Evidence for Triangular $\mathcal{D}_{3h}$ Symmetry in ^{12}C

D. J. Marín-Lámbarri,[1] R. Bijker,[2] M. Freer,[1] M. Gai,[3,4] Tz. Kokalova,[1] D. J. Parker,[1] and C. Wheldon[1]

[1]*School of Physics and Astronomy, University of Birmingham, Birmingham B15 2TT, United Kingdom*
[2]*Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, A.P. 70-543, 04510 México, D.F., Mexico*
[3]*LNS at Avery Point, University of Connecticut, Groton, Connecticut 06340-6097, USA*
[4]*Wright Lab, Department of Physics, Yale University, New Haven, Connecticut 06520-8124, USA*

We report a measurement of a new high spin $J^{\pi} = 5^-$ state at 22.4(2) MeV in ^{12}C which fits very well to the predicted (ground state) rotational band of an oblate equilateral triangular spinning top with a $\mathcal{D}_{3h}$ symmetry characterized by the sequence $0^+, 2^+, 3^-, 4^{\pm}, 5^-$ with almost degenerate 4^+ and 4^- (parity doublet) states. Such a $\mathcal{D}_{3h}$ symmetry was observed in triatomic molecules, and it is observed here for the first time in nuclear physics. We discuss a classification of other rotation-vibration bands in ^{12}C such as the (0^+) Hoyle band and the (1^-) bending mode band and suggest measurements in search of the predicted ("missing") states that may shed new light on clustering in ^{12}C and light nuclei. In particular, the observation (or nonobservation) of the predicted ("missing") states in the Hoyle band will allow us to conclude the geometrical arrangement of the three alpha particles composing the Hoyle state at 7.654 MeV in ^{12}C.

Geometrical equilateral triangular configurations [1,2] have been identified in the triatomic H_3^+ molecule [3] where the predicted spectrum of a triangular oblate spinning top with a $\mathcal{D}_{3h}$ symmetry was observed [1–3]. It was suggested [1,2] that the three alpha particle system of ^{12}C should lead to similar "triatomic like" structure in nuclei. The application to ^{12}C of the $\mathcal{D}_{3h}$ symmetry, a mathematical tool that was developed to describe molecular structure, emphasizes the role of symmetry across very different energy scales, and it leads to a model of ^{12}C that correctly predicts several new observations that we report for the first time in this Letter. Such a polyatomiclike description of light nuclei should lead to a better understanding of the clustering phenomena in light nuclei.

In this Letter, we demonstrate that this $U(7)$ model [1,2] as applied to ^{12}C predicts all known low spin (cluster) states below 15 MeV, as well as the measured $B(E\lambda)$ and form factors measured in electron scattering [4]. But perhaps more importantly, the model predicts new ("missing") states, the observation (or nonobservation) of which will allow us to resolve a major problem of current concern on the geometrical arrangement of the three alpha particles in the Hoyle state at 7.654 MeV. We demonstrate that the observed rotation-vibration spectrum of ^{12}C in of itself already indicates the geometrical structure of ^{12}C.

We report the observation of a new $J^{\pi} = 5^-$ state that fits very well the rotational $J(J + 1)$ trajectory of the ground state band of ^{12}C as predicted by the $U(7)$ model. In addition, the 4^- state recently observed by some of us at 13.35 MeV in ^{12}C [5,6] confirms the $J^{\pi} = 4^{\pm}$ parity doublet predicted by this $U(7)$ model for the ground state band of ^{12}C. The ground state rotational band including the

states of $J^{\pi} = 0^+, 2^+, 3^-, 4^{\pm}$, and 5^- is a strong signature of a $\mathcal{D}_{3h}$ symmetry, and it is observed here for the first time in a nucleus.

The triatomic $U(7)$ mixed-parity structure observed in ^{12}C resembles the diatomic $U(4)$ mixed parity structure [7] observed in ^{18}O [8] and the tetrahedral symmetry (T_d) mixed parity structure recently observed in ^{16}O [9].

