Quantum Spectral Curve of the $\mathcal{N}=6$ Supersymmetric Chern-Simons Theory

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Recently, it was shown that the spectrum of anomalous dimensions and other important observables in planar $\mathcal{N}=4$ supersymmetric Yang-Mills theory are encoded into a simple nonlinear Riemann-Hilbert problem: the $\mathbf{P}\mu$ system or quantum spectral curve. In this Letter, we extend this formulation to the $\mathcal{N}=6$ supersymmetric Chern-Simons theory introduced by Aharony, Bergman, Jafferis, and Maldacena. This may be an important step towards the exact determination of the interpolating function $h(\lambda)$ characterizing the integrability of this model. We also discuss a surprising relation between the quantum spectral curves for the $\mathcal{N}=4$ supersymmetric Yang-Mills theory and the $\mathcal{N}=6$ supersymmetric Chern-Simons theory considered here.

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Introduction.—The Aharony-Bergman-Jafferis-Maldacena (ABJM) model [1] is a unique example of three-dimensional gauge theory which may be completely solvable in the planar limit. Up to now, the full power of integrability has been exploited only to study the spectrum of anomalous dimensions of single-trace operators. In particular, echoing the developments in the study of $\mathcal{N}=4$ supersymmetric Yang-Mills (SYM) theory in 4D, an exact description of the spectrum has been obtained by combining information from two-loop perturbation theory [2] and on the strong coupling limit, corresponding to the classical limit of type IIA superstring theory on $\mathrm{AdS}_4\times CP^3$ [3–5]. This led to the asymptotic Bethe ansatz conjectured in Ref. [6], describing operators with large quantum numbers, and ultimately to the thermodynamic Bethe ansatz (TBA) equations [7,8], an infinite set of nonlinear integral equations encoding the anomalous dimension spectrum as a function of a dressed coupling constant $h(\lambda)$. The exact dependence of h on the 't Hooft coupling λ is still a missing ingredient in the integrability approach to the ABJM theory (see Ref. [9] for a review).

It is expected that other important observables can be studied with integrable model tools. In the case of $\mathcal{N}=4$ SYM theory, it was shown in Refs. [10,11] that a system of boundary thermodynamic Bethe ansatz equations describes the (generalized) cusp anomalous dimension $\Gamma(\phi,\lambda)$ characterizing the logarithmic UV divergences of Wilson lines forming a cusp of angle ϕ. In some near-BPS limits, the cusp anomalous dimension can also be studied with independent localization techniques (see, for example, Ref. [12]), leading to nonperturbative exact results which nicely agree with integrability computations [13,14].

For the ABJM model, the bremsstrahlung function $B(\lambda)$ characterizing the leading small-angle behavior $\Gamma(\phi,\lambda)\sim\phi^2B(\lambda)$ was recently computed in Ref. [15] (see also Ref. [16]). As already put forward in Ref. [10], obtaining the same quantity with integrability methods would allow one to fix the exact relation between h and λ.

An important development in $\mathcal{N}=4$ SYM theory was the discovery of an alternative formulation of the TBA as a nonlinear matrix Riemann-Hilbert problem, known as a $\mathbf{P}\mu$ system or a quantum spectral curve. It is a finite set of universal functional relations, believed to encode not only all states of the anomalous dimension spectrum but also, with an appropriate change in the asymptotics, the cusp spectrum [14,17]. This new tool also proved to be much more efficient than the TBA for extracting exact results. In particular, it led to the nine-loop prediction for the Konishi dimension at weak coupling [18], three loops at strong coupling [19], as well as to new results in the study of the Balitsky-Fadin-Kuraev-Lipatov Pomeron.

In this Letter, we present the $\mathbf{P}\mu$ system for the ABJM theory and discuss a surprising link with the quantum spectral curve equations for $\mathcal{N}=4$ SYM theory.

While here we only discuss the application of this new set of equations to the spectrum of anomalous dimensions, we believe that it will play an important role in fixing the $h-\lambda$ relation.

