



Harnessing Buckling to Design Tunable Locally Resonant Acoustic Metamaterials

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We report a new class of tunable and switchable acoustic metamaterials comprising resonating units dispersed into an elastic matrix. Each resonator consists of a metallic core connected to the elastomeric matrix through elastic beams, whose buckling is intentionally exploited as a novel and effective approach to control the propagation of elastic waves. We first use numerical analysis to show the evolution of the locally resonant band gap, fully accounting for the effect of nonlinear pre-deformation. Then, we experimentally measure the transmission of vibrations as a function of the applied loading in a finite-size sample and find excellent agreement with our numerical predictions. The proposed concept expands the ability of existing acoustic metamaterials by enabling tunability over a wide range of frequencies. Furthermore, we demonstrate that in our system the deformation can be exploited to turn on or off the band gap, opening avenues for the design of adaptive switches.

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Artificially structured composite materials that enable manipulation and control of elastic waves have received significant interest in recent years [1], not only because of their rich physics, but also for their broad range of applications, including wave guiding [2–7], cloaking [8] and noise reduction [9–11]. An important characteristic of these heterogeneous systems is their ability to tailor the propagation of elastic waves due to the existence of band gaps—frequency ranges of strong wave attenuation. In phononic crystals, band gaps are generated by Bragg scattering [12], whereas in acoustic metamaterials, localized resonance within the medium is exploited to attenuate the propagation of waves. Analogous to the case of photonic crystals where split-ring resonators have been embedded within a medium to excite electromagnetic resonances [13,14], in acoustic metamaterials the internal resonance is typically realized by dispersing heavy inclusions coated with a soft layer into a matrix [15]. Interestingly, acoustic metamaterials are capable of manipulating waves with wavelengths much larger than the structural features of the system [15] and have been successfully exploited for vibration control [16,17], imaging [18], design of exotic elastic solids [19,20], and thermal management [21].

Most of the phononic and acoustic metamaterial configurations proposed to date are characterized by a passive response and operate at fixed frequency ranges, limiting the number of possible applications. In an effort to design tunable systems, it has been shown that Bragg-type band gaps can be controlled by instability-induced pattern transformations [22–24]. On the other hand, for acoustic metamaterials, tuning of functionalities has been achieved by altering the resonant frequency via piezoshunting [25],

adaptive connectivity [26], and fluid-structure interactions [27].

Here, we report a new class of adaptive acoustic metamaterials whose response is controlled by mechanical deformation. In the proposed metamaterial, the elastomeric coating that typically surrounds the resonating mass is replaced by easy-to-buckle elastic beams, as shown in Fig. 1(a). When the system is compressed statically, these beams buckle at a relatively low level of applied uniaxial

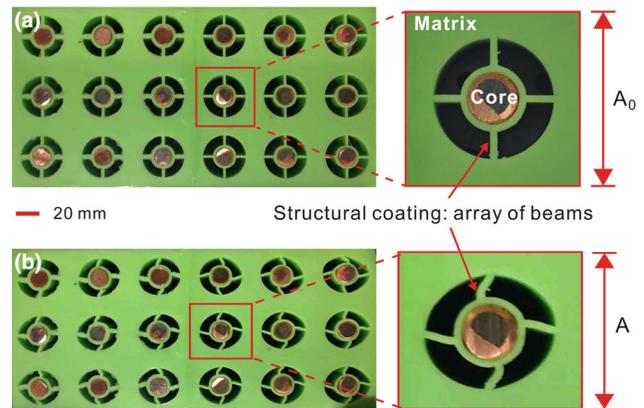


FIG. 1 (color online). Tunable acoustic metamaterial: (a) The undeformed configuration comprises resonating units dispersed into an elastomeric matrix. Each resonator consists of a metallic mass connected to the matrix through elastic beams, which form a structural coating. The black regions in the picture indicate voids in the structure. The unit cell size is $A_0 = 50.0$ mm. (b) When a compressive strain $\varepsilon = -0.10$ is applied in the vertical direction, buckling of the beams significantly alters the effective stiffness of the structural coating, which in turn changes the band gap frequency.

strain ϵ , as seen in Fig. 1(b) for $\epsilon = -0.1$. Interestingly, buckling dramatically alters the stiffness of the beams and consequently the natural frequency of the resonating units, which in turn determines the frequency range of the band gap.