The structure of ^{12}C has recently attracted much theoretical attention due to the availability of *ab initio* no-core shell model calculations [10], the no-core symplectic model [11], and effective field theory calculations on the lattice [12]. These calculations attempt to provide a microscopic description of cluster states that are well described in the traditional clustering model [13] and antisymmetrized molecular dynamics [14], as well as in the more modern fermionic molecular dynamics model [15] and more exotic cluster models [16]. However, thus far, *ab initio* shell model calculations have failed to predict [10] the Hoyle state at 7.654 MeV in ^{12}C that is known to be one of the best examples of alpha clustering in nuclei.

The effective field theory lattice calculations [12] and the fermionic molecular dynamics model [15] predict an equilateral arrangement of the three alpha particles in the ground state of ^{12}C and hence, they provide the microscopic foundation of the conjectured $\mathcal{D}_{3h}$ symmetry of the ground state of ^{12}C. But these models are currently unable to predict the high spin 5^- state reported here or the $4^{\pm}$ parity doublet that we observe in ^{12}C.

The identification of the rotational excitations of the Hoyle state with 2^+ [17] and 4^+ [18] raises an intriguing question of current concern regarding the geometrical structure of the Hoyle state, whether it is a bent-arm

configuration [12] or rather an equilateral triangular con-
figuration just as the ground state. In this Letter, we point
out that future measurements of predicted ("missing")
rotation-vibration states in ^{12}C will allow us to understand
the geometrical arrangement of the three alpha particles in
the Hoyle state—a problem which is as old as the discovery
of the Hoyle state itself.

The present measurements were performed at the
Birmingham MC40 cyclotron facility. A beam of ^{4}He
nuclei at an energy of 40 MeV was incident on a
100 μg/cm^2 carbon target. The reaction of interest was
^{12}C(^{4}He, 3α)^{4}He, in which the ^{12}C nuclei were excited
above the α-decay threshold through the inelastic scattering
process. An array of four, 500 μm thick, silicon strip
detectors was used in order to detect three of the four
final state α particles. Each detector had a surface of
5×5 cm^2 subdivided into 16 horizontal and 16 vertical
strips, front and back, respectively. The detectors were
placed at distances 13.0, 11.0, 11.0, 13.0 cm from the target
at angles 62.0°, 32.0°, −32.0°, −62.0°, respectively (the
signs indicate opposing sides of the beam axis). The array
covered an angular range of $\theta_{\text{lab}} = 20°$ to 75°. The
detection system was calibrated with a triple α-particle
source. These detectors allowed the energy and emission
angle of each particle to be determined and hence, the
momentum, assuming each to be an α particle.

The 4th undetected α-particle's properties were recon-
structed using conservation of energy and momentum.
Events in which any two of the three detected α particles
resulted from the decay of ^{8}Be$_{gs}$ were selected. In order to
reconstruct the origins of the final state particles, a Dalitz
plot was created as shown in Fig. 1. Here the excitation
energy in ^{12}C was calculated by reconstructing $E_x(^{12}$C)
from the ^{8}Be and either the detected (horizontal axis) or
undetected (vertical axis) α particle. The horizontal and
vertical loci correspond to ^{12}C excited states and the weak
diagonal loci of states in ^{8}Be.

The projection of the Dalitz plot onto the vertical axis is
shown in Fig. 2. States at 7.654 (0$^+$), 9.641 (3$^-$), 10.844
(1$^-$), 14.083 MeV (4$^+$) are observed. In Figs. 1 and 2, it is
also possible to observe a peak which would correspond to
a state at 22.4(2) MeV which has not been previously
reported.