Outline of the derivation.—Conceptually, the $\mathbf{P}\mu$ system is equivalent to other reformulations of the TBA as a set of functional relations, such as the Y or T system. In particular, it can be derived from the Y system [20] supplemented by the discontinuity equations [21,22] describing the monodromies of the Y functions around infinitely many branch points in the complex domain of the spectral parameter u. These branch points are located at positions $u=\pm2h+in/2$, $n\in\mathbb{Z}$. However, these functional relations are very intricate, while the $\mathbf{P}\mu$ system involves only a finite number of objects, with the transparent analytic properties shown in Fig. 2 [17]: the $\mathbf{P}_a$ functions are defined on a Riemann sheet with a single cut running from $-2h$ to $+2h$, while the functions μ_{ab}, although still having an infinity of branch cuts for $(-2h, +2h) + in$, $n \in \mathbb{Z}$, satisfy the simple relation

$$\tilde{\mu}_{ab}(u) = \mu_{ab}(u + i), \tag{1}$$

where $\tilde{\mu}$ and $\tilde{\mathbf{P}}$ denote the values of the $\mathbf{P}\mu$ variables analytically continued around one of the branch points on the real axis. Equation (1) means that, on a Riemann section defined with long cuts, $\mu_{ab} \to \check{\mu}_{ab}$ is simply an i-periodic function: $\check{\mu}_{ab}(u) = \check{\mu}_{ab}(u + i)$ [17]. The two sections μ and $\check{\mu}$ coincide for $0 < \mathrm{Im}(u) < 1$.

To reveal this hidden structure, one can start from the analytic properties of the T functions. The latter are in one-to-one correspondence with the nodes of the T-hook diagram of Fig. 1 [20] and satisfy the discrete Hirota equation (T system)

$$T_{a,s}^{[+1]} T_{a,s}^{[-1]} = \prod_{(a' \sim a)_\updownarrow} T_{a',s} + \prod_{(s' \sim s)_\leftrightarrow} T_{a,s'}, \tag{2}$$

where the products are over horizontal ($\leftrightarrow$) and vertical ($\updownarrow$) neighboring nodes and $T^{[n]} := T(u + (i/2)n)$.

In Ref. [23], a beautiful fundamental set of analyticity conditions for the T functions was discovered, and this was adapted to the ABJM case in Ref. [22]; see Appendix C of that paper. Exploiting the gauge invariance of the Hirota equation, it is possible to introduce two very special gauges, denoted as $\mathbf{T}$ and $\mathbb{T}$. For $s \geq a$, in an appropriate analyticity strip just above the real axis [24], the $\mathbb{T}_{a,s}$ functions can be parametrized as

$$\mathbb{T}_{1,s} = \mathbf{P}_1^{[+s]} \mathbf{P}_2^{[-s]} - \mathbf{P}_2^{[+s]} \mathbf{P}_1^{[-s]}, \qquad \mathbb{T}_{0,s} = 1,$$
$$\mathbb{T}_{2,s} = \mathbb{T}_{1,1}^{[+s]} \mathbb{T}_{1,1}^{[-s]}, \qquad \mathbb{T}_{3,2}/\mathbb{T}_{2,3} = \mu_{12}, \tag{3}$$

where $\mathbf{P}_1$, $\mathbf{P}_2$, and μ_{12} have the simple properties discussed above and will be part of the $\mathbf{P}\mu$ system. Furthermore, the $\mathbf{T}$ gauge can be introduced with the transformation

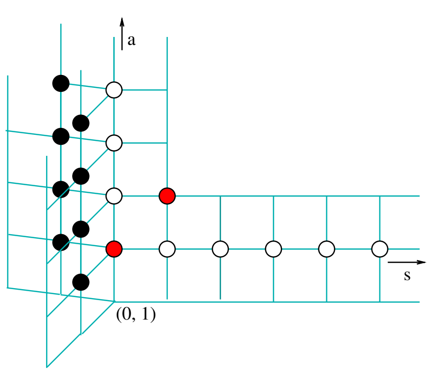

FIG. 1 (color online). T hook for the ABJM T system.