Through a combination of analyses and experiments, we demonstrate that elastic instability and deformation can be exploited to effectively tune and even completely suppress the band gaps of the system. Hence, the metamaterial can be used as an acoustic switch that provides on or off capabilities.

To fully understand the effect of deformation on the propagation of small amplitude elastic waves in the proposed acoustic metamaterial, we focus on the elastic system shown in Fig. 1 (the structure is uniform in the out-of-plane dimension with height of 50.0 mm). Here we consider a square array of locally resonant unit cells with center-to-center distance $A_0 = 50.0$ mm, but the response of the system is not affected by the spatial arrangement of the resonators [28,29]. Each resonating unit comprises (i) a portion of the elastomeric matrix, (ii) a metallic circular core with radius $R_0 = 7.9$ mm, and (iii) four beams with length $L_0 = 16.9$ mm and thickness $t_0 = 1.8$ mm that connect the metallic core to the matrix.

Both the beams and the matrix are made of silicon rubber (Elite Double 32, Zhermack) with measured shear modulus $\mu_0 = 4.44 \times 10^5$ N/m², Poisson's ratio $\nu_0 = 0.499$, and density $\rho_0 = 1050$ kg/m³ (so that in the undeformed homogeneous material the elastic wave speeds are $c_T = 20.6$ m/s and $c_L = 460.1$ m/s for shear and pressure waves, respectively). Differently, the metallic cores are built using copper rods (with shear modulus $\mu_{Cu} = 4.78 \times 10^{10}$ N/m², Poisson's ratio $\nu_{Cu} = 0.34$ and density $\rho_{Cu} = 8960$ kg/m³).

We first investigate the effect of uniaxial compression on the static and dynamic responses of the system via finite element (FE) simulations using the commercial software ABAQUS/STANDARD. In the simulations we focus on a representative volume element comprising a single resonating unit [see Fig. 2(a)] and use periodic and Bloch-type boundary conditions for the static and dynamic analyses [23], respectively. Two-dimensional (2D) models are constructed using quadrilateral plane strain hybrid quadratic elements (Abaqus element type CPE8 H) and the response of the elastomeric matrix is described by a nearly incompressible Neo-Hookean model with initial shear modulus μ_0 and Poisson's ratio ν_0 .

To begin with, we focus on the static response of the unit cell and perform a linear perturbation analysis to investigate the stability of the beams. A local instability is detected at $\epsilon_{cr} = -0.0247$, associated with buckling of the two vertical beams. The postbuckling response of the system is then simulated by introducing small random imperfections into the initial geometry and performing a nonlinear static analysis. Snapshots of the deformed configurations at

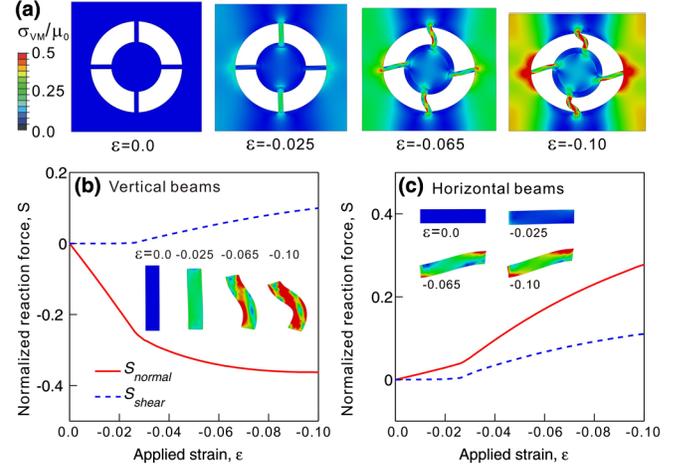


FIG. 2 (color online). Static response: (a) Distribution of the normalized Von Mises stress, σ_{VM}/μ_0 at different levels of applied strain ϵ . (b) Effect of the applied deformation on the reaction force transmitted by the vertical beams to the matrix. The normalized reaction forces acting both in axial (normal component), $S_{normal} = R_{normal}/(t_0\mu_0)$, and tangential (shear component), $S_{shear} = R_{shear}/(t_0\mu_0)$, are reported, R_{normal} and R_{shear} denoting the total reaction force measured at the end of the beam in the normal and tangential directions, respectively. (c) Effect of the applied deformation on the force transmitted by the horizontal beams to the matrix.