In order to achieve a characterization into the nature of
the proposed new state in ^{12}C, we have used the angular
correlations technique, a method in which the distribution
pattern of the products are analyzed [19]. This method
yields a model independent spin determination when the
initial and final state particles are all spin zero, and is
described in more detail in [18,19]. Using the Dalitz plot in
Fig. 1, it is possible to set a window around the specific data
of interest (shown by the tilted rectangular box) and then
generate the angular correlation plot for the selected events
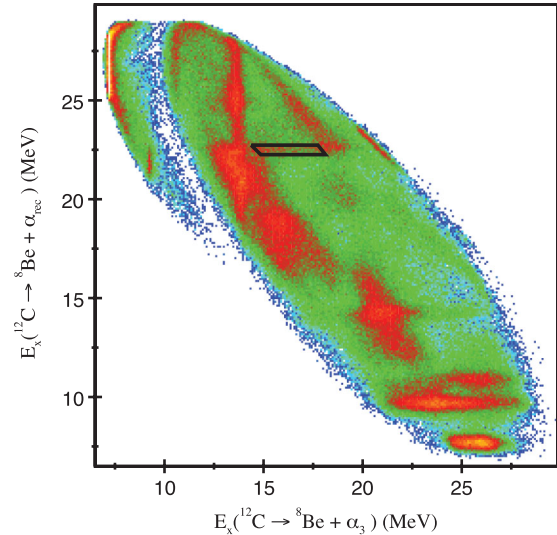(similar to those in Ref. [18]) for the proposed
22.4 MeV state.

FIG. 1 (color online).   Dalitz plot for the ^{12}C(^{4}He, ^{12}C*)^{4}He
reaction. Excitation energies of ^{12}C are plotted on the horizontal
axis in which three α particles were detected. On the vertical axis,
the ^{12}C excitation energies are calculated by the reconstruction of
the undetected α particle and the ^{8}Be nuclei. The tilted rectan-
gular box shows the region selected for the angular distribution
analysis.

In the ^{12}C(^{4}He, ^{12}C*)^{4}He reaction, there are two center-
of-mass frames, the first corresponding to the inelastic
excitation, the second to the decay of ^{12}C into ^{8}Be + α. The
emission angle of the ^{12}C decay process with respect to the
beam axis is described by the angle ψ which is explicitly in
the center of mass of the $\alpha + {}^8$Be system, hence, arising
from a state in ^{12}C and not in ^{16}O. For a ^{12}C state of spin J,
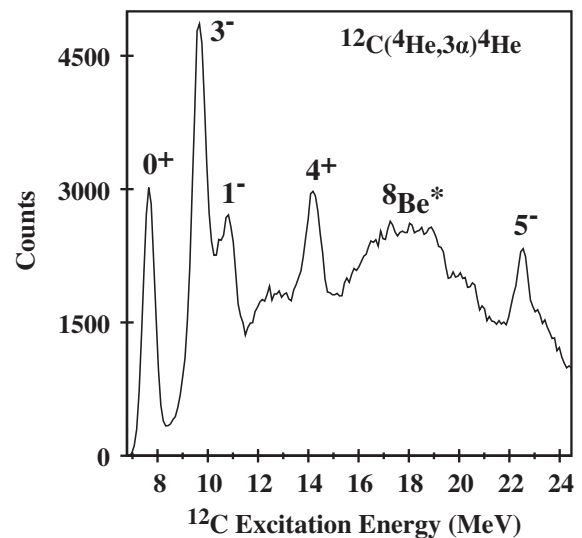

FIG. 2.   Projection of the Dalitz plot in Fig. 1 onto the vertical
axis. In addition to the known states, a peak is observed at 22.4
(2) MeV. The broad background (between 12 and 20 MeV) is due
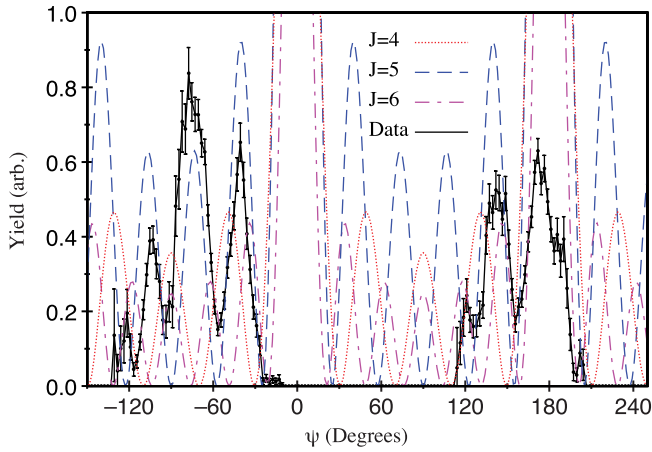to "leaking" of excited states of ^{8}Be into the projected region.

