at the output. The overall effect is essentially based on the pairing of a coherently lasing metasurface, synchronized to the impinging wave, and a perfectly coherent absorbing metasurface, one being the time reversal of the other. Its stability may be seen as a particular case of lasing death via asymmetric gain [34].

The PT-symmetric condition on the metasurface resistances depends on the incidence angle, but with a relatively weak cosinusoidal variation. Therefore, a homogeneous metasurface pair may support negative refraction also in the case of a Gaussian beam excitation with finite waist, as shown in Fig. 3(c), and in animation S3 [33]. The beam is indeed negatively bent by the PT-symmetric pair, without the need of metamaterial effects or strongly nonlinear response. Unlike previous methods, this concept is immune from all loss-related issues that inherently characterize passive metamaterials, since the active metasurface exactly compensates the intrinsic loss of the passive one, and it is free from conversion efficiency and pumping issues typical of nonlinear wave-mixing schemes. Besides, the metasurfaces employed here are less challenging to realize than highly nonlinear metasurfaces required for phase conjugation. They can be realized at radio frequencies using arrays of dipole antennas loaded with complementary positive and negative resistors, readily obtained at microwaves with tunnel diodes or other semiconductor devices. It should be also stressed that this effect can be inherently broadband, since it does not require any reactive element in the metasurface, which usually restrict the bandwidth.

While the proposed metasurface pair may support partial focusing due to its weak angular dispersion, for ideal planar focusing all-angle negative refraction is required. In this case, the required surface impedance $R = \pm 0.5\eta_0/\cos\theta$ should become nonlocal, as it depends on the incidence angle. This may be realized by properly interconnecting the above-mentioned dipoles, or with other suitable spatial dispersion engineering strategies [35]. An alternative and more convenient way to realize ideal focusing, tailored for a specific location of the focal point, consists of letting the surface impedances Z_{left} and Z_{right} be dependent on the transverse coordinate y. Let us assume that the field $E_z(y)$ on the source plane is known, and it is placed at a distance L from the passive metasurface, on the left side. After expanding it in plane waves with complex amplitude $\tilde{E}_z(k_y)$, we can calculate the field everywhere inside and outside the metasurface pair based on the previous analysis. An ideal negative-index planar lens is required to avoid reflections, to compensate the phase of the propagating portion of the spectrum on the image plane, and to similarly compensate the decay of the evanescent portion of the spectrum [5]. By enforcing these requirements, we can find the required condition on the two surface impedances to be able to sustain this field distribution:

$$Z_{\mathrm{left}}(y) = i\frac{\eta_0}{2}\frac{\int_{-\infty}^{+\infty} dk_y \tilde{E}_z(k_y) e^{ik_y y} e^{\sqrt{k_y^2-k_0^2}L}}{\int_{-\infty}^{+\infty} dk_y \sqrt{\frac{k_y^2}{k_0^2}-1}\,\tilde{E}_z(k_y) e^{ik_y y} e^{\sqrt{k_y^2-k_0^2}L}}, \quad (5)$$

$$Z_{\mathrm{right}}(y) = -i\frac{\eta_0}{2}\frac{\int_{-\infty}^{+\infty} dk_y \tilde{E}_z(k_y) e^{ik_y y} e^{\sqrt{k_y^2-k_0^2}(L-d)}}{\int_{-\infty}^{+\infty} dk_y \sqrt{\frac{k_y^2}{k_0^2}-1}\,\tilde{E}_z(k_y) e^{ik_y y} e^{\sqrt{k_y^2-k_0^2}(L-d)}}. \quad (6)$$

These general formulas provide the surface impedances required to reconstruct $E_z(y)$, with infinite resolution, at a distance $d - L$ from the active metasurface on the right side. In the above expressions, the square roots have positive imaginary parts.

If the source is a plane wave incident at an angle θ, $\tilde{E}_z(k_y) = \delta(k_y - k_0 \sin\theta)$, and we obtain $Z_{\mathrm{left}} = -Z^*_{\mathrm{right}} = 0.5\eta_0/\cos\theta$, as in the case studied above. We should stress that Eqs. (5), (6) do not necessarily describe PT-symmetric pairs: first, PT symmetry requires $d = 2L$, which is a necessary condition to have identical effects of the P and T operators on the field outside the lens (the focal distances on both sides are then equal). After applying this condition, Eqs. (5), (6) show that such a lens is PT symmetric, with $Z_{\mathrm{left}}(y) = -Z^*_{\mathrm{right}}(y)$, as long as we truncate the integrals to the propagating portion of the spectrum, in the range $|k_y| \le k_0$. In the general case, involving subwavelength resolution associated with the evanescent portion of the spectrum, $Z_{\mathrm{left}}(y) \ne -Z^*_{\mathrm{right}}(y)$; i.e., perfect focusing requires breaking in part the PT-symmetric nature of the device. This is consistent with the fact that evanescent fields are not time reversible, leading to non PT-symmetric field distributions.