$$\mathbf{T}_{n,s} = (-1)^{n(s+1)} \mathbb{T}_{n,s} \left(\check{\mu}_{12}^{[n+s-1]} \right)^{2-n}, \qquad s \geq 1,$$
$$\mathbf{T}_{n,0}^{\alpha} = (-1)^n \mathbb{T}_{n,0}^{\alpha} \left(\sqrt{\check{\mu}_{12}^{[n-1]}} \right)^{2-n},$$
$$\mathbf{T}_{n,-1}^{\alpha} = \mathbb{T}_{n,-1}^{\alpha} = 1, \qquad \alpha = I, II, \tag{4}$$

and the $\mathbf{T}_{n,s}$ functions are required to satisfy

$$\mathbf{T}_{n,0}^{\alpha} \in \mathcal{A}_{n+1}, \qquad \alpha = I, II, \qquad n \geq 0,$$
$$\mathbf{T}_{n,1} \in \mathcal{A}_n, \qquad n \geq 1, \tag{5}$$

where we denote with $\mathcal{A}_n$ the class of functions free of branch cuts in the strip $|\mathrm{Im}(u)| < n/2$.

The strategy to derive the $\mathbf{P}\mu$ system is then the following (see also Ref. [25]): starting from the Hirota equation and the gauge transformation (4), it is possible to compute any $\mathbf{T}_{n,s}$ function in terms of the only variables $\mathbf{P}_1$, $\mathbf{P}_2$, and μ_{12}, evaluated on different Riemann sheets. Surprisingly, when rewritten in terms of these functions, the conditions (5) show precisely how the system can be closed, introducing only a finite number of fundamental variables, each with one of the two types of cut structures shown in Fig. 2. The simplest nontrivial example is provided by the condition $\mathbf{T}_{2,1} \in \mathcal{A}_2$. Computing $\mathbf{T}_{2,1}$ as described above, and imposing that it has no cut on the real axis, we find the constraint

$$0 = \mathbf{T}_{2,1} - \tilde{\mathbf{T}}_{2,1} = (\mathbf{P}_1^{[+2]} \mathbf{P}_2^{[-2]} - \mathbf{P}_2^{[+2]} \mathbf{P}_1^{[-2]}) \times (\tilde{\mu}_{12} - \mu_{12} - \mathbf{P}_1 \tilde{\mathbf{P}}_2 + \mathbf{P}_2 \tilde{\mathbf{P}}_1). \tag{6}$$

The first factor equals $\mathbb{T}_{1,2}$, which is different from 0, and this leads to a new relation:

$$\tilde{\mu}_{12} - \mu_{12} = \mathbf{P}_1 \tilde{\mathbf{P}}_2 - \mathbf{P}_2 \tilde{\mathbf{P}}_1. \tag{7}$$

As we will show in detail in a more extended work, the structure of the $\mathbf{P}\mu$ system is already revealed just by inspecting a few of the other conditions in Eq. (5). The main results are presented in the next section.

The $\mathbf{P}\mu$ *system.*—The $\mathbf{P}\mu$ system for the ABJM model involves a vector of six functions $\mathbf{P}_i$, $i = 1, \ldots, 6$, and an antisymmetric 6×6 matrix μ_{ab}, with the analytic properties

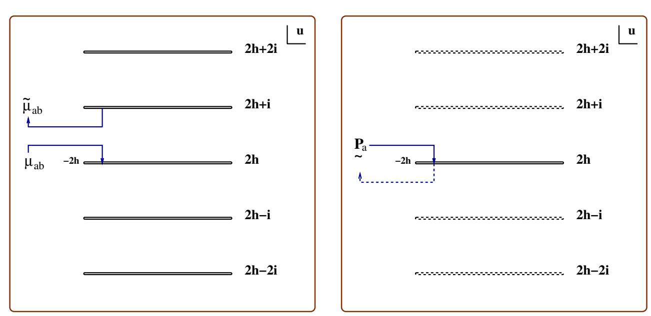

FIG. 2 (color online). Analytic structure for the two types of variables in the quantum spectral curve.