different levels of the applied strain are shown in Fig. 2(a). As predicted by the stability analysis, when the structure is compressed uniaxially in the vertical direction, the two vertical beams buckle at ϵ_{cr} . Moreover, the buckling of vertical beams is also accompanied by rotation of the metallic core that, in turn, results in the stretching of the two horizontal beams. In Figs. 2(b) and 2(c) we show the significant effect of the applied deformation on the axial and tangential forces transmitted by the beams to the matrix. Initially, most of the load is carried by the two vertical beams in their axial direction. However, after the onset of instability the axial stress in the vertical beams is found to plateau, while both the axial and tangential force components transmitted by the horizontal beams significantly increase due to their stretching induced by the rotation of the metallic core. Hence, our results indicate that the applied deformation significantly alters the effective stiffness of the resonator.

Next, we numerically investigate the effect of such change in effective stiffness on the propagation of small amplitude elastic waves. For this, we calculate the dispersion relations for both the undeformed and deformed configurations using frequency domain analyses, fully accounting for the effect of precompression [23,29].

In Fig. 3(a) we report the dispersion relation of the metamaterial in the undeformed configuration, while in Fig. 3(b) the Bloch mode shapes of the four lowest bands at the high-symmetry points, X , M , and Y of the Brillouin zone are shown [29]. For clarity, in Fig. 3(a) we use red

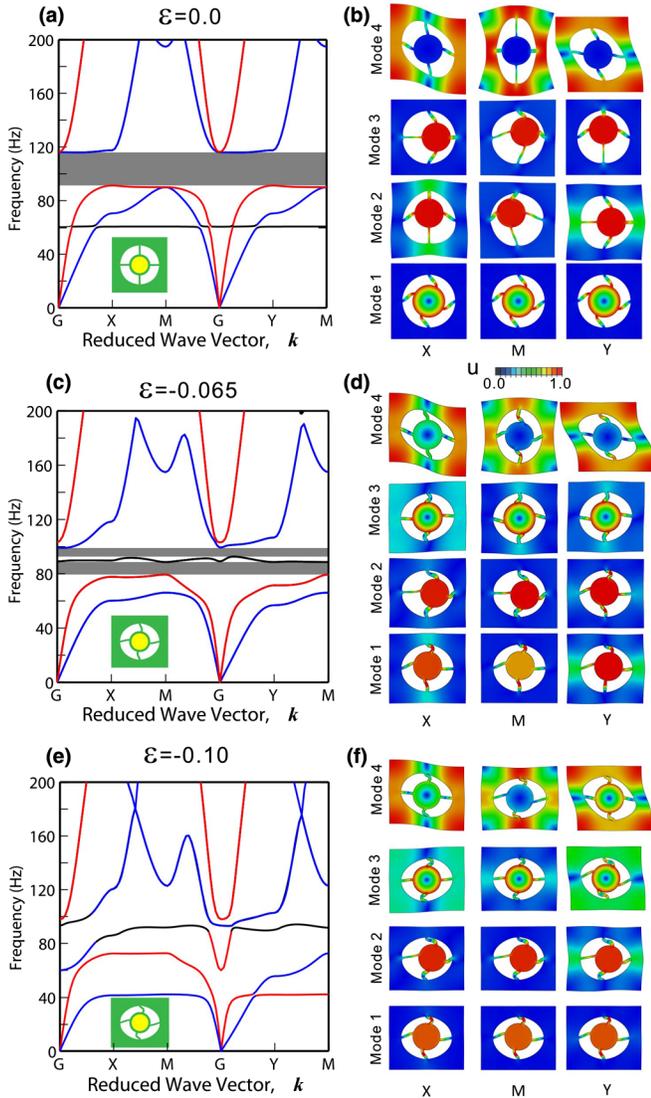


FIG. 3 (color online). Effect of the applied deformation on the band structure: Dispersion relations from Bloch-wave analysis for the infinite metamaterial in (a) the undeformed configuration and under uniaxial compressive strain (c) $\varepsilon = -0.065$ and (e) $\varepsilon = -0.10$. Shear dominated bands are colored in blue, pressure dominated bands in red and locally rotational bands in black. The grey region in (a) and (c) highlights the band gap induced by local resonance. The Bloch modes of the lowest four bands at high-symmetry points of the Brillouin zone (X , M , and Y) [29] are shown in (b), (d), and (f) for $\varepsilon = 0.0, -0.065$, and -0.10 , respectively. The distribution of the magnitude of the modal displacement field is shown.