FIG. 3 (color online). The projection onto the ψ axis of the angular correlations for the 22.4 MeV state. The data points are corrected for the acceptance of the detectors and connected with a (continuous black) line to guide the eye. They are compared with the Legendre polynomials $|P_5[\cos(\psi)]|^2$ (dashed blue line) as well as for $\ell = 4$ (dotted red line) and $\ell = 6$ (dotted-dashed red line). Note that due to the unknown m-substate population of the $J^\pi = 5^-$ state, the height of the oscillations cannot be predicted, but the oscillatory phase determines the angular correlation to arise from a $J^\pi = 5^-$ state.

it would be expected that the angular correlation should oscillate with a period given by $|P_J[\cos(\psi)]|^2$. As described in Refs. [18,19], it is possible to infer from the oscillation pattern of the data, the spin of the excited state. The dependence of the yield on the angle ψ is shown in Fig. 3, in which the data are compared with several Legendre polynomials. The measured alpha spectrum and angular correlation clearly point to the existence of a state at 22.4 (2) MeV with $J^\pi = 5^-$.

In Fig. 4, we show the rotational band structure in ^{12}C. The ground state rotational band consisting of the levels 0^+, 2^+, 3^-, $4^\pm$, and the newly measured 5^- state, follow a $J(J+1)$ trajectory. Also, the recently identified rotational
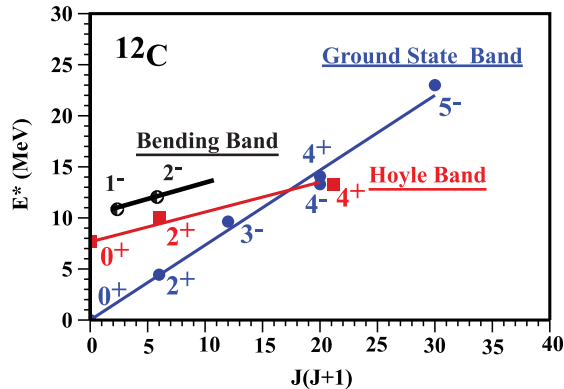

FIG. 4 (color online). Rotational band structure of the ground-state band, the Hoyle band, and the bending vibration in ^{12}C.

excitations with 2^+ [17] and 4^+ [18] of the Hoyle state form a $J(J+1)$ sequence, albeit with a larger moment of inertia. Finally, as we discuss below, the negative parity states 1^- and 2^- shown in Fig. 4 are assigned as members of the bending vibration with almost the same moment of inertia as the Hoyle band.

We present an analysis of the cluster states in ^{12}C in terms of oblate symmetric top which is a special case of the algebraic cluster model [1,2]. In this approach, the three alpha particles are located at the corners of an equilateral triangle. Their relative motion is described by two perpendicular Jacobi vectors, $\vec{\rho}$ and $\vec{\lambda}$, one vector connecting two points on the triangle and the second one along the half angle perpendicular to it. The corresponding algebraic model describing such a system is based on the $U(6+1) = U(7)$ spectrum-generating algebra [1,2].

Of particular interest is the oblate symmetric top limit which corresponds to the geometric configuration of three α particles located at the vertices of an equilateral triangle. The rotation-vibration wave functions of a triangular configuration can be written as [1,2]