of Fig. 2. These variables, moreover, satisfy the nonlinear constraints

$$\mathbf{P}_5\mathbf{P}_6 = 1 + \mathbf{P}_2\mathbf{P}_3 - \mathbf{P}_1\mathbf{P}_4, \tag{8}$$

$$\mu\chi\mu\chi = 0. \tag{9}$$

Here, χ is a 6×6 symmetric matrix whose only nonzero entries are

$$\chi^{14} = \chi^{41} = -1, \qquad \chi^{23} = \chi^{32} = 1, \qquad \chi^{56} = \chi^{65} = -1.$$

The fundamental Riemann-Hilbert relations read

$$\tilde{\mathbf{P}}_a = \mathbf{P}_a - \mu_{ab}\chi^{bc}\mathbf{P}_c, \tag{10}$$

$$\mu_{ab} - \tilde{\mu}_{ab} = -\mathbf{P}_a\tilde{\mathbf{P}}_b + \mathbf{P}_b\tilde{\mathbf{P}}_a. \tag{11}$$

By appropriately constraining the asymptotics, all states of the spectrum can be described by Eqs. (8)–(11). The anomalous dimensions are also encoded in the asymptotics. Below, we will discuss the description of a specific subsector, while the general case will be presented in a future work.

Even-parity states: For many states, it is sufficient to consider a reduced system of equations. The parity invariant sector of the spectrum is identified by the conditions $\mathbf{P}_5 = \mathbf{P}_6$ and $\mu_{5a} = \mu_{6a} = -\mu_{a5} = -\mu_{a6}$.

$\mathbf{Q}\omega$ system: Finally, we remark that, similarly to the case of $\mathcal{N} = 4$ SYM theory, there is a complementary set of conditions, named the $\mathbf{Q}\omega$ system, which is formally the same as Eqs. (8)–(11), with the replacements

$$\mathbf{P}_a \to \mathbf{Q}_a, \qquad \mu_{ab} \to \omega_{ab}, \tag{12}$$

but with all the branch cuts reversed. Namely, the functions $\mathbf{Q}_a$ have a single branch cut for $u \in (-\infty, -2h) \cup (+2h, +\infty)$, while ω_{ab} are i-periodic functions (with the additional interchange of some components in the nonsymmetric case) on a Riemann sheet defined with short cuts, which can be rewritten as

$$\omega_{ab}(u+i) = \omega_{\bar{a}\bar{b}}(u), \tag{13}$$

where $a = \bar{a}$ for $a = 1, \ldots, 4$ and $\bar{5} = 6$, $\bar{6} = 5$. We expect this second system to play an important role in recovering the structure of the asymptotic Bethe ansatz in the limit of long operators [25].

Identification with $\mathcal{N} = 4$ SYM theory: An interesting formal identification is possible between Eqs. (8)–(11) and the $\mathbf{P}\mu$ system previously derived for the $\mathcal{N} = 4$ SYM spectral problem [17,25]. This can be found by parametrizing the ABJM matrix μ_{ab} in terms of eight functions ν_i, $\bar{\nu}_i$, $i = 1, \ldots, 4$, as follows:

$$\mu_{ab} = \begin{pmatrix} 0 & \nu_1\bar{\nu}_1 & \nu_2\bar{\nu}_2 & \bar{\nu}_2\nu_3 - \bar{\nu}_1\nu_4 & \nu_1\bar{\nu}_2 & \bar{\nu}_1\nu_2 \\ -\nu_1\bar{\nu}_1 & 0 & \bar{\nu}_2\nu_3 + \nu_1\bar{\nu}_4 & \nu_3\bar{\nu}_3 & \nu_1\bar{\nu}_3 & \bar{\nu}_1\nu_3 \\ -\nu_2\bar{\nu}_2 & -\bar{\nu}_2\nu_3 - \nu_1\bar{\nu}_4 & 0 & \nu_4\bar{\nu}_4 & -\bar{\nu}_2\nu_4 & -\nu_2\bar{\nu}_4 \\ \bar{\nu}_1\nu_4 - \bar{\nu}_2\nu_3 & -\nu_3\bar{\nu}_3 & -\nu_4\bar{\nu}_4 & 0 & -\bar{\nu}_3\nu_4 & -\nu_3\bar{\nu}_4 \\ -\nu_1\bar{\nu}_2 & -\nu_1\bar{\nu}_3 & \bar{\nu}_2\nu_4 & \bar{\nu}_3\nu_4 & 0 & \bar{\nu}_2\nu_3 - \nu_2\bar{\nu}_3 \\ -\bar{\nu}_1\nu_2 & -\bar{\nu}_1\nu_3 & \nu_2\bar{\nu}_4 & \nu_3\bar{\nu}_4 & \nu_2\bar{\nu}_3 - \bar{\nu}_2\nu_3 & 0 \end{pmatrix}, \tag{14}$$