In Fig. 4, we demonstrate the potential of a PT-symmetric metasurface pair to ideally focus the propagating portion of the spectrum of an arbitrary source. We consider two different PT-symmetric scenarios in which a pair of surfaces is excited by a line current oriented along z and placed at a distance $L = d/2 = 6\lambda_0$ on the left of the passive (left) metasurface. In panel (a), we choose a homogeneous surface pair with $R = \pm 0.5\eta_0$, as in Fig. 3(a). This pair supports ideal negative refraction for normal incidence only, but, due to the relatively weak angular dependence of the required resistance, even a local

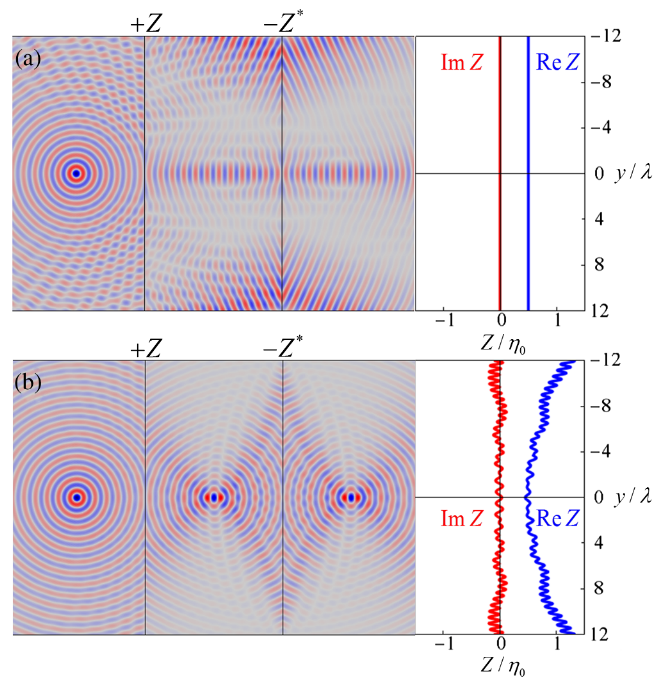

FIG. 4 (color online). Planar focusing using a pair of PT-symmetric metasurfaces for a line current source placed on the left of the structure. We consider two different cases for the surface impedance Z, shown as a function of the transverse coordinate y in the right panels. A constant surface impedance $Z = 0.5\eta_0$ focuses a point source to a spot whose transverse size is close to the wavelength (a). An inhomogeneous surface impedance focuses all propagating spatial harmonics, resulting in a spot size with a transverse size equal to $\lambda_0/2$(b).

homogeneous impedance can produce partial planar focusing, as demonstrated in the figure. We get efficient focusing of the propagating part of the spectrum with this homogeneous surface pair, with a hot spot on the image side whose transverse size is comparable to the wavelength.

In panel (b), we make the PT surface pair inhomogeneous following Eq. (5) with $\tilde{E}_z(k_y) = 1/\sqrt{k_y^2 - k_0^2}$. The integral is truncated for $|k_y| < k_0$, and the required inhomogeneous surface impedance is shown in the figure. In this case, the PT-symmetric pair ideally focuses the propagating spectrum, reaching a focus with transverse size $\lambda_0/2$. Resolutions well below the diffraction limit may be obtained by using Eqs. (5), (6) with larger spectral windows, but our results show that efficient planar focusing is already obtained in the case of homogeneous PT-symmetric metasurfaces. In [33] (animations S4 and S5) we show the time-harmonic evolution of these two cases, visually demonstrating that we can indeed realize the ray picture in Fig. 1(b) in a simple, robust, and loss-free setup.

To conclude, we have proposed a novel concept to achieve negative refraction without the need for bulk DNG metamaterials or conjugating surfaces. Our approach is completely immune to material losses, as it is based on loss-compensated PT-symmetric metasurfaces. Our theory provides efficient negative refraction and planar focusing with a simple pair of homogeneous metasurfaces with negative and positive surface resistances, which may be implemented linearly at microwave or optical frequencies, or for acoustic waves [36]. The system can be designed to be fully stable and the dispersion of gain and loss elements tailored to have a broadband response, for instance, using non-Foster elements [37]. Even though this work focuses for simplicity on 2D problems and a single polarization, similar considerations apply to 3D setups, by generalizing Eqs. (5), (6). This theoretical result opens new possibilities for loss-immune strategies in imaging and unconventional electromagnetic wave manipulation based on PT-symmetric metamaterials.

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