and blue lines to represent pressure-dominated (longitudinal) and shear-dominated (transversal) wave modes, respectively. Interestingly, the first three modes at X , M , and Y show a strong localization of vibration, with only the metallic core vibrating and the matrix mostly at rest. By contrast, the fourth mode at X , M , and Y points has a global nature, since the entire matrix vibrates while the modal displacement of metallic core is negligible. We note that

similar modal patterns have been observed not only in acoustic metamaterials [15,34], but also in photonic crystals [35]. Further inspection of the modes reveals that the first localized mode [black line in Fig. 3(a)] is purely rotational, so that it has a very weak coupling with the propagating shear and pressure waves and does not open a band gap [34]. Differently, in the second and third modes the resonators interact with the shear and pressure waves, respectively. This coupling between the vibration of the metallic core and the propagating waves in the media generates a band gap, highlighted by the grey area in Figs. 3(a) and 3(c). Finally, we note that, although our numerical results indicate an anisotropy in the modes [see mode 2 in Fig. 3(b)], the locally resonant band gap is an omnidirectional complete gap.

The effect of deformation on the propagation of elastic waves is shown in Figs. 3(c) and 3(d) for $\varepsilon = -0.065$ and in Figs. 3(e) and 3(f) for $\varepsilon = -0.10$. Under small ε , the softening of the two vertical beams induced by buckling is found to dominate over the stiffening of the two horizontal beams induced by their stretching, so that the band gap frequency decreases, as shown in Fig. 3(c). However, due to the increase in tangential stiffness induced by instability, the rotational mode rises above the shear and pressure modes [see Fig. 3(d)] and splits the band gap. If we further increase the applied strain to $\varepsilon = -0.10$, the band gap then completely closes because of the modal transformation of the rotational band near the G point [see Fig. 3(e)]. Therefore, our results clearly demonstrate that the proposed metamaterial acts as a reversible phononic switch, since the applied deformation can be exploited to tune and switch the band gap on or off.

To validate the numerical predictions, a sample of the locally resonant metamaterial comprising 6×3 unit cells (see Fig. 1) is fabricated using silicon rubber (Elite Double 32, Zhermack) and a mold-casting process with the mold prepared by 3D rapid prototyping. We first apply the desired level of strain ε using a fixture made of acrylic plates and nylon bolts or nuts. Next, wave propagation in the sample is excited by an electrodynamic shaker (model K2025E013, Modal Shop), which is directly connected to one end of the sample to provide a white noise input signal over a broadband frequency range. The dynamic response at different levels of applied strain ε is recorded using two miniature accelerometers (352C22, PCB Piezotronics) attached to both ends of the sample and the transmittance is computed as the ratio between the output and input acceleration signals (i.e., $\|A_{\text{out}}(\omega)/A_{\text{in}}(\omega)\|$).

In Fig. 4(a) we report the transmittance of the sample measured at $\varepsilon = 0.0, -0.065$, and -0.10 . In the undeformed configuration (i.e., $\varepsilon = 0.0$) a significant drop in the transmittance is observed between 80 Hz and 100 Hz with a pronounced minimum at 95 Hz, which corresponds to the resonance frequency of vibration localization. Note