with the additional requirement that

$$\nu_1\bar{\nu}_4 - \bar{\nu}_1\nu_4 = \nu_2\bar{\nu}_3 - \bar{\nu}_2\nu_3. \tag{15}$$

By definition, ν_i and $\bar{\nu}_i$ have the same analytic properties as μ_{ab}, namely, $\tilde{\nu}_i(u) = \nu_i(u+i)$. The parametrization [Eqs. (14) and (15)] is introduced in order to resolve the constraint $\mu\chi\mu\chi = 0$. Moreover, as we discuss below, we expect that $\nu_1^{[+1]}$ and $\bar{\nu}_1^{[+1]}$ will play the role of fundamental Q functions at weak coupling. Remarkably, it is possible to rewrite Eqs. (10) and (11), eliminating μ_{ab} completely. In fact, one can check that all conditions (11) are satisfied, provided ν_i and $\bar{\nu}_i$ transform in the following simple way under analytic continuation:

$$\tilde{\nu}_i = U_i^j\bar{\nu}_j, \qquad \tilde{\bar{\nu}}_i = \bar{U}_i^j\nu_j, \tag{16}$$

where

$$U_a{}^b = \begin{pmatrix} \mathbf{P}_5 & -\mathbf{P}_2 & \mathbf{P}_1 & 0 \\ \mathbf{P}_3 & -\mathbf{P}_6 & 0 & \mathbf{P}_1 \\ \mathbf{P}_4 & 0 & -\mathbf{P}_6 & \mathbf{P}_2 \\ 0 & \mathbf{P}_4 & -\mathbf{P}_3 & \mathbf{P}_5 \end{pmatrix},$$

$$\bar{U}_a{}^b = \begin{pmatrix} \mathbf{P}_6 & -\mathbf{P}_2 & \mathbf{P}_1 & 0 \\ \mathbf{P}_3 & -\mathbf{P}_5 & 0 & \mathbf{P}_1 \\ \mathbf{P}_4 & 0 & -\mathbf{P}_5 & \mathbf{P}_2 \\ 0 & \mathbf{P}_4 & -\mathbf{P}_3 & \mathbf{P}_6 \end{pmatrix}.$$

Finally, the discontinuity relations for $\mathbf{P}_i$ can be rewritten as

TABLE I. The single-cut ↔ periodic mapping between the ABJM and $\mathcal{N}=4$ SYM theories (here shown in the symmetric case), where we have denoted $\mathbf{P}_5=\mathbf{P}_6=\mathbf{P}_0$.

$\mathcal{N}=4$ SYM	ABJM
$\mu_{ij},\ i,j=1,\ldots,4$	$\begin{pmatrix} 0 & -\mathbf{P}_1 & -\mathbf{P}_2 & -\mathbf{P}_0 \\ \mathbf{P}_1 & 0 & -\mathbf{P}_0 & -\mathbf{P}_3 \\ \mathbf{P}_2 & \mathbf{P}_0 & 0 & -\mathbf{P}_4 \\ \mathbf{P}_0 & \mathbf{P}_3 & \mathbf{P}_4 & 0 \end{pmatrix}$
$\mathbf{P}_i,\ i=1,\ldots,4$	ν_i