$$|N, (v_1, v_2^{\ell_2}), K, L^P\rangle. \tag{1}$$

Here, N is the total number of bosons. The energy spectrum consists of a series of rotational bands labeled by $(v_1, v_2^{\ell_2})$. Here, v_1 corresponds to the breathing vibration with A symmetry and v_2 to the doubly degenerate bending vibration with E symmetry; ℓ_2 denotes the vibrational angular momentum of the doubly degenerate vibration, L the angular momentum, K its projection on the symmetry axis, and P the parity. Since we do not consider the excitation of the α particles, the wave functions describing the relative motion have to be symmetric, i.e., $|K \mp 2\ell_2| = 3m$ a multiple of 3 [1,2]. This imposes some conditions on the allowed values of the angular momenta and parity. For vibrational bands with $(v_1, 0^0)$, the allowed values of the angular momenta and parity are $L^P = 0^+, 2^+, 4^+, \ldots,$ with $K = 0$ and $L^P = 3^-, 4^-, 5^-, \ldots,$ with $K = 3$. The threefold symmetry excludes states with $K = 1$ and $K = 2$ and leads to the lowest predicted $L^P = 4^\pm$ parity doublet in the $(v_1, 0^0)$ vibrational band. The predicted $L^P = 4^\pm$ parity doublet both in the ground band and the Hoyle band is a strong signature of this model. For the bending vibration with $(0, 1^1)$, the rotational sequence is given by $L^P = 1^-, 2^-, 3^-, 4^-, \ldots,$ with $K = 1$, $L^P = 2^+, 3^+, 4^+, \ldots,$ with $K = 2$ and $L^P = 4^+, \ldots,$ with $K = 4$. The degeneracy of the states with the same value of the angular momentum L but different value of K is split by the κ_2 term in Eq. (2) [2]. Since in the application to the cluster states of ^{12}C, the vibrational and rotational energies are of the same order, we expect sizeable rotation-vibration couplings.

In the $U(7)$ algebraic cluster model, the energy eigenvalues of the oblate top, up to terms quadratic in the rotation-vibration interaction, are given by:

$$E = E_0 + \omega_1\left(v_1 + \frac{1}{2}\right)\left(1 - \frac{v_1 + 1/2}{N}\right)$$
$$+ \omega_2(v_2 + 1)\left(1 - \frac{v_2 + 1}{N + 1/2}\right)$$
$$+ \kappa_1 L(L+1) + \kappa_2(K \mp 2\ell_2)^2$$
$$+ \left[\lambda_1\left(v_1 + \frac{1}{2}\right) + \lambda_2(v_2 + 1)\right]L(L+1). \quad (2)$$

This formula includes both the anharmonicities which depend on N and the vibrational dependence of the moments of inertia. In Fig. 5, we show a comparison of the cluster states of ^{12}C with the spectrum of the oblate top according to the approximate energy formula of Eq. (2). The coefficients κ_1, λ_1, and λ_2 are determined by the moments of inertia of the ground state band, the Hoyle band, and the bending vibration. The value of κ_2 term is determined from the relative energies of the positive and negative parity states in the ground state band. The vibrational energies ω_1 and ω_2 are obtained from the excitation energies of the 0^+ Hoyle state and the 1^- state, respectively. Whereas in molecules, the anharmonicities are small and hence N is large, in ^{12}C the situation is completely different. The rotation-vibration couplings and anharmonicities are large and, therefore, N is small. Here, it is taken to be $N = 10$ [1,2]. The large anharmonicities lead to an increase of the rms radius of the vibrational excitations relative to that of the ground state.

In this analysis, the ground state rotational band of ^{12}C is composed of both positive parity states with $J^\pi = 0^+$, 2^+, 4^+ with $K = 0$ and negative parity states with $J^\pi = 3^-$, 4^-, 5^- with $K = 3$. Since these states belong to a single rotational structure, the electromagnetic transition
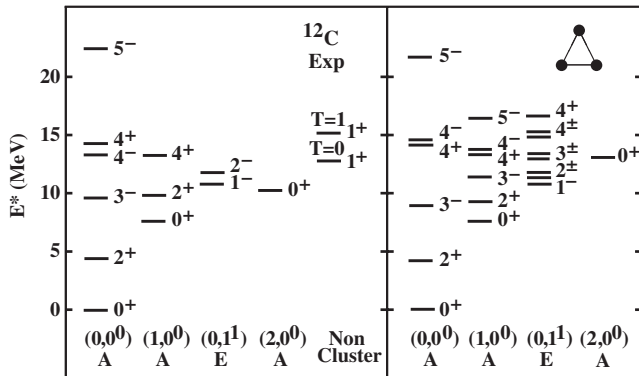

FIG. 5.   Comparison between the low-lying experimental spectrum of ^{12}C and the energies of the oblate symmetric top calculated using Eq. (2) with parameters that are discussed in the text. The levels are organized in columns corresponding to the ground state band and the vibrational bands with A and E symmetry of an oblate top with triangular symmetry. The last column on the left-hand side, shows the lowest observed non-cluster (1^+) levels.