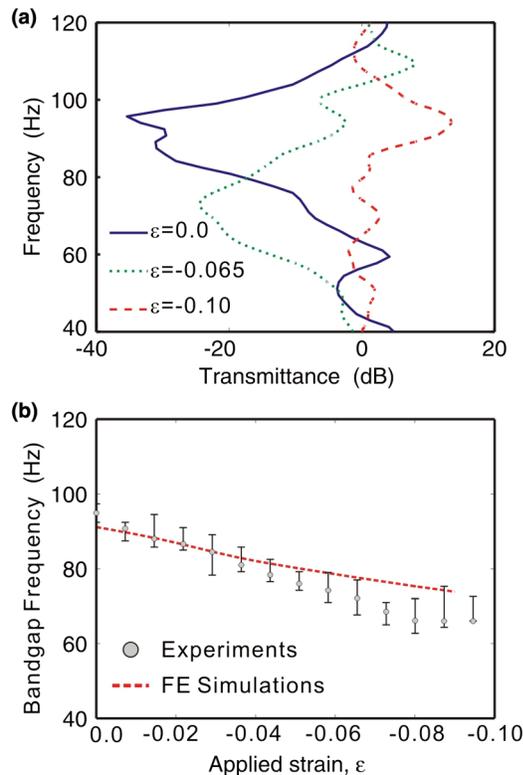


FIG. 4 (color online). Effect of the applied deformation on the band gap frequency: (a) Experimentally measured transmittance in a sample with 6×3 unit cells (see Fig. 1) at different levels of applied prestrains. (b) Evolution of the resonant band gap frequency as a function of the applied strain, ϵ . The markers and error bars represent the lowest transmittance frequency and the ± 3 dB band gap width measured experimentally. The dashed line indicates the lower edge of the band gap (i.e., frequency of resonance) predicted by the FE simulations for the corresponding infinite system.

that in finite-size locally resonant metamaterials the lowest transmittance is usually observed at the resonant frequency of the embedded resonators, which corresponds to the lower edge of the band gap predicted by the dispersion relation [15]. This result not only proves the existence of the band gap predicted by the simulations, but also quantitatively agrees with the FE results in which the lower edge of the band gap (i.e., the frequency with strongest resonance) was observed at 92 Hz [see Fig. 3(a)]. Moreover, as predicted by the numerical results, when a uniaxial strain ϵ is applied, the experimentally measured band gap is found to shift towards lower frequencies. In particular, for $\epsilon = -0.065$ the drop in the transmittance is shifted to the range of 60–85 Hz, with the minimum at 76 Hz. We also note that the band gap splitting predicted at $\epsilon = -0.065$ by the FE simulations [see Fig 3(c)] is not observed in the measured transmittance. This discrepancy is attributed to finite-size effects such as boundary effects and nonuniform distribution of the predeformation, which are not fully

accounted for in the simulations. Lastly, for $\epsilon = -0.10$, the measured transmittance shows no band gap, confirming that the proposed metamaterial can be utilized as a phononic or acoustic switch.

The comparison between the resonant band gap frequency predicted by simulations and experiments as a function of the applied strain ϵ is presented in Fig. 4(b). The markers and error bars represent the lowest transmittance frequency and the ± 3 dB band gap width measured experimentally, respectively. Moreover, the dashed line indicates the frequency of the lower edge of the band gap calculated numerically for the corresponding infinite system. Remarkably, we find excellent agreement between the two sets of data. Both experiments and simulations predict the band gap frequency first to decrease linearly as a function of ϵ and then the band gap to completely disappear as ϵ approaches -0.10 .

In summary, we demonstrated both numerically and experimentally that large deformation and local instability can be exploited to effectively control the response of locally resonant acoustic metamaterials. This remarkable behavior is achieved by introducing a structural coating comprising an array of elastic and highly deformable beams. Our results indicate that under externally applied load the stiffness of the beams varies significantly due to their buckling, altering the resonant frequency of the unit and providing a wide range of tunability for the band gap ($\sim 30\%$ in frequency). Furthermore, we showed that the proposed metamaterial can be utilized as an on or off acoustic switch, since a moderate level of applied uniaxial strain ($\epsilon \sim -0.10$) is enough to entirely suppress the band gap. Although in this Letter we only verified the concept with one specific set of material and geometric parameters, the response of the system is robust. Numerical parametric studies have also been performed by changing the arrangement of the resonating units, the number of beams in the structural coating, their slenderness, and the constitutive behavior of the matrix [29]. Our results indicate that the proposed mechanism works over a wide range of parameters, opening avenues for the design of smart systems that control the wave propagation depending on the applied deformation.

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