$$\tilde{\mathbf{P}}_1-\mathbf{P}_1=\nu_2\tilde{\nu}_1-\nu_1\tilde{\nu}_2,\qquad \tilde{\mathbf{P}}_2-\mathbf{P}_2=\nu_3\tilde{\nu}_1-\nu_1\tilde{\nu}_3,$$
$$\tilde{\mathbf{P}}_3-\mathbf{P}_3=\nu_4\tilde{\nu}_2-\nu_2\tilde{\nu}_4,\qquad \tilde{\mathbf{P}}_4-\mathbf{P}_4=\nu_4\tilde{\nu}_3-\nu_3\tilde{\nu}_4,$$
$$\tilde{\mathbf{P}}_5-\mathbf{P}_5=\nu_4\tilde{\nu}_1-\nu_1\tilde{\nu}_4,\qquad \tilde{\mathbf{P}}_6-\mathbf{P}_6=\nu_3\tilde{\nu}_2-\nu_2\tilde{\nu}_3. \quad (17)$$

To present the identification with $\mathcal{N}=4$ SYM theory, for simplicity, let us restrict ourselves to the parity-symmetric sector, by taking $\nu_i=\bar{\nu}_i$ and $\mathbf{P}_5=\mathbf{P}_6$. Defining $\mathbf{P}_i^{\mathcal{N}=4}:=\nu_i$ for $i=1,\ldots,4$ and organizing the components $\mathbf{P}_j$ into a 4×4 antisymmetric matrix $\mu_{ab}^{\mathcal{N}=4}$ as shown in Table I, one can see that, on the algebraic level, Eqs. (16) and (17) are identical to the quantum spectral curve equations for the left- or right-symmetric sector of $\mathcal{N}=4$ SYM theory [17]. Even the constraints perfectly match: in fact, notice that Eq. (8) translates into the constraint of Ref. [17]:

$$(\mu_{23}{}^{\mathcal{N}=4})^2=1+\mu_{13}{}^{\mathcal{N}=4}\mu_{24}{}^{\mathcal{N}=4}-\mu_{12}{}^{\mathcal{N}=4}\mu_{34}{}^{\mathcal{N}=4}. \quad (18)$$

The identification can be extended to the nonsymmetric case. The $\mathbf{P}\mu$ system for the most general sector of $\mathcal{N}=4$ SYM theory is described in Ref. [25]. It is remarkable that the two theories differ only in the analytic properties. As summarized in Table I, it is possible to map the ABJM system into the $\mathcal{N}=4$ SYM one by exchanging the two types of cut structures presented in Fig. 2, so that i-periodic functions ↔ functions with a single cut.

Description of the spectrum.—In this section, we provide the information needed to study the subsector of the ABJM model which includes the states dual to a folded spinning string with angular momenta L in CP^3 and S in AdS_4. The subsector is completely characterized by the pair of integers (L,S) and by the conformal dimension Δ. In the $\mathbf{P}\mu$ system, these quantum numbers are encoded in the asymptotics. As observed in Ref. [23] in the $\mathcal{N}=4$ case, Δ appears in the large-u behavior of the product of Y functions $Y_{1,1}Y_{2,2}$:

$$\ln Y_{1,1}Y_{2,2}(u)=2i\frac{\Delta-L}{u}+O(1/u^2). \quad (19)$$

This quantity can be computed as

$$\ln Y_{1,1}Y_{2,2}(u)=\ln\mu_{12}(u+i)-\ln\mu_{12}(u)\simeq i\partial_u\ln\mu_{12}(u),$$

and this implies that

$$\nu_1(u)=\sqrt{\mu_{12}(u)}\sim u^{\Delta-L}. \quad (20)$$

The asymptotics of $\mathbf{P}$ functions is related to the CP^3 momentum L as

$$\mathbf{P}_a(u)\simeq(A_1u^{-L},A_2u^{-L-1},A_3u^{L+1},A_4u^L), \quad (21)$$

with $\mathbf{P}_5=\mathbf{P}_6=\sqrt{1+\mathbf{P}_2\mathbf{P}_3-\mathbf{P}_1\mathbf{P}_4}$. To complete the description of the state, we need the following relations between the coefficients A_i:

$$A_1A_4=\frac{[(\Delta-S+1)^2-L^2][L^2-(\Delta+S)^2]}{L^2(2L+1)},$$
$$A_2A_3=\frac{[(\Delta-S+1)^2-(L+1)^2][(L+1)^2-(\Delta+S)^2]}{(L+1)^2(2L+1)}. \quad (22)$$