probabilities from the ground state to the 2^+ and 3^- (and 4^+) states are related. The measured strong electromagnetic transitions $B(E2; 2^+ \to 0^+) = 4.9 \pm 0.3$ W.U. and $B(E3; 3^- \to 0^+) = 12.9 \pm 1.7$ W.U. indicate collectivity. The agreement with the predicted $B(E\lambda)$ values for the 2^+ and 3^- states of the ground state rotational band [2,4] and the agreement of the predicted form factors measured in $^{12}C(e, e')$ scattering for $2^+, 3^-$, and the 4^+ state [2,4] indicates that the $K = 0$ and $K = 3$ bands which are usually considered as separate bands, coalesce to form a single ground state rotational structure. As we discussed above, the merging of the $K = 0$ and $K = 3$ bands leads to the predicted $J^\pi = 4^\pm$ degenerate parity doublet which is a strong signature of the $\mathcal{D}_{3h}$ symmetry.

The 0^+ Hoyle state in ^{12}C at 7.654 MeV is interpreted as the band head of the A symmetric stretching vibration or breathing mode of the triangular configuration with the same geometrical arrangement and rotational structure as for the ground state rotational band, as shown in Fig. 5. The nonharmonicity of the potential discussed above leads to larger rms radii for higher vibrational states; hence, the Hoyle rotational band is predicted to have a moment of inertia larger than the ground state band (by a factor of 2). Recent measurements revealed the existence of the 2^+ [17] and 4^+ [18] members of the Hoyle rotational band which raises the question of the identification of the predicted negative parity states shown in Fig. 5. The 4^- state which is predicted to be nearly degenerate with the 4^+ state, can be measured, for example, in $180°$ electron scattering off ^{12}C [20]. We note that a (broad) 3^- state was suggested to lie between 11 and 14 MeV [5] which is close to the predicted energy shown in Fig. 5. In order to distinguish between different geometric configurations of the Hoyle band, e.g., equilateral triangular or bent arm, the identification of the negative parity states 3^- and 4^- is crucial which is a strong motivation for a dedicated experimental search [20].

The 1^- state at 10.84 MeV is assigned as the bandhead of the vibrational bending mode whose lowest-lying rotational excitations consist of nearly degenerate parity doublets of $2^\pm$ and $3^\pm$ states. So far, only the 2^- has been identified.

In addition to the cluster states, there are other (non-cluster) states in ^{12}C. In particular, with 3α configurations, no 1^+ can be formed. The two 1^+ ($T = 0$) and 1^+ ($T = 1$) states at 12.71 MeV and 15.11 MeV, respectively, shown in Fig. 5, are therefore clearly noncluster states and indicate the energy above which the identification of cluster or noncluster low spin states becomes difficult. The cluster states are characterized by large alpha widths ($\Gamma_{\alpha 0}$, the decay to the ground state of ^8Be or $\Gamma_{\alpha 1}$, the decay to the first excited 2^+ state of ^8Be) with reduced widths that exhaust a large fraction of the Wigner limit.

Of particular interest [21] is the Hoyle state with a geometrical arrangement of the alpha particles that may be deduced from the rotational band built on top of the Hoyle

state [17,18,21]. While the observed moment of inertia of the Hoyle band excludes the proposed linear chain structure of the Hoyle state [17], two geometrical alternatives of either equilateral triangular arrangements or obtuse triangular arrangement are considered for the arrangement of the three alpha particles in the Hoyle state of ^{12}C and can be resolved by the future measurements we propose here.

In conclusion, we presented evidence for triangular $\mathcal{D}_{3h}$ symmetry in the arrangement of the three alpha particle in the ground state of ^{12}C. Such a symmetry is now established in molecular physics and nuclear physics. Another interesting application would be to odd-mass nuclei. Finally, the algebraic cluster model predicts several additional states. The predicted broad and overlapping states require accurate data as measured for the second 2^+ Hoyle state [17]. The selectivity of gamma-ray beams [17] as well as electron beams [20] would aid in populating the states of interest and resolve the broad interfering states. These new capabilities should initiate an extensive experimental program for the search of the predicted ("missing") states and promises to shed new light on the clustering phenomena in light nuclei.

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