Equations (22) can be derived as discussed in Refs. [17,25]. It is interesting that, as remarked in Ref. [19], the quantization of S appears naturally through the nonlinearity of the $\mathbf{P}\mu$ system. The identifications above involve some guesswork, but they can be checked by recovering the correct weak coupling result, as shown below. We expect that Eqs. (20)–(22), together with the pole-free property for the $\mathbf{P}$ and μ functions, are the *only* physical input needed for the computation of Δ at any value of h.

A weak coupling test: As a test of our results, let us show that they reproduce the two-loop Baxter equation. At leading order at weak coupling, we expect that

$$\Delta=L+S+O(h^2), \quad (23)$$

and we see from Eq. (22) that $A_2A_3=O(h^2)$. Therefore, we assume that $\mathbf{P}_2\to0$, and we see that as a consequence, the equations for ν_1 and ν_3 decouple:

$$\begin{pmatrix}\tilde{\nu}_1\\ \tilde{\nu}_3\end{pmatrix}=\begin{pmatrix}\nu_1^{[+2]}\\ \nu_3^{[+2]}\end{pmatrix}=\begin{pmatrix}\mathbf{P}_0 & \mathbf{P}_1\\ \mathbf{P}_4 & -\mathbf{P}_0\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_3\end{pmatrix}. \quad (24)$$

Making the identification $\nu_1^{[+1]}=Q$, the system (24) implies the Baxter equation

$$\left(\frac{\mathbf{P}_0^{[+1]}}{\mathbf{P}_1^{[+1]}}-\frac{\mathbf{P}_0^{[-1]}}{\mathbf{P}_1^{[-1]}}\right)Q=\frac{Q^{[+2]}}{\mathbf{P}_1^{[+1]}}-\frac{Q^{[-2]}}{\mathbf{P}_1^{[-1]}}. \quad (25)$$

Generalizing the argument of Ref. [17], one can go further and reproduce the expected two-loop result [2]:

$$\Delta=L+S+2ih^2\partial_u\ln\frac{Q^{[+1]}}{Q^{[-1]}}\bigg|_{u=0}+O(h^4). \quad (26)$$

Conclusions.—In this Letter, we have recast the spectral problem for the ABJM model as a finite system of coupled Riemann-Hilbert equations: the $\mathbf{P}\mu$ system. The similarity with the $\mathcal{N}=4$ SYM case suggests that an analogous formulation should also exist for the still partly mysterious, integrable models related to AdS_3/CFT_2. Studying other examples would probably help to understand the hidden algebraic structures underlying these systems. It would be particularly interesting to investigate how the analytic properties of the $\mathbf{P}\mu$ system are modified under the q deformation discussed in Ref. [26]. This may help to clarify the physical meaning of the formal map between the quantum spectral curve equations for the $\mathcal{N}=4$ SYM and ABJM theories presented in this Letter.

Let us summarize some of the potential applications to ABJM theory. Adapting the methods of Refs. [17,19], our results should allow one to study the weak and strong coupling expansions and nonperturbative near-BPS regimes such as the small-spin limit described by the slope function [27]. An interesting open problem would be to find numerical solution methods valid at generic values of the coupling. We believe that our equations can also be applied to study the spectrum of cusped Wilson lines.

Finally, one can hope that studying the $\mathbf{P}\mu$ system in the ABJM context would reveal some structures which are harder to see in the case of $\mathcal{N}=4$ SYM theory and help to clarify the nature and the role of this intriguing mathematical object both in the AdS/CFT correspondence and in the general theory of integrable models. Hopefully, this can also teach us something new about nonperturbative gauge theories and AdS/CFT.

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